Inflation and the Forward Premium Anomaly

(Job Market Paper)

James R. Young*
Department of Economics
University of Notre Dame
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Abstract

This paper studies the cause of the forward premium anomaly. I first document a new empirical fact, that the excess currency returns associated with deviations from uncovered interest parity are positively correlated with the lagged inflation differential. I next develop a dynamic stochastic general equilibrium model with nominal rigidities, incomplete international markets, and monetary policy in the form of a Taylor rule to explain this fact and its role in generating the forward premium anomaly. The model generates positive excess currency returns for the home country in periods of relatively high inflation. The anomaly arises, because transient foreign technology shocks cause both the domestic-foreign inflation differential and the real interest rate differential to increase while the real domestic exchange rate immediately depreciates. Over time, because of sticky prices, the real exchange rate appreciates to restore purchasing power parity. This expected appreciation of the real exchange rate coupled with a positive interest rate differential is the forward premium anomaly. Simulations of the model are able to qualitatively reproduce the forward premium anomaly and can explain thirty percent of the volatility of the excess currency returns.

*Correspondence address: Department of Economics, University of Notre Dame, 4th Floor Flanner Hall, Notre Dame, IN 46556. E-mail: jyoung9@nd.edu. I would like to thank my advisors Nelson Mark, Robert Flood, and Thomas Cosimano as well as the participants of the Notre Dame Macroeconomics Seminar for many helpful comments. All remaining errors are my own.

1 Introduction

This paper studies the cause of the forward premium anomaly. The anomaly refers to the violation of uncovered interest parity (UIP) in the form of a negative slope estimate from the regression of the future percent change in the spot rate on the interest rate differential. The result is anomalous since UIP predicts the slope estimate is one[1]. I arrive at the cause of the forward premium anomaly by way of a new empirical regularity, that excess currency returns associated with deviations from UIP are positively correlated with the lagged domestic-foreign inflation differential between countries. To understand this new fact and its connection to the forward premium anomaly, I study a two country dynamic stochastic general equilibrium model (DSGE) with nominal rigidities. I find that the model is capable of reproducing this empirical regularity and that it can provide a resolution to the forward premium anomaly.

While many explanations have been proposed, there is no consensus on the cause of the forward premium anomaly in the literature.[2] The debate in the literature over the correct theoretical model can partially be explained by the lack of a strong empirical co-variate with the excess currency return. For example Burnside, Eichenbaum, Kleshchelski, and Rebelo (2006) conduct an extensive empirical analysis including evaluating such factors as consumption growth, industrial production and stock market returns, but they do not find a factor that is both statistically significant and systematically related to the excess return. However, one variable they did not study is relative inflation. In contrast this paper presents a significant positive relationship between lagged inflation and the excess return, which is reproduced in a DSGE model.

The model features incomplete international risk sharing, nominal rigidities, and a Taylor rule type monetary policy. Incomplete international risk sharing allows for larger violations of UIP when agents have standard preferences with constant relative risk aversion than is obtainable under complete markets.[3]Sticky prices are useful in generating persistent deviations in the real

[1]For a survey of the forward premium anomaly see Lewis (1995)

[2]Examples of current competing theoretical explanations include habits, Verdelhan (2010) and rare disasters, Farhi and Gabaix (2007).

[3]Using a similar framework, Benigno (2004) demonstrates that UIP will be satisfied when international markets are complete. However, in general UIP can be violated even in a world with complete international markets. To get quantitatively large deviations it takes specialized preferences for example either of Campbell and Cochrane (1999) or Epstein and Zin (1989) form. For examples of the use of each class of preferences concerning violations of UIP see Verdelhan (2010) and Backus, Gavazzoni, Telmer and Zin (2010) respectively.

exchange rate from purchasing power parity (PPP).[4] This persistent deviation corresponds to, on average, an appreciating real exchange rate that is coupled with a positive real interest rate differential. These comovements are consistent with a common phrasing of the forward premium anomaly: the relatively high interest rate country's currency appreciates on average.

To summarize my main findings, the excess return in the model arises as compensation for risk. Within the model it is risky to borrow money domestically and lend abroad, since the payoff to such a strategy is positively correlated with domestic consumption. In addition, the model reproduces two important stylized facts. First, multiple simulations of the model over a time frame equivalent to available data produce a distribution of negative slope coefficient estimates, from the change in the real exchange rate regressed on the real interest rate differential, that are consistent with empirical observations for multiple country pairs. Second, the model is able to produce a distribution of positive coefficients from the excess return regressed on lagged inflation, which are consistent with the data. The relationship between inflation and the excess return is first documented empirically in this paper. Finally, the model is able to explain thirty percent of the volatility of the excess return.

The literature on the forward premium anomaly goes back several decades. For a survey of classic works on the topic see Lewis (1995). Recently, there has been renewed interest in modeling the forward premium anomaly. For example, Verdelhan (2010) proposes habit based preferences as an explanation while Farhi and Gabaix (2007) suggest rare disasters. More closely related to this paper are Hollifield and Yaron (2001) and Backus, Gavazzoni, Telmer and Zin (2010). Hollifield and Yaron explore real and nominal factors as explanations of the excess currency return and conclude that nominal factors do not play a large role, which contrasts with the results contained in this paper. Backus et al explore the role of monetary policy, specifically Taylor rules, as a mechanism driving the forward premium anomaly. A major difference between their model and the one contained in this paper is that their model does not allow for nominal rigidities, so that consumption can influence inflation, but inflation cannot impact consumption. They conclude that asymmetric Taylor rules where only one country responds to exchange rate variation can reproduce the anomaly. In contrast, this paper can reproduce the anomaly with symmetric Taylor rules where neither country responds to exchange rate variation.

[4]For example see Benigno (2004)

The remainder of the paper is as follows: The next section presents empirical regularities. The third section introduces a theoretical model of the currency risk premium, while the fourth section provides results from simulations of that model and the fifth section examines the nature of currency risk in the model. Finally, the sixth section provides concluding remarks.

2 Motivation

The purpose of this section is threefold. First, it presents the forward premium anomaly and its connection to violations of UIP. Second, it argues and provides empirical support that inflation is a factor that gives rise to violations of UIP and the forward premium anomaly. Finally, it provides motivation for exploring the anomaly in real terms.

A common regression test of UIP is ,

$$s_{t+1} - s_t = \alpha_1 + \beta_1 \left(i_t - i_t^*\right) + \epsilon_{t+1} \tag{1}$$

where s_t is the log spot rate in units of home currency per foreign currency, i_t is the domestic interest rate, i_t^* is the foreign interest rate,and ϵ_{t+1} is the error term. This equation is equivalent, by covered interest parity, to testing whether the forward rate is an unbiased predictor of the future spot rate. If investors are risk neutral with rational expectations, then the forward rate should be an unbiased predictor of the future spot rate and UIP should hold, or in other words, the null hypothesis is that $\beta_1 = 1$. However, many authors, beginning with Bilson (1981) and Fama (1984), and most recently Verdelhan (2010), have not only rejected the null hypothesis, but have found point estimates that are negative and statistically different from zero. The forward rate does in fact have predictive power of the future spot rate, but it predicts the opposite of what UIP suggests.

This paper presents evidence that relative inflation is an important factor in the violation of UIP. To develop this claim, note that the deviation from uncovered interest parity is an excess currency return. Equation (2) defines the forward rate as comprised of the expected log spot rate next period and an excess return, τ_t that is allowed to vary over time.

$$f_t = E_t(s_{t+1}) + \tau_t \tag{2}$$

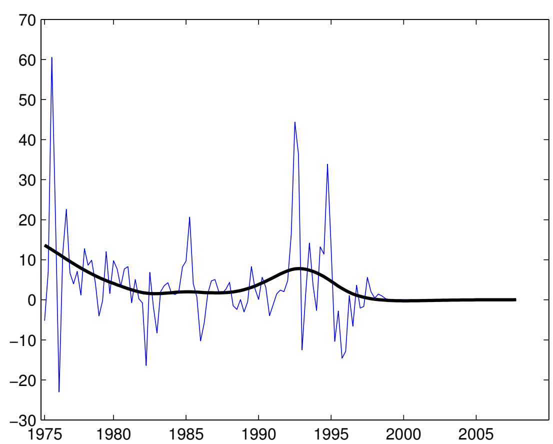
where f_t is the log forward rate

Next, covered interest parity can be applied and agents are assumed to have rational expectations, so that the expected spot is equal to the realized spot next period plus a rational expectations error. The result in equation (3) can be thought of as an expost currency return, τ_t^{EX}, which includes the excess return in equation (2) and the rational expectations error.

$$\tau_t^{EX} = i_t - i_t^* - (s_{t+1} - s_t) \tag{3}$$

Figures 1 through 6 presents motivation for the expost return relating to relative inflation. Specifically, figure 1 plots τ_t^{EX} and its HP filtered trend for Italy relative to France from 1975 onwards.[5]

Figure 1: τ_t^{EX} for Italy relative to France (in percent per annum)



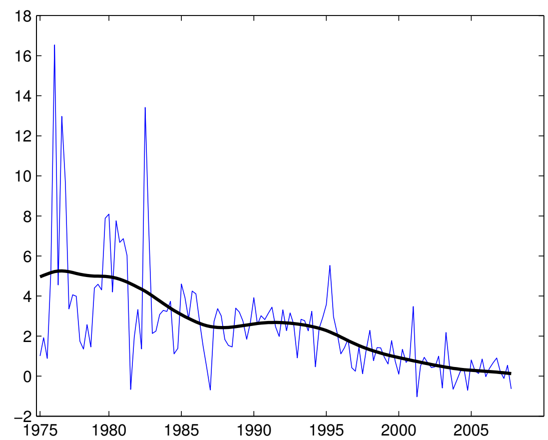
The series has a general downward trend, except for an increase in the mid nineties, as it approaches the introduction of Euro on January 1, 1999.[6] This decrease is more consistent with the excess return arising as a risk premium

[5]The spot rate data is a combination of end of month quotes from the Harris data-set between 1980 to 1993 and Datastream for after 1993. Libor rates from Datastream and one month Eurorates from the Harris data set are used for the interest rates. For all countries except Netherlands the price data, in the form of consumer price indexes, comes from the International Monetary Fund's IFS database. Netherlands' CPI contained a large discontinuity spanning several years in the IFS series, so data from the Organization for Economic Cooperation and Development's SourceOECD database is used.

[6]The general downward trend holds for other country pairs that adopted the Euro.

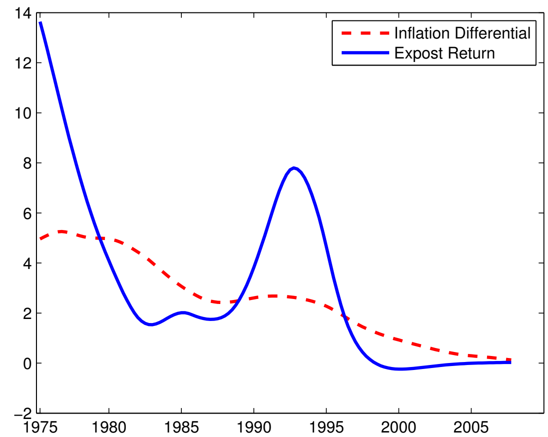
from a nominal risk factor such as inflation, than a real risk factor which would not subside just because the two countries were planning to adopt a common currency. In fact, the inflation differential, displayed in Figure 2, also has a general downward trend except for a slight increase in the nineties

Figure 2: Inflation Differential for Italy relative to France (in percent per annum)



Finally, Figure 3 plots the HP trends from both τ_t^{EX} and the inflation differential. The curvature in the trends are in fact contemporaneous, though the excess return has larger swings.

Figure 3: τ_t^{EX} and Inflation Differential Trends for Italy relative to France (in percent per annum)

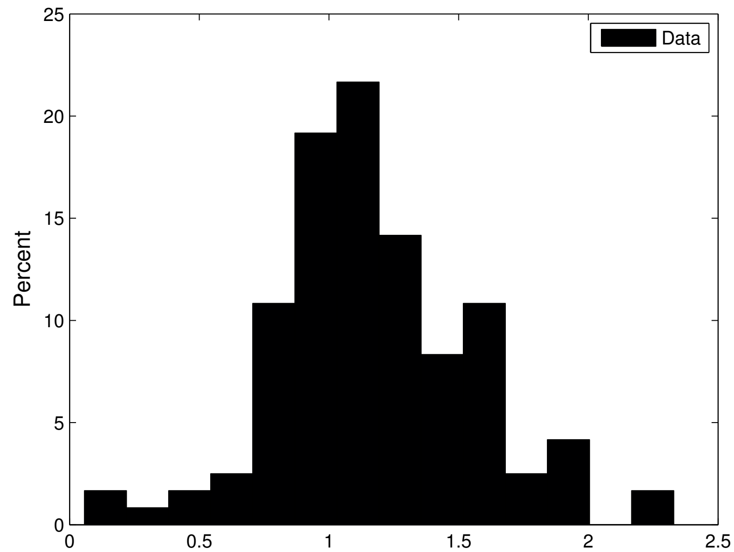


To investigate the effect of inflation further, the expost excess return is regressed on relative inflation between two countries,

$$\tau_t^{EX} = \alpha + \beta_2(\pi_t - \pi_t^*) + \epsilon_{t+1} \tag{4}$$

The results of this regression are contained in Figure 2 for 120 pairwise combinations of a sample of 16 countries that have data available over the last 20 to 35 years. The estimates of β_2 fall around one and many of the estimates are statistically significant using Newey West standard errors. This is a surprising fact since the empirical literature has had a hard time finding a significant co-variate with the excess return, for example see Burnside, Eichenbaum, Kleshckeski and Rebelo (2006).

Figure 4: Histogram of Estimates of β_2 from the Data



Since inflation is a relevant factor, it is proper to look at the anomaly in real terms. To motivate this claim, the expost inflation differential and the expected inflation differential are added and subtracted from the specification in equation (1),

$$\Delta s_{t+1} - \tilde{\pi}_{t+1} = \alpha_1 + \beta_1 \tilde{i_t} - \tilde{\pi}_{t+1} - \beta_1 E_t \tilde{\pi}_{t+1} + \beta_1 E_t \tilde{\pi}_{t+1} + \epsilon_{t+1} \qquad (5)$$

where variables with tildes represent domestic-foreign differentials. Rearranging this specification results in the following,

$$\Delta q_{t+1} = \alpha_1 + \beta_1 \tilde{r}_t - (\tilde{\pi}_{t+1} - \beta_1 E_t \tilde{\pi}_{t+1}) + e_t \qquad (6)$$

where Δq_{t+1} is the change in the real exchange rate and $\tilde{r}_t$ is the real interest rate differential. If the change in the real exchange rate is regressed on the real interest rate differential alone, then the regression coefficient could be biased if the term $(\tilde{\pi}_{t+1} - \beta_1 E_t \tilde{\pi}_{t+1})$ is correlated with the real interest rate differential. To determine if such a bias occurs, a similar specification is examined,

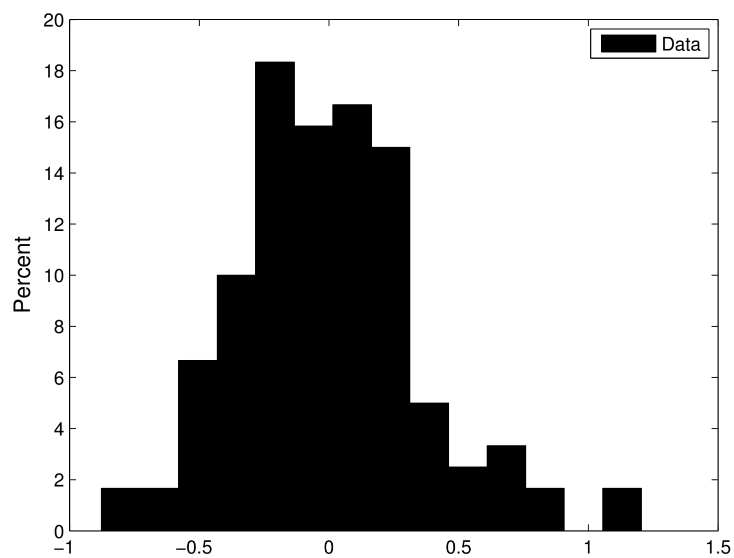
$$\Delta q_{t+1} = \alpha + \beta_3 \left(r_t^{EX} - r_t^{EX*} \right) + \nu_{t+1} \tag{7}$$

where r_t^{EX} is the expost realization of the real interest rate, i.e. $i_t - \pi_{t+1}$, and ν_{t+1} is a composite error term including a inflation forecast error orthogonal to $r_t^{EX} - r_t^{EX*}$and an iid error,

$$\nu_{t+1} = \beta_3 \left(\tilde{\pi}_{t+1} - E_t \tilde{\pi}_{t+1} \right) + \epsilon_{t+1} \tag{8}$$

Figure 3 displays the distribution of the estimated β_3. Roughly half of the country pairs exhibit the anomaly, once again a negative coefficient estimate, and the mean of the distribution is slightly negative.

Figure 5: Histogram of Estimates of β_3 from the Data



An additional interpretation of a negative estimate of β_3 can be gained by first defining the log spot rate as the relative log price levels between countries and the real exchange rate, which is equivalent to a deviation from PPP.

$$s_t = p_t - p_t^* + q_t \tag{9}$$

In light of the fact that the estimate of β_3 is negative, an interpretation that can be gained is that a country that has a relatively high real rate and positive deviation from PPP should expect that the deviation from PPP should decrease over time. In other words, if a country is observed with a positive premium, then it is because both the real interest rate differential is positive and deviations from PPP decreases over time. The central question of the anomaly in these terms is as follows: why is a positive real interest rate accompanied by decreasing deviations in PPP?

This paper argues that a shock to an economy that increases relative inflation between the current and prior periods leads to a higher relative real interest rate and an on average decrease in deviations from PPP which comprises the anomaly. Below follows a model that will not only reproduce the forward premium anomaly, but also the positive correlation between the excess returns and inflation.

3 Model

The model featured here is similar to Benigno (2004). It contains two countries with home agents indexed by h on the interval $[0, n]$ and foreign agents indexed by f on the interval $(n, 1]$. Each agent maximizes utility based on consumption produced both domestically and abroad, and manages a monopolistically competitive firm.

3.1 Consumer's Problem

A home agent maximizes expected lifetime utility, $U(\cdot)$, of the following form:

$$U(C_t, L_t) = E_t\left[\sum_{t=0}^{\infty} \beta^t \left(u(C_t) - v(L_t)\right)\right] \tag{10}$$

where β is the discount factor, E_t is the expectations operator, C is consumption, and L is the labor supplied. Typical assumptions for $u(\cdot)$ and $v(\cdot)$ apply: $u'(\cdot) > 0$, $u''(\cdot) < 0$, $v'(\cdot) > 0$ and $v''(\cdot) > 0$. Foreign agents maximize a utility of the same functional form.

Consumption C is a composite good of home and foreign goods with a index of

$$C = \left[(\alpha n)^{1/\mu} C_H^{(\mu-1)/\mu} + ((2-\alpha)(1-n))^{1/\mu} C_F^{(\mu-1)/\mu}\right]^{\mu/(\mu-1)} \tag{11}$$

where C_H is the home agent's consumption of the domestically produced good, C_F is the home agent's consumption of the foreign produced good, and μ is the elasticity of intertemporal substitution between home and foreign produced goods. The parameter α measures the strength of the home bias present in the model. For $1 < \alpha \leq 2$ the countries will exhibit positive home bias. The price index that corresponds to the consumption index is

$$P = \left[\alpha n P_H^{1-\mu} + (2-\alpha)(1-n) P_F^{1-\mu}\right]^{1/(1-\mu)} \tag{12}$$

With home bias, the domestically produced goods' prices are weighted more heavily in the overall price index than the foreign goods' prices. The home and foreign consumption goods are each composite goods comprised of a set of goods which are produced by monopolistically competitive firms owned by agents indexed h or f in the home or foreign country respectively. Each of these indices have a similar form

$$C_H = \left[\left(\frac{1}{n}\right)^{1/\sigma} \int_0^n c(h)^{(\sigma-1)/\sigma} dh\right]^{\sigma/(\sigma-1)} \tag{13}$$

$$C_F = \left[\left(\frac{1}{1-n}\right)^{1/\sigma} \int_n^1 c(f)^{(\sigma-1)/\sigma} df\right]^{\sigma/(\sigma-1)} \tag{14}$$

where $c(h)$ is a domestically produced variety of good, $c(f)$ is a foreign produced variety of good, and σ is the elasticity of substitution between goods produced within the same country. For each composite good there is a corresponding price index:

$$P_H = \left[\left(\frac{1}{n}\right) \int_0^n p(h)^{1-\sigma} dh\right]^{1/(1-\sigma)} \tag{15}$$

$$P_F = \left[\left(\frac{1}{1-n}\right) \int_n^1 p(f)^{1-\sigma} df\right]^{1/(1-\sigma)} \tag{16}$$

The demand for a specific variety $c(h)$ or $c(f)$ is derived by finding the lowest cost bundle of goods for a given C, which results in

$$c(h) = \left(\frac{p(h)}{P_H}\right)^{-\sigma} \left(\frac{P_H}{P}\right)^{-\mu} aC \tag{17}$$

$$c(f) = \left(\frac{p(f)}{P_F}\right)^{-\sigma} \left(\frac{P_F}{P}\right)^{-\mu} (2-a)C \tag{18}$$

With home bias the home agent will demand more of a good produced in the home country all else equal.

I allow the international asset markets to be incomplete. This fact is made evident in the budget constraint that a typical home agent faces in equation (19). The sources of income the agent has at her disposal at time t are labor income, W_tL_t, where W_t is the wage, profits from her monopolistically competitive firm Π_t, the payoff of a domestic state contingent nominal bond $B_H(s_t)$, and the payoff from a foreign nominal bond $B_{F,t}$ free of default risk converted to home currency at the current nominal exchange rate S_t. The agent spends income on consumption, a portfolio of domestic state contingent nominal bonds at the appropriate price $Q(s_{t+1}|s_t)$, and a foreign bond free of default risk with the rate of return of i_t^* for which she must pay a participation cost $\Gamma(\cdot)$.

$$W_tL_t + \Pi_t + B_H(s_t) + S_tB_{F,t} =$$

$$P_tC_t + \sum_{s_{t+1}} Q(s_{t+1}|s_t)\, B_H(s_{t+1}) + S_t\left(\frac{1}{1+i_t^*}\right) B_{F,t+1} + \Gamma(B_{F,t+1}) \tag{19}$$

The participation cost will prevent the agent from continuously accruing debt, and is necessary to keep the model stationary. The cost takes the following form:

$$\Gamma(B_{F,t+1}) = \left(\frac{\phi}{2}\right)\left(\frac{(S_tB_{F,t+1})^2}{P_tY_t}\right) \tag{20}$$

where ϕ is a constant and Y_t is total output for the domestic country. The ability of domestic agents to trade domestic state contingent claims allow them to insure against profits lost from not being chosen to change prices a la Calvo price setting, which is further discussed below in the section covering the firm's problem. Therefore, ex post home agents will be identical. However, the two countries cannot insure against country specific aggregate shocks using the nominal bond traded between the countries.

The Euler equations for the consumer's problem are for the most part standard. The marginal utility of consumption weighted real wage is equated to the marginal disutility of labor.

$$u'(C_t)\left(\frac{W_t}{P_t}\right) = v'(L_t) \tag{21}$$

Summing over the first order conditions for consumption and the state contingent claim for each state results in the domestic country's risk free rate, i_t, being equated to the intertemporal rate of substitution.

$$\frac{1}{1+i_t} = \frac{P_t}{u'(C_t)} E_t\left[\frac{\beta u'(C_{t+1})}{P_{t+1}}\right] \tag{22}$$

However, the risk free rate in the foreign country is similar, but it takes into account a wedge that occurs from the international bond market participation cost.

$$\frac{1}{1+i_t^*} = E_t\left[\frac{S_{t+1}P_t\beta u'(C_{t+1})}{S_t P_{t+1} u'(C_t)} - \frac{\phi S_t B_{F,t}}{P_t Y_t}\right] \tag{23}$$

The wedge can be thought of as a fraction, ϕ, of the ratio of the international bond measured in the home consumption to total output of the domestic economy.

3.2 Producer's Problem

The production side of the economy is characterized by monopolistically competitive firms in both the domestic and foreign countries. Each agent's firm in the domestic country produces a unique variety of a good according to a linear production function

$$y_t(h) + y_t^*(h) = X_t\left[L_t(h)\right] \tag{24}$$

where X_t is total factor productivity (TFP) which is common to all domestic producers, $y_t(h)$ is the total domestic demand for variety h, and $y_t^*(h)$ is the total foreign demand for variety$h$. Foreign firm owners produce their variety according to a similar production function

$$y_t(f) + y_t^*(f) = X_t^*\left[L_t^*(f)\right] \tag{25}$$

where X_t^* is foreign TFP, $y_t(f)$ is the total domestic demand for variety f, and $y_t^*(f)$ is the total foreign demand for variety f. The firm owners face Calvo pricing. Each period an agent may be chosen to change the price of the good they produce with a probability of $(1-\delta)$. The producer's problem is then to choose a price for each country in its local currency to maximize the discounted sum of profits over time.

$$E_t \sum_{s=0}^{\infty} \delta^s \theta_{t,t+s} \left\{ p_{t+s}(h)\, y_{t+s}(h) + S_{t+s} p^*_{t+s}(h)\, y^*_{t+s}(h) - W_{t+s} L_{t+s} \right\} \qquad (26)$$

where producers discount future profits by the stochastic discount factor:

$$\theta_{t,t+s} = \beta^s \frac{U'(C_{t+s})}{P_{t+s}} \frac{P_t}{U'(C_t)} \qquad (27)$$

In addition firms set the current price based on the likelihood, δ^s, that that price will still be used s periods into the future. The assumption of a linear production function allows the pricing decisions for the domestic and foreign markets to be taken separately. The optimal price for the firm to choose for the domestic market is

$$p_t(h) = \frac{\sigma E_t \sum_{s=0}^{\infty} \delta^s \theta_{t,t+s} \varphi_{t+s} y_{t+s}}{(\sigma-1)\, E_t \sum_{s=0}^{\infty} \delta^s \theta_{t,t+s} y_{t+s}} \qquad (28)$$

where $\varphi_t = W_t/X_t$ is the marginal cost of the firm. In the special case where prices are flexible, i.e. the probability that the current price will be in use tomorrow is $\delta = 0$, the optimal price simplifies to

$$p_t(h) = \frac{\sigma}{(\sigma-1)} \varphi_t \qquad (29)$$

The price set by the monopolistic competitive firms if prices are flexible is the traditional markup over marginal cost.

3.3 Monetary Policy

Each country has a monetary authority that sets policy independently of the actions of the authority in the other country. Each authority employs a Taylor rule, where the Taylor rate $\hat{i}_t$ is based on the steady state interest rate, $\bar{i}$, inflation, π_t, and the output gap compared to the economy with flexible prices, $\tilde{y}_t$.

$$\hat{i}_t = \bar{i} + \omega_1 \pi_t + \omega_2 \tilde{y}_t \qquad (30)$$

The parameters $\omega_1$and ω_2 are the weights the authority puts on inflation and the output gap respectively.

There is no price inflation at the model's steady state, so the authority's inflation target is zero. This is reflected in equation (30) by the lack of an inflation target term. Empirical studies of monetary policy, for example Rudebusch (2002) and English et al (2002), have found interest rate smoothing to be an important factor in explaining the observed monetary policy. To reflect this fact, the monetary authority sets the nominal interest rate as a linear combination between the previous period's interest rate and the Taylor rate:

$$i_t = \omega_3 i_{t-1} + (1 - \omega_3)\,\hat{i}_t + \epsilon_{m,t} \tag{31}$$

where ω_3 is the weight the authority puts on the previous period's interest rate and $\epsilon_{m,t}$ is a shock to monetary policy.

3.4 Steady State, Solution and Calibration

I solve the model using a second order approximation around the steady state. To describe the steady state note that the price index evolves according to

$$P_{H,t}^{1-\sigma} = \left[\left(\frac{1}{n}\right)\int_0^n p_t(z)^{1-\sigma}dz\right] = \delta P_{H,t-1}^{1-\sigma} + (1-\delta)\,p_t(h)^{1-\sigma}$$

This implies that at the steady state the price chosen by firms is the same as the index,

$$P_H = p(h) \tag{32}$$

where the steady state is denoted by the dropping of the time subscripts. Taking advantage of this relationship and that consumption and demand is constant at the steady state, the optimal price a domestic firm chooses for the domestic market simplifies from equation (30) to

$$P_H = \frac{\sigma}{(\sigma-1)}\frac{W}{X} \tag{33}$$

The corresponding price set by the domestic firm in the foreign country is

$$SP_H^* = \frac{\sigma}{(\sigma-1)}\frac{W}{X} \tag{34}$$

There are also corresponding equations for the prices set by the foreign firms.

These equations imply that the nominal exchange rate is

$$S = \frac{P_H}{P_H^*} = \frac{P_F}{P_F^*} \tag{35}$$

Furthermore, the relative prices in the two countries are equal, so that the law of one price holds in the steady state.

$$\frac{P_H}{P_F} = \frac{P_H^*}{P_F^*} \tag{36}$$

Next note that the steady state will be symmetric for equal size countries, $n = 0.5$, which implies that the consumption of the composite good is the same in each country and the consumption of the good produced and consumed locally, i.e. C_H and C_F^*, are the same across countries so that

$$\left(\frac{P_H}{P}\right)^{-\varsigma} \alpha n C = C_H = C_F^* = \left(\frac{P_F^*}{P^*}\right)^{-\varsigma} \alpha n C \tag{37}$$

Which further implies that the ratio of the price level to the price of the locally produced good is the same across the countries.

$$\left[\alpha n + (2-\alpha)(1-n)\left(\frac{P_F}{P_H}\right)^{1-\mu}\right]^{1/(1-\mu)} = \frac{P}{P_H} =$$

$$\frac{P^*}{P_F^*} = \left[(2-\alpha) n \left(\frac{P_H}{P_F}\right)^{1-\mu} + \alpha(1-n)\right]^{1/(1-\mu)} \tag{38}$$

Which implies that the ratio of the price of the home produced good to the foreign produced good equals one, thereby implying that all the ratios of the price level to the sub price indices are equal to one. Therefore, the real exchange rate is

$$S\frac{P^*}{P} = \frac{P_H}{P_H^*}\frac{P^*}{P} = 1 \tag{39}$$

Furthermore using the optimal prices, demand functions and production function to eliminate labor input and the wage in the consumption-labor Euler equations for home and foreign consumption.

$$\left(\frac{\sigma-1}{\sigma}\right) X U'(C) = V'\left(\frac{C}{X}\right) \tag{40}$$

$$\left(\frac{\sigma-1}{\sigma}\right) X^* U'(C^*) = V'\left(\frac{C^*}{X^*}\right) \tag{41}$$

The model is calibrated to a quarterly horizon. A summary of the parameter values chosen to accomplish this are found in Table 1. The probability that a firm does not change its prices next period, α, is set so that on average a firm changes its prices once every five quarters. The discount factor is set to a value to obtain a four percent annual interest rate. The countries are of equal size with n equal to 0.5 and have a moderate amount of home bias with α set to 1.5. Both the consumption and labor portions of the utility function are set to have a moderate amount of curvature, where $\gamma = -CU''(C)/U'(C)$ and $\eta = LV''(L)/V'(L)$. The index parameters μ and σ were set to values consistent with the literature with the value of σ corresponding to a markup of fifteen percent. The Taylor rule values follow the estimates of Rudebusch (2002) and English et al (2002) for non-serial correlated errors. The technological process is assumed to be first order autoregressive with persistence, ρ_X, and volatility, $\sigma_{\epsilon X}$, that follow the literature.

Table 1: Calibration Summary

γ	5		β	0.99		ϕ	0.001		ρ_X	0.94
η	3		δ	0.8		ω_1	1.5		$\sigma_{\epsilon X}$	0.01
μ	1.5		α	1.5		ω_2	0.9			
σ	7.66		n	0.5		ω_3	0.7			

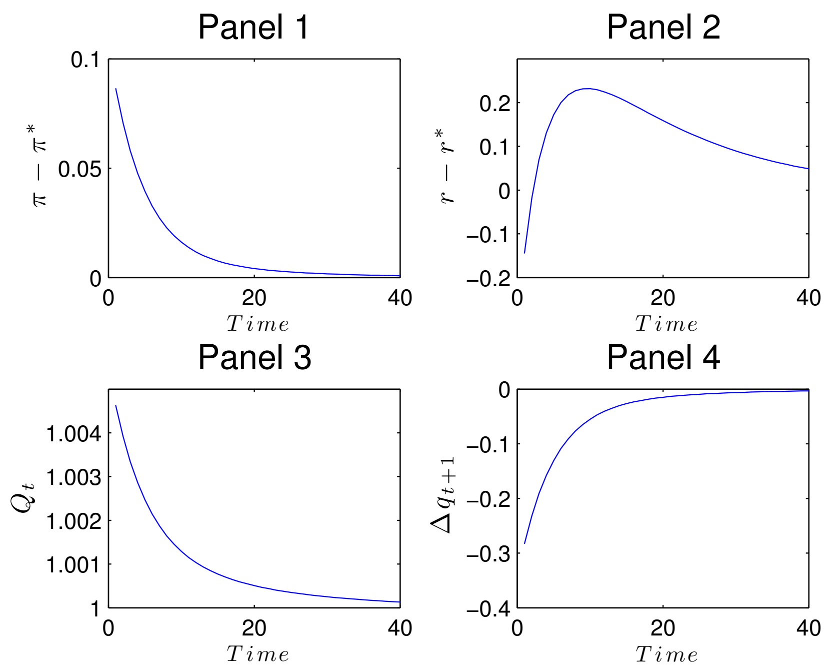
4 Properties of the Model

This section first provides results from simulating the model using two different calibrations: first the model with only technology shocks, and second the model with technology shocks and monetary policy shocks. Impulse responses for relevant variables are displayed for both shocks. Second, The second section evaluates whether the model is able to produce histograms comparable to those contained in figures 2 and 3. The section ends by demonstrating that the excess return generated by the model can be interpreted as a risk premium, and evaluates the size of the risk premium compared to the one seen in the data.

4.1 Impulse Responses and Coefficient Distributions: Technology Shocks

Figure 6 displays the impulse responses for a selection of variables for a positive one standard deviation *foreign* technology shock.

Figure 6: Impulse Response of a One Standard Deviation Foreign Technology Shock



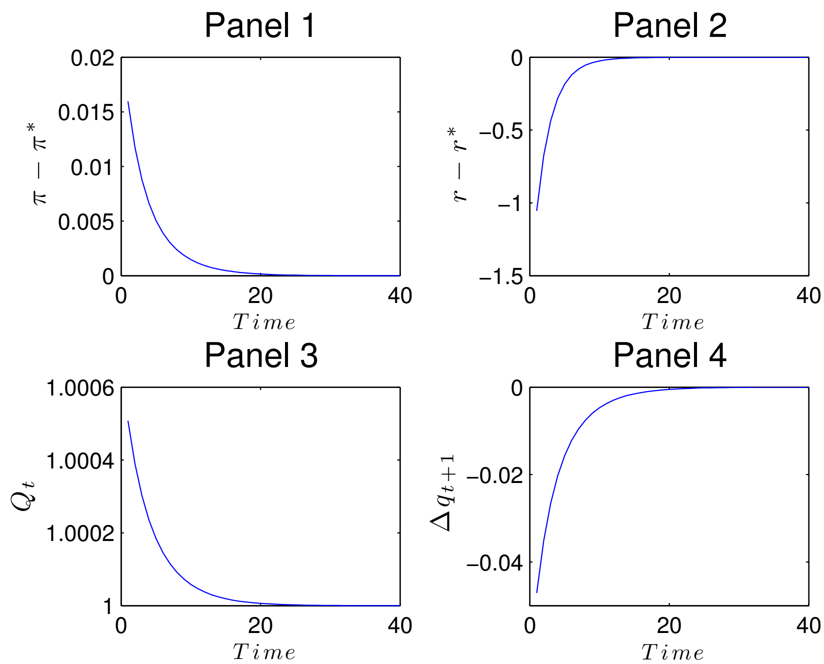
Panel 1 illustrates that the shock increases the inflation rate in the domestic economy relative to the foreign economy. The reason for this is that the positive technology shock abroad decreases the marginal cost of producing the foreign goods, which results in a drop of prices for those that are not rigid. However, since the price of the foreign produced good is more heavily weighted in the price level abroad, the overall price level drops more abroad than at home. With relatively higher inflation in the domestic economy, one would expect the monetary authority at home to set relatively higher interest rates as well. Panel 2 confirms this intuition, the real interest rate differential becomes positive after a few periods and peaks at around the tenth period.[7] This inertia in the real interest rate differential is caused by interest rate smoothing. Finally, Panel 3 displays the level of the real exchange rate. The relative productivity gains abroad has caused the domestic currency to depreciate in real terms, indicated by an increase in the real exchange from its steady state value of one. It then takes approximately 25 quarters of appreciating for the real exchange rate to return to its steady state value and purchasing power parity. Alternatively panel 4 displays the log difference between the real exchange one period into the future and today.

The forward premium is the result of the comovements of the interest differential in Panel 2 and the change of the log real exchange rate in Panel 4. For the first 10 periods the two series are both below their respective mean, but after the interest rate differential peaks the interest rate differential is above its mean, while the real exchange rate is below its mean. This translates into a negative correlation which is consistent with the anomaly. The difference between the series in Panel 2 and Panel 4 is also the excess currency return, which is above its steady state value at the same time as the inflation differential.

Figure 7 displays the impulse responses of a one standard deviation foreign monetary policy shock. The volatility of the monetary policy shock was chosen to hit the target of the mean of the distribution of the empirical data in figure 3, which requires a one standard deviation monetary shock to increase the nominal interest rate by approximately one percent.

[7]It should be noted that there is also a negative output gap abroad, which also puts downward pressure on the nominal interest rate further increasing the domestic-foreign interest differential. Compared to the case of flexible prices, there is less of an increase in output in the economy with rigid prices in response to a positive technology shock since all producers in the flexible price case, will decrease their prices causing a large increase in the quantity of goods demanded spurring output. Therefore, interest rates are counter-cyclical, which is a major distinction between this model and that of Verdelhan (2010), which reproduces the forward premium anomaly with pro-cyclical interest rates.

Figure 7: Impulse Response of a One Standard Deviation Foreign Monetary Shock



In contrast to the technology shock, the comovement of the real interest differential and the change of the log real exchange rate is positive in response to a monetary policy shock, see panels 2 and 4. In addition, the excess return, the difference between the series in Panel 2 and Panel 4, is negatively correlated with the inflation differential after a monetary policy shock. Thus, the introduction of the monetary shock dampens the positive correlation between relative inflation and the excess return and the negative correlation between the change in the real exchange rate and the real interest rate differential.

4.2 Coefficient Distributions

To test whether the model is able to generate distributions of the estimated regression coefficients of the expost return regressed on inflation and the test of UIP in real terms, i.e. the distributions displayed in figures 4 and 5 respectively, the model was simulated 120 times and for a time span equivalent to 25 years to match the pairwise combinations of countries and time frame for available data.

Figure 8 displays the histogram of coefficient estimates of from equation (7), the test of uncovered interest parity in real terms, for simulated and empirical data. The mean of the distribution of regression coefficients is more negative than the results from the data contained in figure 3 by about -0.3.

Figure 8: Histogram of Estimates of β_3 from Simulations with Technology Shocks

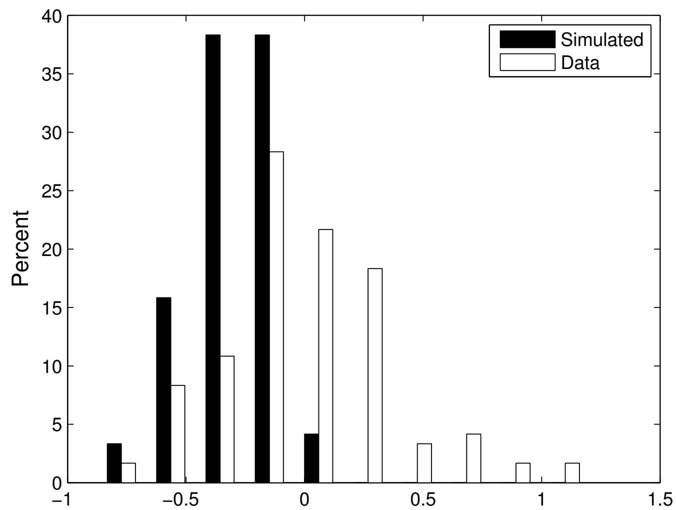
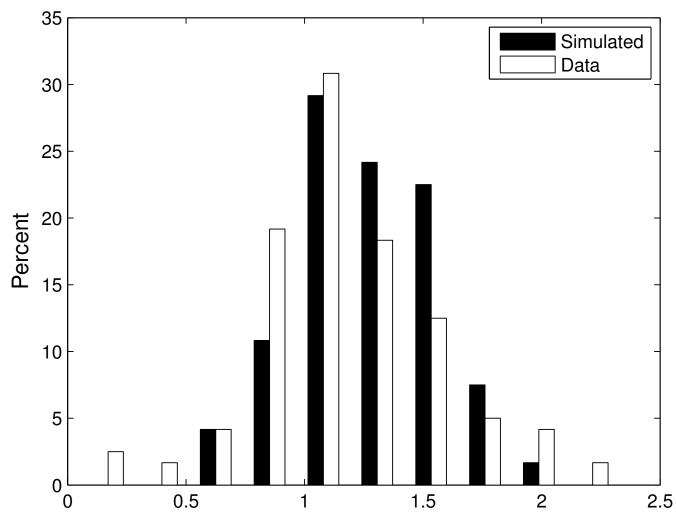


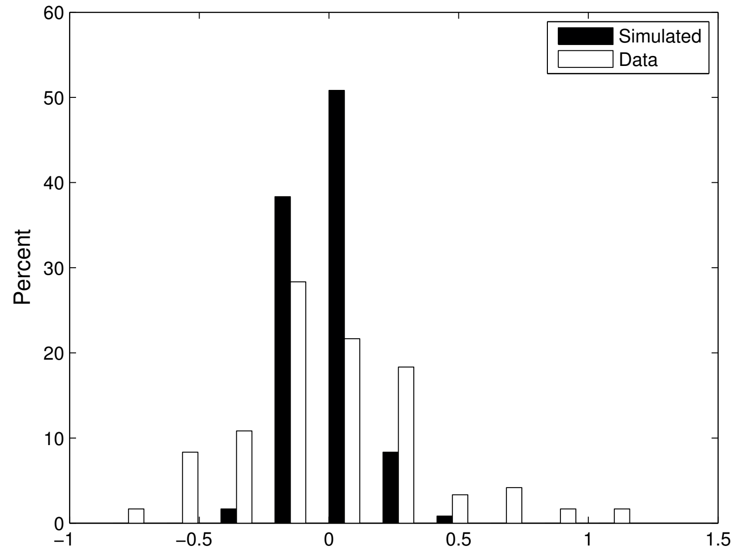
Figure 9 displays the simulated histogram of the regression coefficients for the excess return regressed on inflation, equation (4). The mean in this case is greater than that seen in the data by about 0.10.

Figure 9: Histogram of Estimates of β_2 from Simulations with Technology Shocks



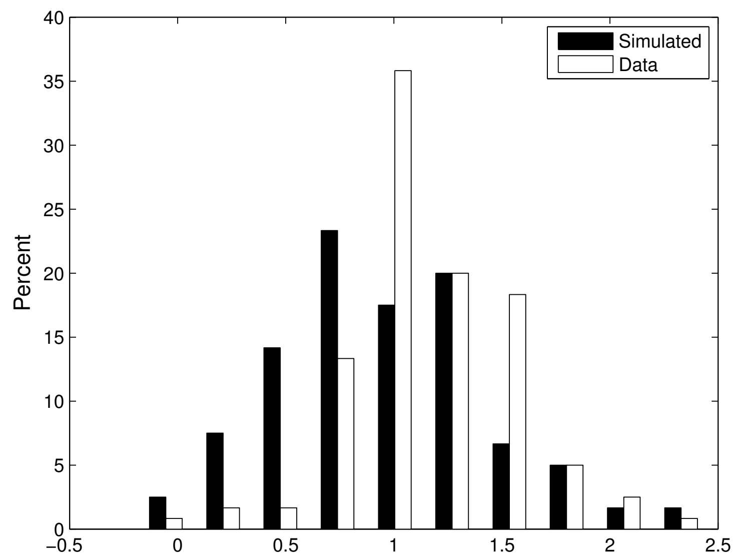
A second calibration of the model includes both technology shocks and monetary policy shocks. The addition of monetary shocks will allow the model to be calibrated in a way to better fit the data. Figure 10 displays the histogram of β_3 for the simulated model with both technology and monetary shocks and the observed empirical distribution. Even with both shocks the simulated histogram still has less spread than seen in the data.

Figure 10: Histogram of Estimates of β_3 from Simulations with Technology and Monetary Shocks



However, the introduction of the monetary shocks has resulted in the simulated distribution of β_2 in Figure 11 to remain fairly closely to that of the data. The mean of the simulated distribution is now a little less than that of the data by 0.15, but the range of the estimated values of the simulated distribution is larger.

Figure 11: Histogram of Estimates of β_2 from Simulations with Technology and Monetary Shocks



4.3 Sensitivity Analysis

Forthcoming.

5 The Nature of Currency Risk

The excess return generated by the model is a risk premium. The premium arises when the relative conditions in the two countries differ. A statistic that summarizes this fact is the correlation generated by the model between the premium and the relative total factor productivity.

$$corr(X_t - X_t^*, \tau_t) = -0.6821 \tag{42}$$

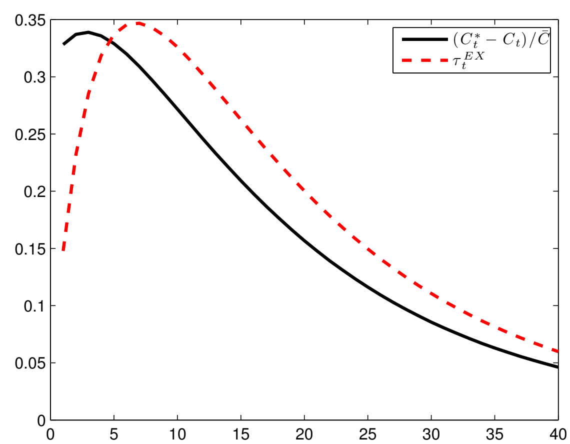
When conditions in the home country are relatively poor compared to the foreign country in terms of TFP, then the home country's currency has a positive excess return. Conditions could be relatively poor in the domestic economy either because it received a bad TFP shock or because the foreign country received a good TFP shock. These possibilities are reflected in table 2, which contains the responses of the excess return, TFP, and domestic and foreign consumption to shocks to domestic and foreign TFP.

Table 2: Risk Summary

Excess Return	Home Shock			Foreign Shock		
τ_t^{EX}	Sign	ΔX_t	$\left(C_{t+1}^{*} - C_{t+1}\right)/\bar{C}$	Sign	ΔX_t^{*}	$\left(C_{t+1}^{*} - C_{t+1}\right)/\bar{C}$
0.147	-	-0.01	0.33%	+	0.01	0.33%

A negative one standard deviation shock to domestic TFP or a positive one standard deviation shock to foreign TFP raises the excess return for the domestic currency by the same amount, almost 15 basis points. In addition, both shocks increase the difference between foreign and domestic consumption by 0.33 percent of steady state consumption. This patterns continues until the effects of the shock dissipate as seen in Figure 12.

Figure 12: Responses to a One Standard Deviation Shock to Foreign TFP



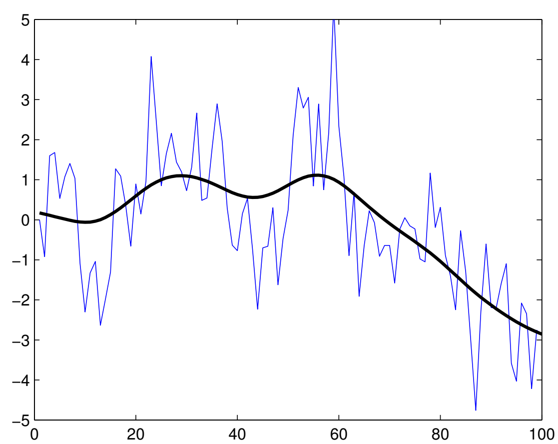
Therefore, on average across realizations of technology shocks, when the excess return is positive, the foreign country's consumption increases relative to the domestic country's consumption. This is reflected in the positive correlation between foreign consumption relative to the domestic country consumption next period and the excess return,

$$corr(\left(C^*_{t+1} - C_{t+1}\right)/\bar{C}, \tau_t) = 0.7913 \tag{43}$$

which indicates that it is a risky strategy for foreign investors to go long in the domestic country's currency and short their own currency. The reasoning for this outcome is as follows: when the domestic country is in relatively poor condition, it desires the foreign investors to buy domestic assets to help alleviate some of the relative shortfall in consumption. However, the next period payoff to the foreign owned domestic assets occurs when the foreign level of consumption is already high. To compensate the foreign investors for this risk, the sellers of domestic assets must pay a risk premium in the form of the excess currency return.

Finally, figure 13 displays one simulated excess return series. The model is able to generate excess return of a significant size, varying between -5 and 5 percent per annum.

Figure 13: Simulated τ_t^{EX} (in percent per annum)



The average standard deviation for the excess return over 120 simulations is 3.7. This is roughly thirty percent of the standard deviation seen in the data at a quarterly horizon. The variance of the excess return can be decomposed into the variance of the change of the exchange rate plus the variance of the interest rate differential minus two times the covariance of the exchange rate and interest rate differential:

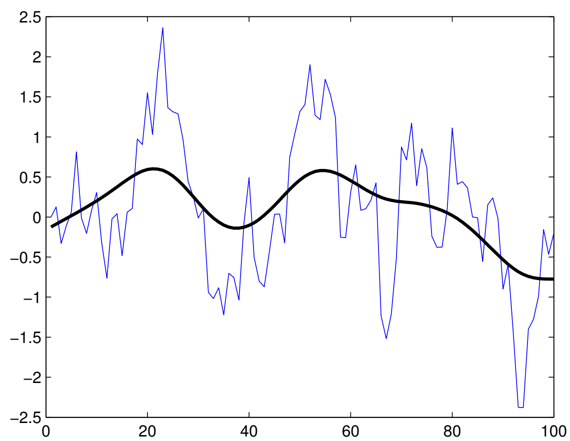
$$\sigma^2_{\tau_t} = \sigma^2_{\Delta q_{t+1}} + \sigma^2_{\tilde{r}_t} - 2 * \sigma_{\Delta q_{t+1}, \tilde{r}_t} \tag{44}$$

$$3.7^2 = 2.8^2 + 2.3^2 - 2 * (-0.2)$$

The standard deviation of the interest rate differential is roughly that found in the data. However, the standard deviation of the change in the exchange rate is four times lower than that seen in the data. The model is unable to match all of the volatility of the excess currency return, because it cannot produce enough volatility in the real exchange rate. In addition, the first order autocorrelation produced by the model is 0.80. However, only few country pairs in the data exhibit positive autocorrelation, ranging between 0.20 and 0.40, that are statistically significant.

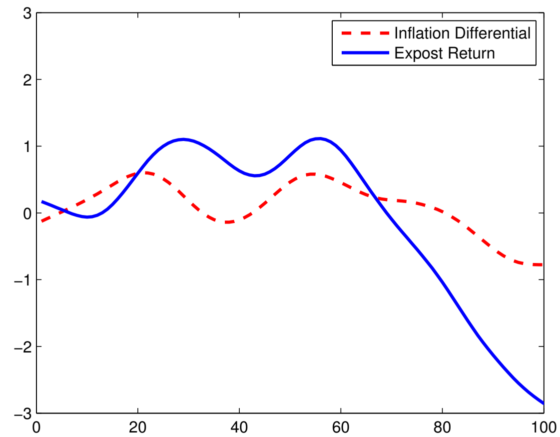
Figure 14 displays the inflation differential.

Figure 14: Simulated Inflation Differential (in percent per annum)



Similar to the relationship seen in data in Figures 1 through 3, the simulated inflation differential's trend closely follows that of the excess return. However, like the data, larger swings are seen in the trend of the excess return than the inflation differential as seen in Figure 15.

Figure 15: Simulated τ_t^{EX} and Inflation Differential Trends (in percent per annum)



6 Conclusion

This paper contributes to the forward premium anomaly literature by documenting a new co-variate, lagged inflation, that is positively correlated with the excess currency return. It then builds a DSGE model that reproduces the relation between inflation and the excess currency return and reproduces the forward premium anomaly itself. The model capitalizes on the fact that sticky price models can generate persistent deviations in the real exchange rate from PPP. Within the model a positive foreign productivity shock causes an immediate depreciation of the domestic real exchange rate, which is expected to slowly appreciate back to PPP. At the same time domestic inflation has increased relative to the foreign country which induces the domestic monetary authority to set a relatively higher interest rate according to a Taylor rule. The expected appreciation of the domestic currency concurrent with a positive interest rate differential comprises the foreign premium anomaly. In addition, the country with the positive excess currency return also experiences relatively higher inflation, which is consistent with empirical findings contained in this paper. The model is able to match these relations in the form of the regression coefficients of the future percent change in real exchange rate regressed on the real interest rate differential and the excess currency return regressed on relative inflation.

In addition, the model is able to produce thirty percent of the volatility of the excess currency return.

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