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Abstract

Patent Assertion Entities (PAEs) are playing an increasingly important role in business strategy, innovation, and litigation. Their strategic advantage comes from the ability to fend off counter-suits. We develop a model to identify channels through which outsourcing patent enforcement to PAEs (“privateering”) can affect the incentives of operating companies. We find that PAEs can reduce transaction costs involved in patent monetization, thus enhancing firms’ incentives to invest in R&D. On the other hand, they can lower these incentives by reducing the defensive value of patents, and also by decreasing total industry profits. In our model, the welfare effects of PAEs on firms and consumers can be positive, even when they increase litigation threats, lower industry profits, and acquire patents only for monetization reasons.

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1 Introduction

The U.S. patent system has changed over the past few years. Firms today have new ways of monetizing their patents: Patent Assertion Entities (PAE), also known as non-practicing entities (NPE), or “patent trolls,” have risen to prominence by buying up significant numbers of patents, bringing alleged infringements to court, and by using the threat of litigation to extract license payments. This recent development has led to much public and academic debate on the merits of PAEs and on their effect on innovation and litigation. There is no doubt that PAE activities significantly impact the way the patent system works. This can be clearly seen from the multitude of bills that have recently been passed or proposed in Congress\(^1\) and President Obama’s public stance on the issue\(^2\).

In practice, PAEs vary significantly in their business strategies\(^3\). One relevant difference is the source of the patents they own. In recent years, they have included universities (including deals whereby a PAE buys the rights to future patents), individual inventors, companies which have at some point invested in R&D but do not (or no longer) produce commercial products using those patents, as well as actively producing firms. The latter source of patents is the most relevant for our paper, which presents a theoretical model of one particular PAE business strategy called Patent Privateering, in which PAEs acquire patents from an operating company to use offensively against rivals of the patent seller.

Examples of producing firms that have sold significant numbers of patents to PAEs include Alcatel-Lucent, British Telecom, Digimarc, Ericsson, Kodak, Micron Technology, Microsoft, Nokia, and Sony. Nokia and Sony, for example, sold some of their portfolios to MobileMedia, a PAE which subsequently sued Apple, HTC, and Research In Motion\(^4\). Another example is Micron Technology, a multinational corporation and one of the largest memory chip makers in the world\(^5\). Micron has sold at least 20% of its patent portfolio to Round Rock, in multiple transactions between 2009 and 2013. Round Rock, a PAE, asserted these patents against SanDisk\(^6\). Although many examples can be found in the high-tech industry, the patent

\(^1\)These include for example the SHIELD Act, the Patent Quality Improvement Act, America Invents Act, and the End Anonymous Patents Act, among others.

\(^2\)See “Patent Assertion and U.S. Innovation,” published in June 2013 by the Executive Office of the President and prepared by the CEA, NEC & OSTP.

\(^3\)See for example Risch (2012).

\(^4\)“Patent Privateers Sail the Legal Waters Against Apple, Google” by Susan Decker (Bloomberg; January 10, 2013).

\(^5\)Micron has recently been named one of Thomson Reuters’s top 100 global innovators.

\(^6\)“Patent ‘Troll’ Tactics Spread” by Asbhy Jones (WSJ; July 8, 2012). Also, see here.
privateering phenomenon is also found in other industries. For instance, in 2006, Nike sold part of its patent portfolio to a company called Cushion Technologies, LLC, who later sued several rivals of Nike in the running shoe market.

Patent Privateering is an important phenomenon and needs to be studied in more detail. On April 5, 2013, Google, BlackBerry, EarthLink, and Red Hat, sent a letter to the Federal Trade Commission and the Department of Justice asking for more scrutiny, specifically on patent privateering. An extract of this letter makes clear the importance of this patent monetization strategy:

“PAEs impose tremendous costs on innovative industries. These costs are exacerbated by the evolving practice of operating companies employing PAE privateers as competitive weapons. The consequences of this marriage on innovation are alarming. Operating company transfers to PAEs create incentives that undermine patent peace. [...] We therefore urge the antitrust agencies to study carefully the issue of operating company patent transfers to PAEs.”

In this paper, we build a theoretical model to assess the effect of patent privateering on innovation, licensing, and litigation. Our model incorporates key features of the patent system today especially relevant to high tech industries, where PAEs have been most active. In our model: litigation is costly and is often resolved through settlement; firms counter-sue using their patents when they are accused of infringement; PAEs cannot be counter-sued (since they do not produce); patent enforcement is noisy; and products use multiple patentable components. The latter point is fundamental for our results and it is often observed in reality. For instance, Apple holds nearly 1,300 patents protecting the iPhone including software, hardware, and design patents.

Our model endogenizes both the innovation and the litigation processes. Firms decide how much to invest in R&D in anticipation of the rewards to patenting, which can come in the form of product sales, patent trade, licensing, and litigation revenue. The goal of the patent system is to provide incentives to invest in R&D and generate innovation, so this is the most appropriate context in which to evaluate the effect of PAEs. This is in contrast to most existing papers that study PAEs, which take R&D investments as exogenous and look at litigation and licensing incentives in a fixed patent landscape (for example, Choi and Gerlach (2013)). As noted, our model also endogenizes the litigation process, which is important as a

http://patentlyo.com/media/docs/2013/06/pae-0047.pdf (Visited on August 20, 2014)
Lloyd et al. (2011) shows evidence of the large amount of patents involved in the legal protection of one product.
key feature of PAEs is the way they change the credibility of litigation threats.

Our main contribution is to identify two effects of outsourcing patent monetization to PAEs. First, since patent monetization involves transaction/litigation costs, when producing firms do not have access to PAEs, the threat of countersuits and the cost of litigation dampen ex-ante innovation incentives. PAEs can help producing firms to overcome transaction costs, enhancing ex-ante incentives to invest in R&D. Second, PAEs can also reduce R&D incentives by decreasing the marginal value of patents that are used defensively, and by potentially extracting rents from the market.

The first effect can be explained by understanding patent enforcement without PAEs. In their absence, competitors with similarly-sized patent portfolios will often engage in a tacit “IP truce,” whereby neither firm is willing to sue its rivals for infringing their patents, as the rivals’ portfolios act as a deterrent. Since going to court is costly for both parties, even when one firm has more patents than its rival (i.e. expects a positive overall payoff if it triggers a “patent war”), the net benefit from enforcing its patents after accounting for the cost of a potential counter-suit may be less than the expected legal costs. This “mutually assured destruction” scenario implies that a firm with a larger patent portfolio may not have a credible litigation threat, which prevents it from monetizing its patents. Hence, some of the value of the firm’s patent portfolio is lost.\footnote{Counter-suing plays a crucial role on litigation strategy when firms can use their patents as defensive weapons. Some salient examples are Apple vs HTC, where HTC counter sued with 2 patents, or Yahoo vs Facebook, where Facebook counter sued with 10 patents.}

When PAEs are present, litigation incentives change. Because PAEs cannot be counter sued, their litigation threats are stronger than those of an operating firm, conditional on having the same patents. Thus, by enforcing patents, PAEs can extract higher licensing payments compared to a producing firm. However, by selling patents to the PAE, a producing firm leaves itself more vulnerable to lawsuits, because it has fewer patents to use in a counter-suit. Thus, PAEs change the bargaining position of the firms. We show that, overall, this benefits the firm with more patents and harms the firm with fewer. In equilibrium, if a PAE extracts additional rents from the rival firm, this excess must somehow be divided among the original owner of the patent and the PAE that monetizes it. Hence, the original inventor’s payoff weakly increases. In fact, when the PAE allows more patent monetization there are two effects: 1) the firm with the smaller portfolio loses more compared to the tacit “IP truce” equilibrium in absence of PAEs; and 2) the firm with the larger portfolio can capture some of the extra surplus generated by the PAE, determined by its bargaining power, while the rest
goes to the PAE as rents. Notice that both of these effects push the incentives for patenting in the same direction: they both make more profitable being the firm with a larger portfolio. Thus, PAEs help overcome transaction costs, which can lead to larger ex-ante incentives to invest in R&D.

The negative effect of PAEs on R&D incentives comes via two channels. First, they reduce the marginal value of “defensive portfolios.” A firm’s portfolio is defensive when it is not large enough to start a lawsuit, even if its rival will not counter sue. When PAEs monetize patents, they eliminate the defensive value of the portfolios (because PAE cannot be counter sued), which lowers ex-ante incentives to invest in R&D. Second, when firms are unable to sign licensing agreements before bilaterally trading patents with the PAE, the PAE may extract positive rents from the producing firms. This rent extraction effect lowers total industry profits, which reduces the incentive to to engage in R&D.

In general, whether PAEs increase or decrease innovation depends on which of these effects dominates. We show that when firms can trade licenses before arrival of the PAE, the positive effect always dominates in symmetric equilibrium. Moreover, there is always an equilibrium in which the positive effect dominates because the PAE manages to extract no rents. This happens because of the “contracting with externalities” structure of the patent acquisition process.

We also consider the welfare effect of PAEs. In our theory, R&D investments determine the random arrival time of discovery for each one of the components of the final product. The times of the discovery determine two elements: the expected time to discover all technologies that are necessary for production, and the patent portfolio of each firm. A firm that invests in R&D more than its rival is more likely to discover and patent more components of the final product. A larger R&D investment also speeds up innovation, which implies that both firms and consumers can capture the rewards from commercialization sooner.

In the model, the social benefit of R&D is to reduce the delay of the introduction of the product in the market. To evaluate the effect of PAEs from a welfare perspective, we characterize conditions under which the firms under-invest and over-invest in equilibrium (in the absence of PAEs) relative to the social planner’s first- and second-best outcome. The model provides some comparative statics. For example, the R&D equilibrium features under-investment when firms are less patient, when consumer surplus is large, when the final product is more complex (i.e. involves more pieces of technology), or when patent protection is weak. Overall, for a range of parameter values, PAEs enhance welfare.
The paper is organized as follows: In Section 2, we review some of the literature on PAEs, and some of the recent papers addressing related research questions. In Section 3 we introduce a model of R&D, licensing, and litigation. In Sections 4 and 5 we solve the licensing and litigation game and present our main results. In Section 6 we solve the endogenous R&D decision. In Section 7 we discuss the welfare implications of our results and, finally, in Section 8 we summarize our findings and offer some policy implications.

2 Literature Review

Khan (2005) shows that commercialization of patents is not a phenomenon particularly tied to high technology products. Companies whose sole business is the monetization of patents have existed for a long time and were called in the past “patent sharks” rather than “patent trolls”. Maglioceca (2006) describes “patent sharks” as entities that extracted money from innocent individual farmers and railroad companies. In recent years, the proliferation of companies focused on the assertion, rather than the commercialization, of patents has opened an important debate. Chien (2010) studies this proliferation of PAEs, and the rise of strategic management of patents. In particular, she emphasizes the importance of holding large portfolios to sustain “patent peace” among operating companies through the threat of counter-suing.

Empirical research on PAEs is scarce, mostly due to the difficulty of finding the data. The majority of the empirical studies have been restricted to a small number firms for which data is available, and some arguments against PAEs have been based on anecdotal evidence or isolated cases. Risch (2012) and Fischer and Henkel (2012) have tried to shed light on the practices of PAEs by analyzing the patent portfolios of a sample of firms. An important finding of these papers is that, in their sample, PAEs acquire patents of relatively good quality (in terms of validity), which goes against the commonly held belief that PAEs try to enforce bad quality patents. Shrestha (2010) finds similar results, when compares the forward citations of patents acquired by PAEs versus those acquired by operating companies. An important empirical finding is that PAEs do not acquire all their patents from individual inventors. Fischer and Henkel (2012) find that about 65% of the patents acquired by PAEs came from operating companies with more than 100 employees.

Bessen et al. (2011) estimates the cost imposed by PAEs on operating companies. Analyzing stock market events around NPE lawsuit filings, they find a loss of about half a trillion dollars to defendants over the period 1990-2010. Bessen and Meurer (2014) estimate that the direct
costs of PAE assertions (not including diversion of resources, delays in new products, and loss of market share) was about $29 billion in 2011. These studies, highly cited in the media for the large amount of rents extracted by PAEs, are not without caveats and critiques to their methodology. Risch (2014) and Cotropia et al. (2013), for example, question the sensitivity of these results to definition used for PAEs. Schwartz and Kesan (2014) claims that the findings in Bessen and Meurer (2014) are based on a biased sample, and that majority of the $29 billion correspond to settlements and licensing, which are transfers and not costs.

PAE business strategies vary a lot, and different companies have found different ways of monetizing patent portfolios. There is no “one-size-fits-all” patent monetization strategy. Lemley and Melamed (2013) discusses different models of patent assertion by practicing and non-practicing firms. Also, Scott Morton and Shapiro (2014) provides a description of the different strategies employed by PAEs to monetize their patents. They provide a simple model of PAE intermediation between an individual inventor and an operating company. In their baseline case, the individual inventor cannot monetize its patent, and the operating company infringes on the patent to produce the final product. The PAE acquires the patent from the individual inventor and it has the ability to enforce it. In their model, when the PAE does not transfer enough rents to the original inventor, PAEs will have a negative impact on welfare. The main difference between their model and ours is the source of the patents.

Cohen et al. (2014) presents a model of PAE formation. In their model, initially two firms try to enter the market and each firm owns one invention of exogenously given quality. If one firm has very low quality, it will stay out of the market and it will act as a patent troll. In our model, we endogenize the quality of the innovation, and also allow for entry decisions. Cosandier et al. (2014) studies defensive patent acquisition services, a strategy that is utilized by, for example, the company RPX Corporation. Finally, Tucker (2014) studies the effect of patent litigation on VC investments, concluding that frequent litigators are associated with a direct and negative effect on VC investments.

In our paper, patent portfolios arise endogenously, which differs from models of exogenously given patents portfolios, as in Choi and Gerlach (2013). We also present a model novel of R&D, which features a “contest for bundles”, related to some extent to the work of Fu and Lu (2012), and Clark and Riis (1998).

In particular, Cotropia et al. (2013) provides a finer classification of PAEs by different types (universities, individual inventor, IP holding companies, etc.) for all patent litigation cases in the years 2010 and 2012.
3 Model Overview

Two firms (A and B) race to discover $N$ pieces of technology, which we call *components*, in order to produce and sell a final product that incorporates all of them. The timing of the model, depicted in Figure 1, is as follows: first, firms invest in R&D and patent their discoveries; second, observing the realization of patent portfolios after the R&D stage, firms have the option to buy or sell patents; third, firms decide whether to enter the final product market; fourth, PAEs may acquire patents; fifth, patent owners and producing firms engage in patent licensing in the shadow of litigation; sixth, if a firm has entered the product market without patents or licenses on all $N$ components, it can potentially be sued for patent infringement.

**Figure 1:** Timing of the events in the model.

In the first stage firms simultaneously make sunk R&D investments to discover the $N$ components, and the discovery of a particular component arrives stochastically, given the R&D investments. We assume as in Loury (1979) that R&D investment is a one-time fixed investment, rather than a flow investment that can be revised upon the realization of uncertainty as in Lee and Wilde (1980). The cost of investing $z$ units of R&D for the firms is $c_I(z)$, where $c_I(\cdot)$ is increasing, convex, differentiable, and $c_I(0) = 0$. Fixing the firm’s R&D investments $x$ and $y$, respectively for firm A and B, firm A is the first to discover any one particular component independently with probability $p(x, y) = \frac{h(x)}{h(x) + h(y)}$, where $h(\cdot)$ is increasing, concave, differentiable, and $h(0) = 0$. This function is derived from independent exponential arrivals. If, for a given level of R&D $z$, each component arrives independently at time $\tau_c(z) \sim \exp(h(z))$, then firm A discovers a component first if $\tau_c(x) < \tau_c(y)$ which occurs with probability $\frac{h(x)}{h(x) + h(y)}$. Hence, the number of patented components for a particular firm follows a binomial distribution, and the probability that firm A discovers exactly $k$ components is given by

$$P(k; x, y) = \binom{N}{k} p(x, y)^k (1 - p(x, y))^{N-k}.$$ 

Discoveries are publicly observable, and the firm which discovers a component immediately and costlessly obtains a patent on it.\footnote{For simplicity we assume away the possibility of trade secrets or strategic delay in patenting.} At the end of the R&D stage the patent portfolio of
each firm is fixed. The expected time to complete the R&D stage is endogenously determined by the level of investment of the firms. The time at which a particular component $i \in \{1, ..., N\}$ is discovered is given by $\tau_i(x, y) = \min\{\tau_c(x), \tau_c(y)\}$. Production can take place only when every component has been discovered by some firm, since firms require every component to produce. The time at which firms will enter the market and produce is therefore given by $\tau(x, y) = \max_{i=1, ..., N}\{\tau_i(x, y)\}$, which is distributed according to $F(\tau; x, y)$.

Once patent portfolios are determined, in stage two, firms can engage in patent trade. We assume that the original inventor of a patent always retains a license for his invention, even after assigning the patent to a new firm. In consequence, the original assignor of a patent cannot infringe on that patent, even after it no longer owns it.

Given the patent portfolios after the patent trade, in stage 3, firms simultaneously decide whether to enter the final product market. Entry is not blocked by the lack of patents or licenses for some components, since firms can freely and immediately imitate any component discovered by any other firm. Industry profits (before accounting for license and litigation costs) are modeled in reduced form: if both firms enter the market, each one of them makes profit $\pi > 0$ by selling the final product.\footnote{Notice that firms have the same market size, despite having potentially asymmetric patent portfolios.} If only one firm enters, it monopolizes the market and obtains $\pi_\text{m}$ in the final product market.

If PAEs are present, in stage 4, firms can decide to trade patents with a PAE after entry and before they license with their rivals. Next, in stage 5, firms engage in patent licensing with any patent owner (including, possibly, PAEs), and licenses are determined under the threat of litigation. We assume that license prices are set through Nash bargaining over the surplus that is generated by a licensing deal, relative to the firms’ outside options of not licensing and potentially going to court.\footnote{See Spier (2007) for a discussion of the role of bargaining power in settlement outcomes. In a separate working paper, Lemus and Temnyalov (2013), we study how bargaining power and injunctions affect the incentives for litigation in the presence and absence of a PAE.}

Finally, in stage 6, if a firm that has entered the product market and does not have a license or patent protection for some of the $N$ components, it may be sued for infringement, and the court will decide whether the final product infringes on unlicensed components. We assume that patents are valid (“strong”), but probabilistic: a firm’s product infringes on each patent with probability $\beta > 0$, which is independent across patents. We further assume that going to court is costly: each side must pay $c > 0$ in legal fees per lawsuit, and that the defendant...
may bring counter-claims (defensive countoursuing) to the court at no additional cost. If a firm is found to infringe on a patent, it must pay the patent-owner a per-patent infringement fee $R > 0$. In the next section we proceed to solve the model by backward induction. We derive the continuation payoffs from the last stage first in an economy without PAEs and then we re-derive the continuation payoffs with PAEs. Then, we examine the R&D decisions when firms forsee these continuation payoffs.

4 Patent Trade, Entry, Licensing, and Litigation without PAEs

We solve the game by backward induction, assuming first that PAEs do not exist. We start by solving the licensing and litigation stages, and we move backwards to entry and patent trade.

4.1 Licensing & Litigation

We analyze the licensing and litigation stages, for any given distribution of patents, after one or both firms has entered the final product market.

Licensing and litigation payoffs when both firms enter

Consider the situation in which firms A and B enter the market for the final product, which is composed of $N$ components, and the R&D investments determined that firm A patented $n$ components, while firm B patented $m$, with $n + m = N$. When $0 < n < N$ both firms have entered the product market with incomplete patent protection. When firm A sues firm B using its complete patent portfolio, given that each infringement claim is evaluated independently, the probability that firm B’s product infringes on exactly $k$ out of the $n$ patents owned by firm A is given by

\[ \binom{n}{k} \beta^k (1 - \beta)^{n-k}, \]

\footnote{We assume that $c$ is independent of the number of patents being litigated.}

\footnote{Because we model industry profits in reduced form, we also focus on per-patent royalties, rather than per-sale royalties. This avoids the complication of how royalties themselves affect pricing, which is not central to this paper.}
and the expected payment in royalties received by A is given by
\[ \sum_{k=0}^{n} \binom{n}{k} (R_k)^k (1 - \beta)^{n-k} = R_n \beta. \]

Because countersuing is free, firm B will counter-sue using its entire portfolio to obtain expected royalties of \(R_m \beta\). Thus, firm A’s expected payoff from going to litigation is \(R\beta(n - m) - c\), and B’s expected payoff is \(R\beta(m - n) - c\). In this situation, should licensing negotiations fail, firm A is willing to initiate litigation if and only if \(R\beta(n - m) > c\), while firm B is willing to initiate litigation if and only if \(R\beta(m - n) > c\) (i.e., these are the cases where one side has a positive-expected-value suit, as discussed for example in Shavell (1982) and Nalebuff (1987)).

If one firm has a credible threat of litigation, firms will bargain to avoid losing a total of \(2c\) in joint surplus due to litigation costs. For simplicity, we assume equal bargaining power in the Nash bargaining solution at this stage. Denoting \(V = R\beta\), \(\hat{c} = \lceil \frac{c}{\beta} \rceil\), and the function
\[ T(n, m) = \begin{cases} V \cdot (n - m) & \text{if } |n - m| > \hat{c} \\ 0 & \text{if } |n - m| \leq \hat{c} \end{cases}, \]
we have three relevant cases to analyze:

1. If firm A has a credible litigation threat: \(n - m > \hat{c}\), firms cross license and firm A receives the transfer \(T(n, m) = V \cdot (n - m) - c + \frac{1}{2} (2c) = V \cdot (n - m)\).

2. If firm B has a credible litigation threat: \(m - n > \hat{c}\), firms cross license and firm B receives the transfer \(T(m, n) = V \cdot (m - n) - c + \frac{1}{2} (2c) = V \cdot (m - n)\).

3. If no firm has a credible litigation threat: \(-\hat{c} \leq n - m \leq \hat{c}\), and \(T(n, m) = 0\).

This means that for fixed portfolio sizes \((n, m)\) either one of the following two cases occur: one of the firms has a relatively large enough portfolio, so the litigation threat is credible and that firm receives a payment from cross-licensing portfolios; or firms have portfolios of similar sizes, so litigation is not credible and firms produce while tacitly agreeing not to sue. The figure below depicts the function \(T(n, m)\) for all possible combinations of patent portfolios.
Firm A pays $V(m - n)$.

No payments

Firm A gets $V(n - m)$.

**Figure 2:** Licensing transfers are shown for different portfolio configurations. Cross-licensing agreements featuring positive transfers occur only when $|n - m| > \hat{c}$.

We denote by $U^A_{E,E}(n, m)$ the payoff of firm A after both firms entered the product market ("E" stands for entry) and bargained over their patent portfolios, whose sizes are $n$ and $m$ for firm A and firm B, respectively. By definition, $U^A_{E,E}(n, m) = \pi + T(n, m)$, or equivalently:

$$U^A_{E,E}(n, m) = \begin{cases} 
\pi + V \cdot (n - m) & \text{if } n - m > \hat{c}, \\
\pi & \text{if } |n - m| \leq \hat{c}, \\
\pi - V \cdot (m - n) & \text{if } m - n > \hat{c}.
\end{cases}$$

By symmetry, $U^B_{E,E}(m, n) = U^A_{E,E}(n, m)$.

**Licensing and litigation when only one firm enters**

Consider now the case in which only one firm enters the product market, say firm A, and A has $n$ patents, and B has $m$ patents. Because $B$ is not producing anything, firm A’s portfolio is worthless in litigation. Firm B’s portfolio, however, can be monetized as long as firm B is willing to litigate, which is the case when $m > \hat{c}$. In this case, the negotiated license fees are given by $T(m, 0) = Vm$. Notice that in this case, firm B is not producing and is actually operating as a PAE, although it invested in R&D.

If $m \leq \hat{c}$, firm B cannot monetize its portfolio and firm A produces as a monopolist without any credible threat of litigation. The total payoffs when only firm A enters the product
market, denoted by $U^A_{E,NE}(n,m)$ and $U^B_{E,NE}(n,m)$, are therefore given by

\[
U^A_{E,NE}(n,m) = \begin{cases} 
\pi_m & \text{if } m \leq \hat{c}, \\
\pi_m - Vm & \text{if } m > \hat{c}
\end{cases}, \quad U^B_{E,NE}(n,m) = \begin{cases} 
0 & \text{if } m \leq \hat{c}, \\
Vm & \text{if } m > \hat{c}
\end{cases}.
\]

4.2 Entry decisions

We now analyze the optimal product market entry strategies, given the continuation payoffs described above, for a fixed patent portfolio, where firm A has $n$ patents and firm B has $m$ patents, and $n + m = N$. Since the problem is symmetric for both firms, we focus on firm B’s optimal entry decision, taking the decision of firm A as given. We assume that duopoly profits are larger than the cost of litigation, i.e. that $\pi > c$, and that monopoly profits are larger than twice duopoly profits, i.e. that $\pi_m > 2\pi$.

**Lemma 1.** When $\pi > NV$ it is a dominant strategy for each firm to enter the final product market.

**Proof.** Consider firm A with $n$ patents, while its rival has $m$ patents. By entering, firm A gets at least $\pi - mV$ and at most $nV$ by not. So entering is better iff $\pi - mV > nV \iff \pi > (n + m)V = NV$.

We will assume $\pi > NV$ for the remainder of the paper.

4.3 Patent Trade

In this section we analyze the possibility of patent trade among firms A and B after the R&D stage is over. We again consider the portfolios $n$ and $m$ for firms A and B, respectively. We assume that an original patentee always retains a license, even after selling the patent to another party. That is, the first firm to discover a component will always have protection over it. We also assume that firms cannot sign a contract that only allows one firm to enter and monopolize the market, even when this might be profitable to do. In other words, we assume that the antitrust authorities will be vigilant and prevent firms from signing these type of contracts.
Under these conditions firms are indifferent about trading patents given that they can license in the following stage. Since the continuation game is efficient conditional on both firms entering the market, there are no gains from patent trade before entry.

4.4 The Return to R&D without PAEs

In this subsection we summarize the returns to R&D in the absence of PAEs. We have seen that in this case, firms do not trade patents, they both enter, and they reach a licensing agreement prior to litigation. When firm A has discovered and patented \( n \) components, and firm B has discovered the remaining \( m \) components, the continuation payoffs are \( U^A(n, m) = \pi + T(n, m) \) for firm A, and \( U^B(n, m) = \pi - T(n, m) \) for firm B. Since \( n + m = N \) we can write \( U^A(n, N - n) = \pi + T(n, N - n) \). Notice that \(|n - m| > \hat{c}\) is equivalent to \(|2n - N| > \hat{c}\), and in that case \( T(n, N - n) = (2n - N)V \). Firm A’s continuation payoff as a function of \( n \) is depicted in Figure 3.

![Figure 3: Continuation payoff without PAEs after the R&D stage for firm A, when it has discovered \( n \) components, while its rival has discovered \( m = N - n \), in the case \( N > 2\hat{c} \).](image-url)
5 Patent Trade, Entry, Licensing and Litigation with PAEs

In this section we introduce a PAE into the model, and derive the returns to R&D with PAEs. The PAE is strategically different from producing firms since it cannot be counter-sued, by definition. Therefore, producing firms have no tools to defend themselves against a PAE. The only risk that PAEs face in litigation is the randomness of court decisions. In our model the PAE begins the game with no patents, since it does not invest in R&D. The only way for the PAE to acquire patents is to buy them from firms that invested in R&D, once the research stage is over, and after the entry decisions have been made. When the PAE acquires \( n' \) patents from a producing firm, that firm is granted a license for the patents it sold, so the PAE cannot sue the original inventor. A producing firm leaves itself more vulnerable to litigation after selling some of its patents, since it cannot use them defensively; but it may also generate additional revenue from the price that the PAE would be willing to pay for the patents, since the PAE can use those patents offensively without the counter-suing threat.

Suppose a PAE has acquired \( n' > \hat{c} \) patents from firm A and is planning to sue firm B. The expected payoff from litigation for the PAE is \( V \cdot n' - c \), and for firm B is \( -V \cdot n' - c \). We again assume symmetric bargaining power in the negotiation of licenses between the PAE and a producing firm. When the litigation threat is credible \( (n' > \hat{c}) \), firms will bargain over the surplus gained by avoiding litigation. Firm B is willing to pay the PAE

\[
T(n', 0) = V \cdot n' - c + \frac{1}{2}(2c) = Vn'
\]

to avoid litigation\(^{16}\).

We adopt the following notation: \( n \) are the number of patents originally invented by firm A, \( n' \) are the number of patents sold by firm A to the PAE. Analogously, \( m \) are the number of patents originally invented by firm B, \( m' \) are the number of patents sold by firm B to the PAE. The values \( n' \) and \( m' \) will be endogenously determined in equilibrium, and we present its full characterization in section 5.1.

Licensing and litigation payoffs when both firms enter

Consider a fixed allocation of patents \((n, m, n', m')\) and suppose that both firms have entered the final product market. We denote by \( p_A(n') \) the price paid by the PAE for the \( n' \) patents

\(^{16}\)Note that the PAE earns its payoff in the event negotiations fail \((Vn' - c)\) plus half of the gains from trade \(\frac{1}{2} \cdot (2c)\).
acquired from firm A, and we define $p_B(m')$ analogously.\footnote{To be rigorous we should write $p_A(n,m,m',n',s)$, but we omit the rest of the arguments for the sake of exposition.}

Then, using the same notation as in section 4, we obtain the following payoffs from licensing and litigation:

$$
\pi_A = \pi + T(n-n', m-m') - T(m', 0) + p_A(n'), \quad \pi_B = \pi - T(n-n', m-m') - T(n', 0) + p_B(m'), \\
\pi_{PAE} = T(n', 0) + T(m', 0) - p_A(n') - p_B(m').
$$

**Licensing and litigation when only one firm enters**

Suppose that only firm A enters the final product market. In this case, firm B effectively acts as a PAE when monetizing its patents. The payoffs from licensing and litigation in this case are:

$$
\pi_A = \pi_m - T(m, 0) - T(m', 0) + p_A(n'), \quad \pi_B = T(m, 0) + p_B(m'), \quad \pi_{PAE} = T(m', 0) - p_A(n') - p_B(m').
$$

In the next section we solve the problem of finding the equilibrium values of $n'$ and $m'$, which arise as the solution of a bilateral bargaining game between the operating firms and the PAE.

### 5.1 Patent Acquisition by the PAE

We adopt the simultaneous and symmetric Nash bargaining approach of Horn and Wolinsky (1988). The PAE simultaneously bargains with firms A and B over the outcomes. An outcome corresponds to the set of patents acquired by the PAE from the producing firms and the prices at which they were bought. Let $S_{PAE}(n', m')$, $S_A(n', m')$, and $S_B(n', m')$ be the payoffs from licensing in the shadow of litigation for the PAE, firm A, and firm B, respectively, after the PAE acquires $n'$ patents from A and $m'$ from B, at prices $p_A(n')$ and $p_B(m')$, respectively. The result of each negotiation is the solution to Nash Bargaining, given the deal reached between the PAE and the other producing firm. In this section we do not assume symmetric bargaining power when firms negotiate patent sales with the PAE. The bargaining power of the producing firms is $s$ and is $(1-s)$ for the PAE, with $s \in [0, 1]$.

For a given profile of initial patent portfolios $(n, m)$, with $n + m = N$, we find the equilibrium of this bargaining game between the producing firms and the PAE. We focus on the case $n \geq m$ since the other case is symmetric.
More formally, given that the producing firms have bargaining power $s$, the equilibrium in the bargaining game is one in which, taking $m'$ as given, firm A and the PAE bargain over their outcome à la Nash

$$ (n', p_A) \in \max_{z, p} \left( S_{PAE}(z, m') - p - S_{PAE}(0, m') \right)^{1-s} \left( S_A(z, m') + p - S_A(0, m') \right)^s. \quad (1) $$

Taking $n'$ as given, firm B and the PAE bargain over their outcome

$$ (m', p_B) \in \max_{z, p} \left( S_{PAE}(n', z) - p - S_{PAE}(n', 0) \right)^{1-s} \left( S_B(n', z) + p - S_B(n', 0) \right)^s. \quad (2) $$

We denote by $J_{A,PAE}(z, m')$ the joint surplus of firm A and the PAE from the litigation stage, when firm A transfers $z$ patents to the PAE and firm B has sold $m'$ patents to the PAE. We define $J_{B,PAE}(z, m')$ analogously for firm B and the PAE. A standard result in bargaining games with lump sum payments is the following:

**Lemma 2.** The outcome of the bilateral negotiation between an operating firm and the PAE maximizes their joint surplus, for a fixed deal between the rival firm and the PAE.

Bilaterally, an operating firm and the PAE trade patents to maximize their joint surplus. Once the allocation of patents maximizes the joint surplus, a monetary transfer splits the surplus between the parties according to their bargain power. Thus, to find the number of patents traded between producing firms and the PAE, we need to examine the allocations that simultaneously maximize the joint surplus of each pair.

**Proposition 1.** It is an equilibrium for each firm to sell its whole portfolio to the PAE in the bargaining game described above. In that equilibrium, the PAE extracts no rents from the producing firms, i.e., $\pi_{PAE} = 0$.

**Proof.** When a producing firm sells its entire portfolio to the PAE, the other producing firm and the PAE achieve the same joint surplus at any allocation that monetizes all patents (when possible). Suppose without loss of generality that firm A sold its entire portfolio to the PAE in their bilateral negotiation (that is, $n' = n$), and consider the negotiation between firm B and the PAE.

Firm B, on its own, can use its entire portfolio and get $T(m, 0)$ from firm A, because firm A has no patents to use defensively. The PAE will use the patents acquired from firm A against firm B, to obtain $T(n, 0)$ in licenses. Hence, firm B’s outside option is $S_B(n, 0) = T(m, 0) - T(n, 0)$ and the PAE’s outside option (from the bilateral negotiation with firm B) is $S_{PAE}(n, 0) = T(n, 0)$.
The joint surplus between firm B and the PAE without agreement is $J_{B,PAE}(n,0) = T(m,0)$. Any number $z > 0$ of patents allocated from firm B to the PAE must generate a weakly lower surplus $J_{B,PAE}(n,z) \leq T(m,0)$, and it attains this level when $z = m$. Therefore, an outcome of the bargain process between firm B and the PAE is $z^* = m$ and $p(m) = T(m,0)$.

Thus, whenever firm A has sold everything to the PAE, the joint surplus of firm B and the PAE is maximized by selling all of B’s patents to the PAE. Analogously, when firm B is selling all its patents to the PAE, an outcome of the bilateral negotiation between firm A and the PAE is to have firm A sell all its portfolio to the PAE at price $Vn$. In each case, the PAE pays each firm an amount exactly equal to its licensing revenue from the acquired patents, and so earns no profit.

Proposition 1 shows one equilibrium outcome of the bargaining game in which both producing firms are selling their entire portfolio to the PAE. The PAE will monetize the patents, but it will not extract rents from the producing firms. The equilibrium payoffs will be:

$$
\pi_A = \pi + T(n,0) - T(m,0), \quad \pi_B = \pi - T(n,0) + T(m,0), \quad \pi_{PAE} = 0. \quad (3)
$$

To understand this result, observe that when firm A sells all of its portfolio to the PAE it loses the ability to counter-sue. This implies that the maximum surplus that firm B and the PAE can achieve jointly equals what firm B can achieve on its own. The PAE does not offer any strategic advantage to firm B, and therefore it does not increase their joint surplus, and firm B is indifferent between selling or not.

Depending on the size of the portfolios there are cases in which the bargaining game has multiple equilibria. The following lemmas are used to characterize the multiple equilibria. We first study the case in which firm B has enough patents to be monetized by the PAE.

**Lemma 3.** Suppose $n > m > \hat{c}$. In any equilibrium of the bargaining game firm B sells all of its patents to the PAE.

**Proof.** Suppose by contradiction there is an equilibrium in which firm B sells $m' < m$ and retains $k = m - m' > 0$. Given $m'$, the strategy that maximizes the joint surplus between firm A and the PAE is for firm A to retain $\ell = \max\{0, k - \hat{c}\}$ and for the PAE to acquire $n - \ell$ patents from firm A. To show this claim, we analyze two cases: $k \leq \hat{c}$ and $k > \hat{c}$.

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18 When $z > 0$, the joint surplus between firm B and the PAE is strictly less than $T(m,0)$ when $T(m,0) = Vm$, and $m - z < \hat{c}$ or $z < \hat{c}$, which means, respectively, that firm B or the PAE cannot credible sue firm A.
When \( k \leq \hat{c} \), firm A does not face a direct litigation threat from firm B. But if firm A were to sue firm B, those \( k \) patents would be used defensively. We distinguish the cases \( m' > \hat{c} \) and \( m' \leq \hat{c} \). When \( m' > \hat{c} \), without an agreement between firm A and the PAE, the PAE gets \( Vm' \) from threatening to sue firm A with the patents acquired from firm B, and firm A gets \( T(n, k) \) from firm B. Thus, the bilateral joint surplus without agreement is \( J_{A,PAE}(0, m') = Vm' + T(n, k) - Vm' \). When \( m' \leq \hat{c} \), without an agreement between firm A and the PAE, firm A gets \( T(n, k) \) by threatening to sue firm B and the PAE gets 0. Thus, for any value of \( m' \), the bilateral joint surplus between firm A and the PAE without agreement is \( J_{A,PAE}(0, m') = T(n, k) \).

When \( k > \hat{c} \) and firm A sells everything to the PAE, firm A leaves itself vulnerable to firm B’s litigation threat. However, firm A can “cancel out” this litigation threat by holding on to some patents. The minimum number of retained patents that are sufficient to deter firm B from litigation is \( \ell = k - \hat{c} \). Again, we distinguish the cases \( m' > \hat{c} \) and \( m' \leq \hat{c} \). When \( m' > \hat{c} \), without an agreement between firm A and the PAE, the PAE gets \( Vm' \) from firm A, using the patents acquired from firm B, and firm A gets \( T(n, k) \) from firm B. The bilateral joint surplus without agreement is \( J_{A,PAE}(0, m') = T(n, k) \). When \( m' \leq \hat{c} \), without an agreement the PAE gets 0 and firm A gets \( T(n, k) \). Therefore, for any value of \( m' \), firm A and the PAE get \( T(n, k) \) as joint surplus without an agreement. Consider firm A keeping \( k - \hat{c} \) patents and selling \( n' = n - k + \hat{c} \) to the PAE. Notice that \( n > m \geq k \) implies that \( n' > \hat{c} \), so the PAE can credibly monetize the patents acquired from firm A. By keeping \( k - \hat{c} \) patents, firm A effectively deters firm B from starting a lawsuit. Thus, the joint surplus between firm A and the PAE in this case is \( J_{A,PAE}(n', m') = Vn' \) which is strictly larger than \( J_{A,PAE}(0, m') \). In fact, this is the largest joint surplus that firm A and the PAE can achieve, because selling more than \( n' \) would imply that firm B has a credible threat against firm A (which lowers the bilateral joint surplus), and selling less than \( n' \) would imply that the PAE extracts less surplus from firm B.

We have shown that firm A and the PAE best respond to \( m' < m \) by playing the strategy \( n'(m') = n - \max\{0, k - \hat{c}\} \). But in this case, firm B and the PAE do not maximize their joint surplus at \( m' < m \), since \( J_{B,PAE}(n'(m'), m') = T(m', 0) < Vm \). By selling everything to the PAE, firm B and the PAE can guarantee a larger joint surplus of \( Vm \). Therefore, selling \( m' < m \) cannot be an equilibrium.
Intuitively, when firm B holds on to some patents, firm A and the PAE have a strategy that prevents firm B from monetizing those patents. When firm A and the PAE are playing this strategy, firm B and the PAE are losing value on the patents held by firm B, since the PAE could monetize them.

Hence, when \( n > m > \hat{c} \) any equilibrium must have firm B selling everything to the PAE. We now study the case in which firm B’s portfolio cannot be monetized by the PAE.

**Lemma 4.** When \( m \leq \hat{c} \), in every equilibrium of the bargaining game firm A sells all of its patents to the PAE.

*Proof.* Suppose there is an equilibrium in which firm A sells \( n' < n \) and retains \( \ell = n - n' > 0 \). If \( \ell \leq \hat{c} \) firm A is not monetizing those patents (firm B cannot sue firm A) so firm A and the PAE could increase their joint surplus by transferring all of those patents to the PAE. Suppose, instead, that \( \ell > \hat{c} \). Since the PAE cannot credibly sue firm A using firm B’s patents, in this case, firm B and the PAE best respond having firm B retain all of its patents and counter-suing firm A. But this cannot be an equilibrium, because when firm B retains all of its patents, firm A and the PAE maximize their joint surplus by allocating all the patents to the PAE and letting it monetize them. \( \square \)

If firm B does not have enough patents to be monetized by the PAE, firm A is “safe” selling all of its patents to the PAE. This in turn avoids counterclaims that would be brought by firm B had firm A sued directly.

Lemmas 3 and 4 characterize the unique equilibrium behavior of one of the firms in the game. The multiplicity arises from the different strategies that the other firm can play. In Proposition 2 we characterize all the equilibrium payoffs.

**Proposition 2.** The equilibrium payoffs of the game are:

1. When \( n = m \) there are multiple equilibria, and in all these equilibria the payoffs are:

   \[ \pi_A = \pi, \quad \pi_B = \pi, \quad \pi_{PAE} = 0. \]

2. When \( n > m > \hat{c} \), firm B sells its entire portfolio. The equilibrium payoffs of any equilibrium are:

   \[ \pi_A = \pi + V(n - m), \quad \pi_B = \pi - V(n - m), \quad \pi_{PAE} = 0. \]
3. When \( m \leq \hat{c} \) Firm A sells its entire portfolio to the PAE and Firm B is indifferent between selling any amount \( m' \in [0, m] \). The equilibrium payoffs depend on how many patents firm B is selling, as they change the outside option in the bilateral bargaining of firm A and the PAE.

\[
\pi_A = \pi + T(n, m-m') + s[Vn-T(n, m-m')], \quad \pi_B = \pi - Vn, \quad \pi_{PAE} = (1-s)[Vn-T(n, m-m')].
\]

**Proof.** In this proof we also find all the equilibria of the game.

1. When \( n = m \leq \hat{c} \), any allocation is an equilibrium, since patents cannot be monetized. When \( n = m > \hat{c} \), any equilibrium features firms either holding on to all their patents on selling them all. Suppose in equilibrium firm A kept \( 0 < \ell < n \) patents. Firm B can “cancel out” this threat by keeping \( k = \ell - \hat{c} \) patents, and the PAE will be able to monetize the rest as long as \( n > \ell \). Thus, firm A will always lose the value of those \( \ell \) patents, unless \( \ell = n \). Therefore, in equilibrium firms either sell all or keep all.

\[
\pi_A = \pi, \quad \pi_B = \pi, \quad \pi_{PAE} = 0.
\]

2. When \( m > \hat{c} \), by lemma 3, any equilibrium features firm B selling everything to the PAE.

Consider an equilibrium where firm A keeps \( \ell > 0 \) patents. Because this is an equilibrium, firm A and the PAE must get the highest joint surplus with this allocation of patents. If firm A kept all of its patents, the joint surplus from the threat of litigation of firm A and the PAE would equal \( Vn \). Any other equilibrium allocation must have all the patents being monetized to achieve this maximal joint surplus \( Vn \). This is the maximal joint surplus, because firm B has sold all its patents to the PAE. Therefore, for any equilibrium with \( \ell > 0 \) we must have \( \ell \in (\hat{c}, n - \hat{c}) \). This implies that when firm A deals with the PAE there is no increase in joint surplus between firm A and the PAE. Thus, the PAE extracts no rents from firm A.

In addition, firm B and the PAE must maximize their joint surplus when firm B sells all of its patents to the PAE. Firm B and the PAE could increase their joint surplus by “cancelling out” firm A’s litigation threat. The minimum number of patents that firm B must keep to prevent firm A from starting a lawsuit is \( k = \ell - \hat{c} \). If the PAE can still monetize the patents bought from firm B, that is \( m - k > \hat{c} \), and \( k < m \), then firm B and the PAE would have a profitable deviation since by cancelling out firm A’s
threat, firm B and the PAE can increase their joint surplus by $V\hat{c}$. But this cannot be an equilibrium, since by lemma (3) firm B sells all its patents in equilibrium. Hence, either firm B does not have enough patents to cancel firm A’s lawsuit ($k > m$) or, by keeping some patents, the remaining patents cannot be monetized by the PAE or firm B ($m - k < \hat{c}$). Suppose firm B has enough to “cancel out” firm A’s litigation threat, but when doing it the remaining patents cannot be monetized by the PAE, which is equivalent to $m < \ell < m + \hat{c}$. In this case, by keeping all of its portfolio, firm B and the PAE have a joint bilateral surplus of $-V(n - \ell)$ which is a profitable deviation. Thus, in equilibrium it must be the case that firm B does not have enough patents to “cancel out” firm A’s litigation threat. That is, $\ell > m + \hat{c}$. When firm A keeps this large amount of patents, firm B cannot avoid the litigation threat by holding on to some patents, and therefore firm B’s outside option is the same as if firm A sold all of its patents to the PAE.

Therefore, for an equilibrium with $\ell > 0$ to exist, it must be the case that $m + \hat{c} < \ell < n - \hat{c}$. In this equilibrium, firm B and the PAE’s joint surplus is $-V(n - m)$, and the PAE does not increase their joint surplus either, so extracts no rents from firm B.

This implies that the PAE extracts no surplus in this case.

3. When $m \leq \hat{c}$ and firm B keeps $k = m - m'$ the outside option of firm A is to use its portfolio to litigate when possible, which only happens when $n - k > \hat{c}$. In this case, firm A obtains $T(n, k)$. By selling all its patents to the PAE, firm A avoids the counterclaims brought by firm B using the portfolio it withheld. Thus, the PAE monetizes all firm A’s patents and firm A does not faces a threat of litigation from firm B. Thus, the joint surplus between firm A and the PAE is in this case $Vn$. The extra surplus from selling everything to the PAE is given by $Vn - T(n, k)$, which is split according to firm A’s bargain power $s$.

Our results show the difference in the continuation payoff between the game with PAEs and without PAEs (section 4). In the figure below, we summarize our main findings, for the case $n > m$, which is the effect of the PAE on the equilibrium continuation payoffs. From the figure

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19This is because by holding on to $k$ patents firm B has to pay $V(n - \ell)$ to the PAE, as the PAE monetizes the patents bought from firm A, and also the PAE gets $V(m - k)$ from monetizing the patents bought from firm B. When firm B sells everything to the PAE, the joint surplus is $-V(n - m)$ as firm B gets $-Vn$ from the litigation against firm A and the PAE, and the PAE monetizes firm B patents getting $Vm$. 

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it is easy to see that the PAE affects the firms’ continuation payoffs only in three regions. The case \( n < m \) is symmetric by changing the roles of firms A and B, and when \( n = m \) the PAE does not change the payoffs.

![Figure 4: Changes produced by the PAE in the firms’ continuation payoffs for a given patent portfolio \((n, m)\), with \( n > m \). The payoffs are symmetric for the case \( n < m \). The changes in payoffs occur only in three regions, and the PAE does not change the payoffs when \( n = m \).](image)

1. **Region (I):** When \(|n - m| \leq \hat{c}, m \leq \hat{c}\), and the PAE did not exist, firms were not suing each other. Even when \( n > \hat{c} \), firm A did not have the ability to monetize its patents, because of the fear of retaliation from firm B. If the PAE exists, it allows firm A to monetize its patents (when \( n > \hat{c} \)) by “cancelling out” firm B’s portfolio. This strategic advantage offered by the PAE comes from its ability to avoid countersuing. Notice that although firm B cannot monetize its patents, its decision of how many patents to keep has an impact on the equilibrium payoffs, as they determine the outside option of firm A in the bilateral bargain with the PAE.

   The effect of the PAE on equilibrium payoffs is to increase firm A’s payoff from 0 to \( sVn + (1 - s)T(n, m - m’) \), and decrease firm B’s payoffs by \( Vn \).

2. **Region (II):** When \(|n - m| > \hat{c}, m \leq \hat{c}\), and although firm A had a credible litigation threat without the PAE, the PAE can offer to “cancel out” firm B’s portfolio, which increases firm A’s surplus and the PAE is able to extract rents. The effect of the PAE on equilibrium payoffs is to increase firm A’s payoff from \( V(n - m) \) to \( V(n - m) + sVm + (1 - s)Vm’ \), and decrease firm B’s payoffs by \( Vm \).

3. **Region (III):** When \(|n - m| \leq \hat{c}, n > m > \hat{c}\), without PAEs firms did not have a credible litigation threat. However, when the PAE acquires all the patents it has two individually rational lawsuits \( n > \hat{c} \) and \( m > \hat{c} \), against firm A and B, respectively. In
this case, patent monetization generates a positive total surplus of $V(n - m)$. However, although aggregate surplus increases, firm A gets all of it and firm B loses all of it. Therefore, comparing to the case of no PAEs, the PAE increases firm A’s continuation payoff by $V(n - m)$, which equals the decrease in the continuation payoff for firm B. The PAE is not able to extract surplus in this case, because in equilibrium the "weak" player (the firm with fewer patents) sells everything to the PAE. This implies that firm A’s outside option is to monetize all its patents without a threat of countersuing, which is the same benefit that the PAE can offer.

4. Other cases: When $|n - m| > \hat{c}$, $n > \hat{c}$, and $m > \hat{c}$ firms A and B were already monetizing all their patents, and therefore the PAE does not offer any strategic advantages. And finally when $n \leq \hat{c}$, and $m \leq \hat{c}$ no firm has enough patents to start a lawsuit. Hence, the PAE does not affect the continuation payoffs.

5.2 Patent Trade before Bargaining with the PAE

In this section we study the patent trade among operating companies, before they engage in bilateral trading with the PAE. In particular, we can think of firms cross-licensing their portfolios rather than trading patents. If, for some reason, firms are unable to bargain for licenses before the bilateral negotiation with the PAE, for some patent allocations the PAE will be able to extract rents in equilibrium. In Appendix E we analyze this important case as an extension. Qualitatively, the results are similar to what we obtain by allowing firms bargaining before they sell to the PAE (avoiding the loss in surplus). The main difference is that, if firms do not bargain before the negotiation with the PAE, then PAE is able to obtain rents in equilibrium. This rent extraction is an important reason why PAEs may hinder ex-ante R&D incentives. We show in Appendix E that, even in this case, PAEs can increase incentives to invest in R&D.

When both firms own more than $\hat{c}$ patents after the R&D stage, the PAE does not extract rents from the bilateral negotiations. In those cases, firms are indifferent between bargaining licenses among them or trading patent with the PAE in bilateral negotiation, because the PAE does not reduce the joint surplus of the producing firms. In other words, when the continuation game has an efficient equilibrium, firms are indifferent between trading patents

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20Similarly to the case without PAEs, firms can either buy licenses or transfer patents. If firms trade patents, they will engage in licensing at the next stage. Since the equilibrium is efficient, we can assume they buy licenses right away.
before or after engaging with the PAE.

In patent allocations such that one of the firms owns fewer than \( \hat{c} \) patents and the other firm has more than \( \hat{c} \) patents after the R&D stage, firms will lose surplus if they do not buy licenses before bargaining with the PAE. In this case, Proposition 2 shows that the PAE is able to extract
\[
(1 - s)\left[ V_n - T(n, m - m') \right]
\]
from the bilateral negotiation, where \( m' \) is the number of patents sold by firm B to the PAE.

By negotiating, firms can avoid the surplus extraction by the PAE. Firm will bargaining over licenses under the threat of bilaterally bargaining with the PAE, if the negotiation fails. We assume without loss of generality that \( n > \hat{c} \geq m \) patents. In the bilateral negotiation with the PAE, firm A splits surplus with the PAE, which depends on the number of patents that firm B sold to the PAE. Therefore, it is optimal for firm B to threaten to sell nothing to the PAE if the licensing negotiation with firm A fails. This strategy maximizes the surplus that is bargained over between firm A and firm B. Using the results in Proposition 2, we find that firms cross-license and the payoffs are given by the Nash bargaining solution:

\[
\pi_A = \pi + T(n, m) + s[V_n - T(n, m)] + b(1-s)[V_n - T(n, m)], \quad \pi_B = \pi - V_n + (1-b)(1-s)[V_n - T(n, m)].
\]

Defining \( \mu = s(1-b) + b \) the payoffs can be written as:

\[
\pi_A = \pi + T(n, m) + \mu[V_n - T(n, m)], \quad \pi_B = \pi - V_n + (1-\mu)[V_n - T(n, m)].
\]

Combining these payoffs with those in Proposition 2 for the rest of the patent allocations, we can find the continuation payoff of the firms after the R&D stage with PAEs.

**Proposition 3.** When firms bargain over licenses under the threat of bilateral trade with the PAE, the equilibrium payoffs for the firm with the largest portfolio are:

\[
\pi_A = \pi + \begin{cases} 
V(n - m) & n > \hat{c} \text{ and } m > \hat{c} \\
T(n, m) + \mu[V_n - T(n, m)] & n > \hat{c} \text{ and } m \leq \hat{c} \\
-Vm + (1-\mu)[Vm - T(m, n)] & n \leq \hat{c} \text{ and } m > \hat{c} \\
0 & n \leq \hat{c} \text{ and } m \leq \hat{c}
\end{cases}
\]

In particular, in the case \( N > 3\hat{c} \), we have that \( T(n, m) = V(n - m) \) for any \( m \leq \hat{c} \), and the
following payoffs:

\[
U_{PAE}^A(n) \equiv \pi_A(n, N-n) = \pi + \begin{cases} 
(2 - \mu)Vn - VN & n \leq \hat{c} \\
V(2n - N) & n > \hat{c} \text{ and } m > \hat{c} \\
(2 - \mu)Vn - (1 - \mu)VN & n \geq N - \hat{c}
\end{cases}
\]

Figure 5 depicts the continuation payoff for firm A in the case \( N > 3\hat{c} \) below.

**Figure 5:** Consider a producing firm which discovers \( n \) components, while its rival discovers \( N - n \), and \( N > 3\hat{c} \). The firm’s payoff from the equilibrium with a PAE (bolded) is superimposed on its payoff from the equilibrium without a PAE. Notice that \( \mu = s(1-b)+b \leq 1 \).

This figure shows the effects of the PAE on continuation payoffs, compared to the case without PAEs. First, the PAE allows firms to overcome transaction costs, enabling patent monetization for cases in which, without PAEs, firms do not have a credible litigation threat. This occurs, for example, when \( |n - m| \leq \hat{c} \), \( n > \hat{c} \), and \( m > \hat{c} \). In Figure 5, we can see this effect in the middle region. Without PAEs payoffs are flat, because transaction costs prevent firms from monetizing their portfolios. Since PAEs allow for monetization, incentives are increased, reflected in a steeper slope. Second, when patent portfolios are very different, PAEs increase the payoff of the firm with more patents, by preventing firms from using their patents defensively. In Figure 5, we can see this effect in the regions \( n \in [0, \hat{c}] \) and \( n \in [N - \hat{c}, N] \). Consider first the case where firm A has portfolio of size \( n \) such that \( n - 1 < \hat{c} \). In the game without PAEs, by obtaining one more patent firm A gains for two reasons: One more patent
to use defensively, and one less patent that firm B could use offensively. Thus, the marginal value of that extra patent was $2V$. With PAEs, the patent cannot be used defensively, so the only gain from having one more patent is to reduce by one the number of patents of firm B. Hence, the marginal value of that patent is now $V$. When firm A has $N - \hat{c} \leq n < N$ patents the argument is reversed. Without PAEs getting one more patent allowed firm A to use one more patent offensively and firm B one less defensively. Thus, the value of an extra patent was $2V$. With PAEs, the defensive value of patents is already taken into account. Therefore, one more patent allows firm A to gain only the offensive value of that patent, which equals $V$. Finally, firms will split the surplus that the PAE will extract if their negotiation fails, and that implies the slope at the extremes are $2 - \mu$. The parameter $\mu = s + b(1 - s)$ measures how the surplus that the PAE can potentially extract from the firms is divided between firm A and firm B. Incentives are decreasing in $\mu$, as the slopes at the extremes decrease. If firms cannot trade patents before the PAE arrives, but in the continuation equilibrium the PAE does not extract rents, firms are indifferent between negotiating with or without the PAE.

5.3 Entry decisions

Analogously to the case without PAEs, as long as duopoly profits are large enough, it is a dominant strategy for both firms to enter the market. We assume in our model that the PAE approaches the firms after they enter the market. If we switch the timing, allowing the PAE to approach firms before they enter the market, the results are the same as long as $\pi$ is large enough. Results could be modified only if trading with the PAE before entering the market affected the entry decision of firms.

5.4 The returns to R&D with PAEs

In this section we summarize the subgame equilibrium: Both firms enter and bargain licenses anticipating the bilateral bargaining with the PAE if no deal is reached. Proposition 2 presents all the equilibria and continuation payoffs when firms do not reach a licensing agreement, and they bilaterally trade patents with the PAE. Proposition 3 characterizes all the equilibria when firms can bargain before trading with the PAE, avoiding the loss in surplus.

In Figure 5 we saw the effect on incentives induced by the presence of PAEs. When firms have patent portfolios of similar size, the PAE allows for more monetization overcoming transaction costs, reflected in the straight line in the middle region, compared to the flat line in the case
without PAEs. We can also see that PAEs reduce incentives at the extremes. A firm with a small portfolio will be unable to use it defensively in presence of PAEs. At the same time, the firm with the largest portfolio also loses incentives when there is no countersuing, because without PAEs discovering one more component implies that the rival firm has one fewer patent to use defensively.

The case \( N \in [2\hat{c}, 3\hat{c}) \) is qualitatively similar to \( N > 3\hat{c} \). The other important case is, \( N \in [\hat{c}, 2\hat{c}) \). In this case, the PAE is ineffective to monetize patents when both firms individually have fewer than \( \hat{c} \) patents, however, it can still help monetization when one firm has more than \( \hat{c} \) patents. In the latter case, the PAE still shields the firm with the largest portfolio from litigation. The details for these two cases are in Appendix B. For the remaining sections, we focus our analysis on the case \( N > 3\hat{c} \).

In the next section, taking the continuation payoffs as given, we study the decision problem of how much to invest in R&D to determine (stochastically) the number of patents discovered by each firm and the moment at which firms start producing the final product.

## 6 Endogenous R&D investments

In this section we study the optimal R&D investments when firms rationally anticipate the subsequent payoffs from entry, trade with the PAE, licensing and litigation. After each firm makes its investments, discoveries arrive stochastically and are patented, and once all \( N \) components are discovered, firms play their optimal strategies in the entry, licensing, and litigation stages of the game. We focus on the case where \( \pi > NV > 3\hat{c} \), which guarantees that Lemma \( 1 \) holds, and also the payoffs with PAEs are the ones depicted in figure 5. The entry condition in this figure corresponds to \( U^{PAE}(0) = \pi - NV > 0 \).

Before investing in R&D firms anticipate the continuation payoffs, which depend on the number of components discovered by each firm. When PAEs do not exist, we denote by \( U(k) \) the continuation payoff of a firm that discovers (and patents) \( k \) out of \( N \) components (see figure 3), while its rival discovers the remaining \( N - k \) components. When PAEs are present, we denote the continuation payoff by \( U^{PAE}(k) \) (see figure 5).

Firms simultaneously make R&D investments to patent the \( N \) components, anticipating the continuation payoffs. We denote by \( x \) and \( y \) the R&D investments by firm \( A \) and firm \( B \), respectively. The cost of \( z \) units of R&D is the same for both firms, given by \( c_I(z) \). Firm \( A \)
discovers any one particular component independently with probability \( p(x, y) = \frac{h(x)}{h(x) + h(y)} \), which implies a binomial distribution for the total number of discovered components.

\[
P(k; x, y) = \binom{N}{k} p(x, y)^k (1 - p(x, y))^{N-k}.
\]

R&D investments not only determine the distribution of patents, but also the time at which production begins. Since only the first version of a component is patentable, the time at which component \( i \) is available is given by \( \tau_i(x, y) = \min\{ \tau(x), \tau(y) \} \), where \( \tau(x) \) and \( \tau(y) \) are random arrival times for firm A and B, respectively, whose distributions depend on the R&D investments. Production can take place only when every component has been discovered, since firms either invent and get a patent, or imitate. The time at which firms will enter the market and produce is given by \( \tau(x, y) = \max_{i=1,\ldots,N} \{ \tau_i(x, y) \} \), distributed according to \( F(\tau; x, y) \). Firms discount profits at rate \( r \).

### 6.1 The investment problem without PAEs

Each firm chooses its R&D investment to maximize its expected payoff, given the investment level chosen by its rival. Since firms are symmetric, we focus on firm A’s problem, which is:

\[
\max_{x \geq 0} \mathbb{E}_{\tau,k}[e^{-r\tau} U(k)|x,y] - c_I(x).
\]

**Lemma 5.** The random variables \( k \) and \( \tau \) are independent for all \( x \) and \( y \).

**Proof.** Consider random variables \( \tau_i^A(x) \sim \exp(x) \), \( i = 1, \ldots, N \) and \( \tau_i^B(y) \sim \exp(y) \), for \( i = 1, \ldots, N \) and assume they are all independent. Define the random variables

\[
k = \sum_{i=1}^{N} 1(\tau_i^A < \tau_i^B), \text{ and } \tau = \max\{\min\{\tau_i^A, \tau_i^B\}\}.
\]

Define also \( Z_i(x, y) = \min\{\tau_i^A(x), \tau_i^B(y)\} \) and \( W_i(x, y) = \tau_i^A(x) - \tau_i^B(y) \). A property of the exponential distribution is that \( Z_i \) and \( W_i \) are independent (see for example [Ferguson (1964)]). Therefore, the vectors of random variables \( Z = (Z_1, \ldots, Z_N) \) and \( W = (W_1, \ldots, W_N) \) are independent. Since \( k \) and \( \tau \) are measurable functions of \( Z \) and \( W \) they are independent. \( \square \)

Lemma 5 implies that \( e^{-r\tau} \) and \( U(k) \) are independent, so we can write:

\[
\max_{x \geq 0} \mathbb{E}_{\tau}[e^{-r\tau}|x,y] \cdot \mathbb{E}_k[U(k)|x,y] - c_I(x).
\]
Let $G(x, y)$ be the expected discount rate, and $\Pi(x, y)$ the expected continuation payoff,

$$
G(x, y) = \int_0^\infty e^{-rt}F(\tau; x, y), \quad \Pi(x, y) = \sum_{k=0}^N \binom{N}{k} p(x, y)^k (1 - p(x, y))^{N-k} U(k).
$$

In Appendix B, we derive an explicit formula for $G(x, y)$ and show its properties. Note that $\Pi(x, x) = \pi$ because, given symmetric R&D investments, firms expect symmetrically each portfolio allocation, expected licensing transfers for each firm net to zero, and the expected reward of entering the market is $\pi$. Thus, the R&D decision problem can be written as:

$$
\max_{x \geq 0} G(x, y)\Pi(x, y) - c_I(x).
$$

This formulation is a generalized rent-seeking contest, because firms derive utility for a bundle of objects and their investments also determine the “size of the pie” through the discount factor. A standard rent-seeking contest usually is a competition for a single prize (see for example Corchón (2007)). Competition over multiple prizes is studied in, for example Clark and Riis (1998), although their setting differs from ours, because in our formulation firms care (non-linearly) about the bundle of components they obtain.

From figures (3) and (5) we can see that the continuation payoff is weakly increasing, non-linear, and not concave. Besides this difficulty, the expected discount factor for $N > 1$ is not a concave function of the R&D investment (See Appendix B). As a consequence, the R&D decision problem is not generally well-behaved (in particular, not pseudo-concave) which does not allow us to use standard results for existence and comparative statics for symmetric games. However, under a stability condition similar to the one in Lee and Wilde (1980), we can show the problem has a unique interior solution $x^* > 0$ that solves:

$$
G_x(x^*, x^*)\pi + G(x^*, x^*)\Pi_x(x^*, x^*) = c'(x^*).
$$

The first order condition characterizes how the payoffs from the continuation game change the incentives to innovate. Investing one more unit of R&D has two effects. First, it brings the expected continuation payoff earlier. This effect is given by the term $G_x(x^*, x^*)\pi$. Because more R&D today brings this payoff sooner, it will be discounted at a smaller rate, captured by the marginal change in the expected discount rate, $G_x$. Second, as the firm with the largest portfolio can capture weakly more rents than $\pi$ through licenses in the continuation stage, firms race to be the firm that discovers more components. This is represented by the second

\footnote{As far as we know, the literature on “contests for bundles” has not been well developed yet.}
term $G(x^*, x^*)\Pi_x(x^*, x^*)$. The marginal gain $\Pi_x(x, y)$ is positive for all $x$ and $y$, by first order stochastic dominance. In Appendix A, we derive an explicit formula for the marginal rent seeking incentive:

$$\Pi_x(x, x) = \frac{h'(x)}{h(x)} \Psi(\hat{c}, V),$$

where $\Psi(\hat{c}, V)$ is decreasing in $\hat{c}$ and increasing in $V$.

Our analysis will restrict attention to parameter values such that a symmetric pure strategy equilibrium exists. In some cases, a symmetric pure strategy equilibrium with a positive level of investment might fail to exists. This happens when $\pi$ is too small or $r$ is too large, in which case there is not much of an incentive to invest in R&D since the expected discounted continuation profits are too small. For an extended discussion of existence and uniqueness of equilibrium, see Appendix D.

### 6.2 The investment problem with PAEs

When firms are allowed to trade patents with the PAE, the continuation payoffs are modified, as described in Proposition 3. We focus our analysis on the case $N > 3\hat{c}$, for which the continuation payoffs are shown in figure 5.

As discussed before, the PAE has two effects on equilibrium payoffs. First, when firms have patent portfolios of similar sizes, and one of the firms has more than $\hat{c}$ patents, PAEs enable patent monetization in cases where otherwise transaction costs would prevent it. Second, when patent portfolios are very different in size, PAEs give a premium to the firm with the largest portfolio. However, although the payoff of this firm increases, it does it in a “flatter” way compared to the case without PAEs. Similarly, the payoff for the firm with the smallest portfolio decreases, but also in a “flatter” way. The reason for this change is that PAEs prevent the firm with fewer patents from using them defensively, by countersuing the other firm. This reduces the incentives to obtain more patents, when having a very small portfolio, because those patents cannot be used defensively. For the firm with the largest portfolio, the incentive to obtain more patents is also hindered by the lack of counter-suing. In this case, the value of one more patent does not take into account its value in reducing by one the number of countersuing patents.

Given these effects, from an ex-ante perspective PAEs give more incentives to firms when

$$\Pi_x(x, y) = \frac{\partial U(p)}{\partial p} \frac{\partial p}{\partial x} = \frac{\partial U(p)}{\partial p} \kappa(x)p(1 - p).$$

By properties of the binomial distribution (FOSD), and using the fact that $U(k)$ is weakly increasing in $k$, we have $\frac{\partial U(p)}{\partial p} > 0$. 

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they anticipate they will end up with patent portfolios of similar sizes, while they dampen incentives when firms anticipate they will end up with very asymmetric patent portfolios.

Denote by $U^{PAE}(k)$ the expected continuation payoff of a firm that discovers $k$ components in presence of the PAE. We define $\Delta(k)$ as the difference in the payoffs from the continuation game with and without PAEs:

$$\Delta(k) = U^{PAE}(k) - U(k).$$

In figure 5, $\Delta(k)$ corresponds to the difference between the bolded line and the gray line. Specifically, for the case $N > 3\hat{c}$ we have:

$$\Delta(k) = \begin{cases} 
-\mu V k & k \in [0, \hat{c}] \\
0 & k \in [\hat{c}, \frac{N-\hat{c}}{2}] \, \text{or} \, \left[ \frac{N+\hat{c}}{2}, N - \hat{c} \right] \\
V(2k-N) & k \in \left[ \frac{N-\hat{c}}{2}, \frac{N+\hat{c}}{2} \right] \\
\mu V(N-k), & k \in [N - \hat{c}, N] 
\end{cases}$$

where $\mu = (1 - s)b + b$ is a parameter that depends on the the bargaining power of the PAE and the producing firms. The figure below illustrates $\Delta(k)$.

![Figure 6: Difference in continuation payoffs induced by the PAE, when $N > 3\hat{c}$.](image)

From Figure 6 we can see the two effects mentioned before. First, in terms of levels, the PAE increases the payoff of the “winner” and decreases that of the “loser’, which is formally given by $\Delta(k) \geq 0$ for $k \geq \frac{N}{2}$ and $\Delta(k) \leq 0$ for $k \leq \frac{N}{2}$. Second, in terms of marginal incentives (slopes), we can see that in the region where the firms used to play the truce equilibrium without PAEs, which is $\frac{N-\hat{c}}{2} \leq k \leq \frac{N+\hat{c}}{2}$, the PAE changes the payoffs strictly since $S'(k) > 0$. However, at the extremes, the effect of the PAE is negative since $S'(k) < 0$. We will show that in a symmetric equilibrium, since firms expect to end up closer to the middle region, the effect on incentives is always positive.
To analyze the effect of the PAE, we define $PAE(x, y)$ to be the expected discounted difference in a firm’s equilibrium payoffs with and without PAEs, given R&D investments $x$ and $y$.

$$PAE(x, y) = G(x, y) \sum_{k=0}^{N} P(k; x, y) \Delta(k).$$

Let $V^{Eq}(x, y)$ be the objective of firm A when it chooses its R&D investment in the absence of PAEs, for a given investment level $y$ of firm B,

$$V^{Eq}(x, y) = G(x, y) \Pi(x, y) - c_I(x).$$

In the presence of PAEs, given the R&D level $y$ of firm B, firm A solves:

$$\max_{x \geq 0} V^{Eq}(x, y) + PAE(x, y).$$

The effect of PAEs on incentives is given by the marginal expression:

$$PAE_x(x, y) = G_x(x, y) \sum_{k=0}^{N} P(k; x, y) \Delta(k) + G(x, y) \sum_{k=0}^{N} P_x(k; x, y) \Delta(k).$$

The first term, the *rent extraction effect*, reflects the marginal effect of R&D investments on the time at which firms obtain the change in levels of expected payoffs, due to the presence of the PAE. Figure 6 shows that the PAE weakly increases the payoff of a firm with more than half of the patents, and it weakly decreases the payoff otherwise. The investments $x$ and $y$ determine the expected number of patents after the R&D stage. Notice that $E[\Delta|x, y] > 0$ if and only if $x > y$. In other words, the rent extraction is positive whenever firm A expects to obtain more patents than firm B. The second term, the *winner premium effect* is the change in the marginal propensity of obtaining more patents, due to the presence of the PAE, i.e., the rent-seeking incentive.

**Proposition 4.** When $N > 3\hat{c}$, for symmetric R&D investments $x = y = x^*_PAE$ we have:

- The *rent extraction effect* (RE) is always zero.

- The *winner premium (WP) effect* is strictly positive and equal to

$$WP(x^*_PAE) \equiv G(x^*_PAE, x^*_PAE) \frac{h'(x^*_PAE)}{2N+1} \cdot \sum_{k=0}^{N} \binom{N}{k} (2k - N) \Delta(k) > 0.$$  

$^{23}$ When $x = y$, $E[\Delta|x, y] = 0$. This is because the PAE does not extract rents. In Appendix E we show that $E[\Delta|x, x] < 0$ when firms cannot avoid the rent extraction from the PAE.
• The marginal effect of PAEs is always positive.

The previous proposition characterizes the PAE effect in a symmetric equilibrium. First, it shows that the rent effect extraction is always zero in a symmetric equilibrium, which comes from the fact that firms expect to obtain half of the patents in equilibrium, and therefore the level of premium induced by the PAE is zero. In Appendix E, we show that this effect is negative when PAEs extract rents. Second, the proposition shows that the PAE has a positive impact on the rent-seeking incentive: despite the fact that the continuation payoff becomes “flatter” at extremely asymmetric patent positions, it becomes “steeper” in the middle (see figure 5). In a symmetric equilibrium, being in the middle region is more likely and this outweighs the potential negative effect on incentives at the extremes. Overall, the marginal effect of PAEs in a symmetric equilibrium is always positive.

Now that we understand the marginal effect of the PAE, we can study how the equilibrium R&D investments compare in the cases with and without the PAE. Under conditions of existence and uniqueness discussed in Appendix D, there exists a unique interior symmetric equilibrium without PAEs, $x^*$ such that

$$foc(x^*) \equiv G_x(x^*, x^*)\pi + G(x^*, x^*)\Pi_x(x^*, x^*) - c'_I(x^*) = 0.$$ 

If we incorporate the PAE, the condition changes to

$$foc(x^*_{\text{PAE}}) + PAE_x(x^*_{\text{PAE}}) = 0$$

We also show in Appendix D that around the symmetric equilibrium the game features strategic substitutes. For that to be true, we needed the condition $h(x) > \frac{r\ln N}{2}$.

**Proposition 5.** If a symmetric equilibrium with and without PAEs exists, and the equilibrium values are such that $h(x) > \frac{r\ln N}{2}$, then the equilibrium with PAEs is larger than the equilibrium without PAEs.

**Proof.** Let $x^*$ be the equilibrium without PAEs, so $foc(x^*) = 0$. By proposition 4 we know that $PAE_x(x) > 0$. Therefore, any $x^*_{\text{PAE}}$ such that $foc(x^*_{\text{PAE}}) + PAE_x(x^*_{\text{PAE}}) = 0$, we have that $foc(x^*_{\text{PAE}}) < 0 = foc(x^*)$. But in the region where $h(x) > \frac{r\ln N}{2}$ we showed that $foc(x)$ is strictly decreasing and therefore we must have $x^* < x^*_{\text{PAE}}$. 

Proposition 5 establishes that PAEs can increase the (symmetric) equilibrium level of R&D investment.
Figure 7: Effect of the PAE on equilibrium R&D investments. Solid lines correspond to the case without a PAE, while dotted lines correspond to the case with a PAE. The winner premium effect dominates the rent extraction effect at higher investment levels, which provides firms extra incentives to invest.

In the next section we explore when it is desirable to increase the level of R&D investment from a social welfare perspective.

7 Welfare Analysis

In this section we compare the symmetric equilibrium solution to what a planner would do if it could control the level of investment in each firm. Notice that with or without PAEs, the continuation game between the firms (and the PAE) is a zero-sum game with total industry profits $2\pi$. The planner does not care about the allocation of patents, as long as both firms enter, even if one of the firms owns all the patents. Hence, the first best solution, in which the planner controls the investment level of the firms and grants a license to every firm, coincides with the second best solution, in which the planner just controls the investment levels and once the patents are allocated among firms, they will bargain over licenses in the shadow of litigation, possibly through PAEs.
The social planner chooses investment levels $x$ and $y$ to maximize the total surplus generated by the commercialization of the final product. Let $W$ be the consumer surplus generated by discovering all the components and selling the final product. Then, the planner solves

$$\max_{x \geq 0, \ y \geq 0} (2\pi + W) G(x, y) - c_I(x) - c_I(y).$$

An interior solution therefore implies the conditions

$$(2\pi + W) G_x(x, y) = c'_I(x) \quad \text{and} \quad (2\pi + W) G_y(x, y) = c'_I(y).$$

By concavity of $h$ and convexity of $c_I$ any interior solution is symmetric. The symmetric solution for the planner’s problem is equivalent to the competitive firm best response when its rival invests zero, after relabeling the parameters values: $\pi \to 2\pi + W$ and $r \to \frac{\pi}{2}$. Therefore, under our assumptions in Appendix D, the planner solution $x_P > 0$ exists and is unique.

The symmetric equilibrium conditions can be written as:

$$G_x(x_P, x_P) \pi - c'_I(x_P) + G_x(x_P, x_P)(\pi + W) = 0 \quad \text{(Planner problem)}$$

$$G_x(x^*, x^*) \pi - c'_I(x^*) + G(x^*, x^*) \Pi_x(x^*, x^*) = 0 \quad \text{(Equilibrium)}$$

By comparing $G_x(x, x)(\pi + W)$ and $G(x, x) \Pi_x(x, x)$ we find when the planner invests more or less relative to the firm equilibrium.

The term $G_x(x, x)(\pi+W)$, which we call the planner incentive, represents the marginal benefit of higher investment that is internalized by the planner but not the firms. It corresponds to the marginal expected discounted consumer surplus and payoff of the rival firm. The term $G(x, x) \Pi_x(x, x)$, which we called competition incentive, represents the winner premium which is taken into account by firms, but not by the planner. These two effects misalign the incentives to invest between the planner and the firms. In the next lemma, we show how these two different components are ordered for different levels of R&D investments.

**Lemma 6.** The planner incentive is larger than the competition incentive if and only if $x > x_M$, where $x_M$ is defined as:

$$h(x_M) = \frac{2^{N-1}(\pi + W)r \ln(N)}{V \cdot \sum_{k:|2k-N|\geq \hat{c}} \binom{N}{k} (2k - N)^2}.$$

**Proof.** We show in Appendices A and D that

$$G_x(x, x) = G(x, x) \frac{r \ln(N)}{4h(x)^2} h'(x) \quad \text{and} \quad \Pi_x(x, x) = \frac{h'(x)}{h(x)} \frac{V}{2^{N+1}} \sum_{k:|2k-N|\geq \hat{c}} \binom{N}{k} (2k - N)^2.$$
Substituting these in and rearranging, we obtain the condition

\[(\pi + W)G_x(x, x) > G(x, x)\Pi_x(x, x) \iff h(x) < \frac{2^{N-1}(\pi + W)r \ln(N)}{V \cdot \sum_{k:|2k-N| \geq \hat{c}} (\frac{N}{k})(2k - N)^2} := h(x_M).\]

Lemma 6 shows that the planner incentive is larger than the competition incentive for levels of R&D investment smaller than \(x_M\). Moreover, the comparative statics of this cutoff point are straightforward: it increases with consumer welfare increases, duopoly profits, discount rate, \(\hat{c}\), and decreases with \(V\). The intuition for this result is simple. The planner incentive only depends on bringing the payoff earlier.

Using lemma 6 we can compare the symmetric equilibrium with the planner solution.

(1) **Under-investment:** If \(x^* < x_M\), then \(x^* < x_P\).

(2) **Over-investment:** If \(x^* > x_M\), then \(x^* > x_P\).

Firms will underinvest only when \(x_M\) is large. The main reason why \(x_M\) is large is because \(W\) is likely to be large to capture R&D spillovers and consumer welfare. Although measuring private versus social returns of R&D is not an easy task, Jones and Williams (1998) and Hall (1996) point out that private R&D investment is lower than the social optimal level of investment. Hence, it is likely \(x_M\) is large and firms are under-investing relative to the planner. In that case, the effect of the PAE attenuates the under-investment problem.

## 8 Conclusions

This paper provides a theoretical framework to understand the effect of Patent Assertion Entities (PAEs) on the incentives for litigation and innovation. In particular, we focus on the practice of “Patent Privateering”, which describes the outsourcing of patent monetization to PAEs by producing firms.

Our main contribution is to identify different channels through which PAE privateers can affect incentives for innovation. In our model, firms decide their level of R&D investment by looking forward to the continuation payoffs. PAEs change these payoffs by enhancing patent monetization, destroying the value of defensive patent portfolios, and, in some cases, extracting rents.
We find that, without PAEs, the fear of retaliation and the cost of litigation may preclude producing firms with similarly-sized patent portfolios from monetization. In these cases, firms remain in a tacit “IP truce” equilibrium, meaning that they do not enforce their patents even when they know their rival infringes on them. When firms decide to invest in R&D, the incentives to obtain more patents decrease if those extra patents will not be monetized. Therefore, transaction costs reduce the rent-seeking incentives of R&D.

With PAEs, firms anticipate better continuation payoffs if they come up ahead after the R&D stage. The strategic advantage of PAEs is that they cannot be counter-sued, implying that their litigation incentives are stronger than those of an operating firm holding the same patents. PAEs are able to disrupt the “IP truce” equilibrium, and they are able to create incentives on the margin for firms to invest more in R&D, in order to capture that value which would otherwise not have been realized. In fact, when firms have large portfolios of similar size, the payoffs they receive in the equilibrium with PAEs are identical to the payoffs they would receive in a world where litigation costs are zero. That is, in some cases, PAEs allow firms to overcome transaction costs.

However, we find that PAEs also affect the continuation payoffs by reducing the value of defensive patent portfolios, and by potentially extracting rents from the producing firms. These effects hinder ex-ante incentives to innovate. By avoiding counter-suits, PAEs destroy the value of patents that otherwise would be used defensively by counter-suing a producing firm. This reduces the incentives to obtain those patents in the first place. We showed that this effect not only matters to the firm with the smallest portfolio, but also the firm with the largest portfolio. Thus, in our model, PAEs reduce the marginal incentive of getting one more patent, when one of the firms owns a very large patent portfolio. Another negative effect of PAEs is their potential to extract rents from the market. When this happens, firms have less incentives to invest in R&D to bring the continuation payoff sooner.

We showed that PAEs can increase R&D incentives even when: they do not invest in R&D, do not use the patents to produce products, do not have any cost advantage in litigation with respect to the producing firms, and could lower total industry profits extracting a positive amount of rents. Perhaps surprisingly, by increasing litigation threats PAE privateers can facilitate rather than obstruct innovation. This occurs when overcoming transaction costs is the dominant effect. In this case when, without PAEs, there is a large fraction of the patent portfolio that cannot be monetized. For example, when the legal costs are significant, and the number of components is very large, transaction costs will be more prominent. On the contrary, if legal costs are significant and the number of components is low, not even PAEs
will have incentives to assert patents, and they will not increase in patent monetization.

Our model is stylized, because we want to isolate some of the channels through which PAE privateers can affect incentives. Extensions of our model could incorporate private information, reputational concerns, contracting issues, selection, other penalty structures, asymmetries in costs, etc.

Finally, our paper provides results that could be explored empirically. For example, do producing firms try to negotiate licenses before they trade patents with the PAE? Our results show that PAEs may be able to extract rents when this is not the case. Thus, it would be interesting to find out how and when PAE privateers earn rents. Another possible question would be: do PAEs approach firms with similarly-sized patent portfolios? Or they approach firms with asymmetric portfolios? Our model shows that PAEs do not extract rents when firms have similar patent portfolios. A detailed examination of the sources of patents for PAEs could shed light on this issue.
References


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A Appendix A: Proofs

Proof of Lemma 2

Proof. Consider the bilateral bargain between firm A and the PAE, taking the outcome of the negotiation between firm B and the PAE as given. Using the change of variables $u = S_{PAE}(z, m') - p - S_{PAE}(0, m')$, the maximization problem (1) can be written as

$$\max_{z,u} u^{1-s}(J_{A,PAE}(z, m') - J_{A,PAE}(0, m') - u)^s.$$

The solution is $z^* \in \arg\max J_{A,PAE}(z, m')$ and $u^* = (1 - s)[J_{A,PAE}(z^*, m') - J_{A,PAE}(0, m')]$. which implies the transfer $p = s[J_{A,PAE}(z^*, m') - J_{A,PAE}(0, m')] - [S_A(z^*, m') - S_A(0, m')]$. The agreement is incentive compatible as long as $J_{A,PAE}(z^*, m') \geq J_{A,PAE}(0, m')$. \hfill \qed

The following Lemma is used in several of the proofs of the propositions:

Lemma 7. Let $f : \{0, 1, ..., N\} \rightarrow \mathbb{R}$. Define $f(N/2)=0$ if $N$ is odd. Then,

$$\sum_{k=0}^{N} \binom{N}{k} f(k) = \sum_{k<\frac{N}{2}} \binom{N}{k} [f(k) + f(N - k)] + \binom{N}{\frac{N}{2}} f(N/2).$$

Proof. By properties of the binomial coefficient $\binom{N}{k} = \binom{N}{N-k}$, for all $k = 0, ..., N$. We have:

$$\sum_{k=0}^{N} \binom{N}{k} f(k) = \sum_{k<\frac{N}{2}} \binom{N}{k} f(k) + \sum_{k>\frac{N}{2}} \binom{N}{k} f(k) + \binom{N}{N/2} f(N/2)$$

$$= \sum_{k<\frac{N}{2}} \binom{N}{k} f(k) + \sum_{k>\frac{N}{2}} \binom{N}{N-k} f(k) + \binom{N}{N/2} f(N/2)$$

$$= \sum_{k<\frac{N}{2}} \binom{N}{k} f(k) + \sum_{s<\frac{N}{2}} \binom{N}{s} f(N - s) + \binom{N}{N/2} f(N/2) \quad \text{(change of variable)}$$

$$= \sum_{k<\frac{N}{2}} \binom{N}{k} [f(k) + f(N - k)] + \binom{N}{N/2} f(N/2).$$

In particular, when $f(k) + f(N - k) = 0$ for all $k = 0, ..., N$, and $f(k) < 0$ for $k < \frac{N}{2}$:

$$\sum_{k=0}^{N} \binom{N}{k} f(k) \geq 0, \quad \sum_{k=0}^{N} \binom{N}{k} kf(k) > 0. \hfill \qed$$

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Derivation of $\Pi_x(x, x)$

Lemma 8.

$$\Pi_x(x, x) = \frac{h'(x)}{h(x)} \frac{V}{2^{N+1}} \sum_{k: |2k-N| \geq \hat{c}} \binom{N}{k} (2k-N)^2 \equiv \frac{h'(x)}{h(x)} \Psi.$$ 

Proof. Let $p(x, y) = \frac{h(x)}{h(x)+h(y)}$. We have that

$$\frac{\partial}{\partial x} P(k; x, y) = \binom{N}{k} \frac{p'}{p(1-p)} p^k (1-p)^{N-k}(k-Np).$$

Therefore, the marginal change in the expected payoff is given by:

$$\Pi_x(x, y) = \frac{p_x}{p(1-p)} \sum_{k=0}^{N} \binom{N}{k} p^k (1-p)^{N-k}(k-Np) \cdot U(k).$$

Using the definition of $p$ we find $p_x = \frac{h'(x)}{h(x)} p(1-p)$. Replacing we obtain:

$$\Pi_x(x, y) = \frac{h'(x)}{h(x)} \sum_{k=0}^{N} \binom{N}{k} p^k (1-p)^{N-k}(k-Np) \cdot U(k).$$

Define $f_N(k) = p^k (1-p)^{N-k}[U(k) - \pi]$, and applying Lemma 7 we obtain,

$$\sum_{k=0}^{N} \binom{N}{k} p^k (1-p)^{N-k}(k-Np)[\pi + f_N(k)] = \pi \sum_{k=0}^{N} \binom{N}{k} p^k (1-p)^{N-k}(k-Np) + \ldots + \sum_{k=0}^{N} \binom{N}{k} p^k (1-p)^{N-k}(k-Np)f_N(k).$$

Therefore, the marginal benefit in terms of expected continuation payoff is given by:

$$\Pi_x(x, y) = \frac{Vh'(x)}{h(x)} \sum_{k: |2k-N| \geq \hat{c}} \binom{N}{k} p^k (1-p)^{N-k}(k-Np)(2k-N).$$

In a symmetric equilibrium, $p^* = \frac{1}{2}$ and therefore the expression above equals:

$$\Pi_x(x, x) = \frac{Vh'(x)}{2^{N+1}h(x)} \sum_{k: |2k-N| \geq \hat{c}} \binom{N}{k} (2k-N)^2 \equiv \frac{h'(x)}{h(x)} \Psi.$$ 

Notice that $\Psi$ measures the intensity of rent seeking incentives, and it is decreasing in $\hat{c}$, and increasing in $V$. \qed
**Proof of Proposition 4**

**Proof.**

- Using symmetry of the payoff function we have $\Delta(k) = -\Delta(N - k)$ for all $k$.

In a symmetric equilibrium $p = \frac{1}{2}$ and by symmetry of the binomial coefficients we have that:

$$\sum_{k=0}^{N} \binom{N}{k} p^k (1-p)^{N-k} \Delta(k) = \frac{1}{2^N} \sum_{k=0}^{N} \binom{N}{k} \Delta(k),$$

and we can decompose the sum as:

$$\sum_{k=0}^{N/2} \binom{N}{k} \Delta(k) + \sum_{k=N/2}^{N} \binom{N}{k} \Delta(N - k).$$

Using the relation described above for different regions between $\Delta(k)$ and $S(N - k)$ (for all $k$) we have that the two terms above cancel out, and we obtain 0.

- We borrow some algebra from the results in “Derivation of $\Pi_x(x, x)$”. Since $\Delta(k) \geq 0$ if $2k > N$ and non-positive otherwise, the winner incentive effect is given by

$$WP(x_{PAE}^*) = G(x_{PAE}^*, x_{PAE}^*) \frac{Vh'(x_{PAE}^*)}{2N+2h(x_{PAE}^*)} \cdot \sum_{k=0}^{N} \binom{N}{k} (2k - N)\Delta(k) > 0.$$ 

Notice we can decompose this effect in two regions: One that increases relative to the case of no PAEs, which is the “middle region” $k \in \left[\frac{N-\hat{c}}{2}, \frac{N+\hat{c}}{2}\right]$, and one that decreases incentives $k < \hat{c}$ or $k > N - \hat{c}$. However, overall the winner premium effect is still positive.

- Adding the two effects, the result is immediate.

\[\square\]

**B Appendix B: Continuation payoffs with PAEs**

In this section we complete the analysis of the continuation payoffs for the cases $N < 3\hat{c}$.

First of all, when $N \leq \hat{c}$ the PAE has no effect in continuation payoffs as the firms will never monetize their portfolios. Here, again we break the indifference by assuming that the firm with the smallest portfolio holds on to all its patents.
Case $N \in [2\hat{c}, 3\hat{c})$:

This case is qualitatively similar to the case presented in the body of the paper $N > 3\hat{c}$. From Proposition 3 we obtained the payoffs for the different cases. If $n \in \left[0, \frac{N-\hat{c}}{2}\right]$, firm A gets $\pi + (2 - \mu)Vn - VN$. If $n \in \left[\frac{N-\hat{c}}{2}, \hat{c}\right]$, firm A gets $\pi + \mu V(n-N)$. For $n \in [\hat{c}, N-\hat{c}]$ firm A gets $\pi + V(2n-N)$. For $n \in \left[N-\hat{c}, \frac{N+\hat{c}}{2}\right]$, firm A’s payoff is $\pi + \mu Vn$. Finally, if $n \in \left[\frac{N+\hat{c}}{2}, N\right]$, firm A’s payoff is $(2 - \mu)Vn - (1 - \mu)VN$.

**Figure 8:** Consider a producing firm which discovers $n$ components, while its rival discovers $N - n$, and $N \in [2\hat{c}, 3\hat{c})$. The firm’s payoff from the equilibrium with a PAE (bolded) is superimposed on its payoff from the equilibrium without a PAE.

Case $N \in [\hat{c}, 2\hat{c})$:

From Proposition 3 we obtained the payoffs for the different cases. If $n \in \left[0, \frac{N-\hat{c}}{2}\right]$, firm A gets $\pi - V(N-n)$. If $n \in \left[\frac{N-\hat{c}}{2}, N-\hat{c}\right]$, similar to the previous case, firm A gets $\pi + \mu V(n-N)$. If $n \in [N-\hat{c}, \hat{c}]$, firm A gets $\pi$, since both $n$ and $m$ are less than $\hat{c}$. If $n \in \left[\hat{c}, \frac{N+\hat{c}}{2}\right]$, the PAE is able to monetize firm A’s portfolio, while firm A on its own cannot. Thus, firm A’s payoff is $\pi + sVn$. Finally, if $n \in \left[\frac{N+\hat{c}}{2}, N\right]$, firm A could monetize its portfolio, although firm B would counter-sue. The PAE allows firm A to avoid the counter-suing, and thus firm A’s payoff is
\[ \pi + V(2n - N) + sV(N - n). \]

**Figure 9:** Consider a producing firm which discovers \( n \) components, while its rival discovers \( N - n \), and \( N \in [\hat{c}, 2\hat{c}) \). The firm’s payoff from the equilibrium with a PAE (bolded) is superimposed on its payoff from the equilibrium without a PAE.

Notice that this case is qualitatively different to the case presented in the paper, because the PAE is not able to break the “IP-truce” when the patent portfolios are of similar size. Therefore, the effect of the PAE is potentially negative in this case.

**C Appendix C: Derivation of** \( G(x, y) \)

In this appendix we derive an explicit formula for \( G(x, y) \) and we study its properties.

**Explicit formula**

Since each component has an independent exponential arrivals we have that

\[ G(x, y) = \int_0^\infty e^{-rt}N(1 - e^{-(h(x)+h(y))t})^{N-1}(h(x) + h(y))e^{-(h(x)+h(y))t}dt \]
Given the symmetry, let’s call \( \lambda = h(x) + h(y) \), so we have

\[
g(\lambda) = N \lambda \int_0^\infty e^{-(r+\lambda)t}(1 - e^{-\lambda t})^{N-1} dt
\]

\[
= N \lambda \int_0^\infty e^{-(r+\lambda)t} \left[ \sum_{k=0}^{N-1} \binom{N-1}{k} (-1)^k e^{-\lambda k t} \right] dt
\]

\[
= N \lambda \sum_{k=0}^{N-1} \binom{N-1}{k} (-1)^k \int_0^\infty e^{-(r+\lambda(k+1))t} dt
\]

\[
= N \sum_{k=0}^{N-1} \binom{N-1}{k} (-1)^k \frac{\lambda}{r + \lambda(k + 1)}
\]

\[
= \sum_{k=1}^{N} \binom{N}{k} (-1)^k \frac{\lambda k}{r + \lambda k} \quad \text{(change of variables)}
\]

\[
= \sum_{k=0}^{N} \binom{N}{k} (-1)^k \frac{r}{r + \lambda k}
\]

In the last step, we added and subtracted \( r \) in the numerator, and we used the fact that \( \sum_{k=0}^{N} \binom{N}{k}(-1)^k = 0 \).

Notice that \( g(\lambda) \) is equivalent (for \( \lambda > 0 \), and using induction) to

\[
g(\lambda) = \sum_{k=0}^{N} \binom{N}{k} (-1)^k \frac{r}{r + \lambda k} = \frac{N!}{(\frac{r}{\lambda} + 1)(\frac{r}{\lambda} + 2) \cdots (\frac{r}{\lambda} + N)} = \frac{\Gamma(N + 1)}{\Gamma(N + 1 + \frac{r}{\lambda})}
\]

where \( \Gamma(z) \) is the Gamma function, which is increasing and convex for \( z > 1 \). It’s easy to see that for \( N > 1 \) we have \( g(0) = 0, g'(0) = 0, \lim_{\lambda \to \infty} g(\lambda) = 1 \) and \( \lim_{\lambda \to \infty} g'(\lambda) = 0 \). Also, we can show that \( g \) is increasing and S-shaped. It is convex and then concave.

Notice that \( \ln(g(\lambda)) = \ln(N!) - \ln(\Gamma(h_N(\lambda))) \), where \( h_N(\lambda) = N + 1 + \frac{r}{\lambda} \). Thus,

\[
g'(\lambda) = -g(\lambda)[\ln(\Gamma(h_N(\lambda)))]' = \frac{g(\lambda) r \Gamma'(h_N(\lambda))}{\lambda^2 \Gamma(h_N(\lambda))} > 0.
\]

\[
g''(\lambda) = -\left\{g(\lambda)[\ln(\Gamma(h_N(\lambda)))]'ight\}'
\]

\[
= -g'(\lambda)[\ln(\Gamma(h_N(\lambda)))]' - g(\lambda)[\ln(\Gamma(h_N(\lambda)))]''
\]

\[
= g(\lambda) \left\{ \left([\ln(\Gamma(h_N(\lambda)))]'ight)^2 - [\ln(\Gamma(h_N(\lambda)))]'' \right\}
\]
After some algebra we can show that
\[ g''(\lambda) = \frac{g(\lambda)}{1^2} \left\{ [\Gamma']^2 - [\Gamma']^2 + \frac{r}{\lambda^2} [\Gamma'] \right\} |_{h_N(\lambda)}. \]
The convex region of \( g \) is where \( \left\{ [\Gamma']^2 - [\Gamma']^2 - \frac{r}{\lambda^2} [\Gamma'] \right\} |_{h_N(\lambda)} > 0. \)

**Approximation**

In order to get more tractable properties of \( G(x, y) \) we will use the Stirling approximation, which is a highly precise approximation for the Gamma function, even for small values of \( N \):

\[
\frac{\Gamma(x + 1 + \beta)}{\Gamma(x + 1 + \alpha)} \approx \frac{\sqrt{2\pi(x + \beta)} \left( \frac{x + \beta}{e} \right)^{x + \beta}}{\sqrt{2\pi(x + \alpha)} \left( \frac{x + \alpha}{e} \right)^{x + \alpha}} = \left( 1 + \frac{\beta - \alpha}{x + \alpha} \right)^{x + \alpha + 1/2} \left( 1 + \frac{x}{\beta} \right)^{\beta - \alpha} \left( \frac{x}{e} \right)^{\beta - \alpha} \approx e^{\beta - \alpha} \left( x/e \right)^{\beta - \alpha} = x^{\beta - \alpha}.
\]
Therefore, assuming that \( N \) is large (larger than 6 is highly precise approximation), a good approximation is:
\[
\hat{g}(\lambda) = N^{-\frac{\lambda}{r}} = e^{-r \frac{\ln(N)}{\lambda}}.
\]
The properties if \( g \) are easily derived using this approximation:

\[
\hat{g}'(\lambda) = \hat{g}(\lambda) \frac{r \ln(N)}{\lambda^2},
\]
\[
\hat{g}''(\lambda) = \hat{g}(\lambda) \frac{r \ln(N)}{\lambda^4} (r \ln(N) - 2\lambda).
\]
It’s easy to see that \( \hat{g} \) is increasing and S-shaped: Initially convex, for \( \lambda < \bar{\lambda} = \frac{r \ln(N)}{2} \), and for \( \lambda > \bar{\lambda} \) is always concave.

**D Appendix D: Existence and Uniqueness of Equilibrium**

**Existence**

We cannot apply the standard results of existence of equilibria, despite the game being symmetric, the payoffs are continuous, and the actions are chosen from a convex and compact
Consider firm A’s problem
\[
\max_{x \in [0, M]} u_A(x, y) = G(x, y)\Pi(x, y) - c(x)
\]

When \(\pi > NV\) both firms always enter. We first impose a participation condition: even when firm B does not invest in R&D, firm A still wants to invest in R&D. Equivalently, if we denote A’s best response to B by \(x^*(y)\), we can write this as \(x^*(0) > 0\), where:
\[
x^*(0) \in \arg \max_{x \in [0, M]} u_A(x, 0) = G(x, 0)\Pi(x, 0) - c(x).
\]

As long as \(\pi\) is large enough we have that \(x^*(0) > 0\), since \(\Pi(x, 0) = \pi + NV\) for all \(x > 0\), so we have \(u_A(x, 0) = G(x, 0)(\pi + NV) - c(x)\). Clearly there exists \(\pi\) large enough such that the optimum is strictly positive.

Next, notice that \(u_A(0, 0) = 0\) and \(u_A(\infty, 0) = -\infty\). Under general conditions it can be shown that \(u_A(x, 0)\) has at most two zeros in \((0, \infty)\). The next figure depicts the typical shape of \(u_A(x, 0)\), showing that in fact is not quasi-concave for small values of \(x\).

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24 Clearly the game is symmetric game, and the payoffs are continuous. Since \(G(x, y) \leq 1\) and \(\Pi(x, y) \leq \pi + NV\), no firm will choose an investment level above \(M = c^{-1}(\pi + NV)\). Thus, strategies are chosen from the interval \([0, M]\).

25 \(u_A(x, 0) = 0\) is equivalent to \(\pi + NV = e^{\frac{\ln(N)}{r}}\), \(c(x) \equiv K(x)\), and \(K(\cdot)\) is always positive and its derivative is zero whenever \(\frac{c'(x)h^2(x)}{c(x)h'(x)} = \frac{r\ln(N)}{2}\). If there a unique \(x\) (or none) that satisfies this equation, then by continuity we can have at most two solutions. This is the case, for example, when \(h(x) = x^\alpha\) and \(c(x) = x^\beta\) with \(\alpha < 1 < \beta\).
Figure 10: Simulation of the objective function for parameter values $N = 8$, $\pi = 2.5, 3.5, 4.5$, $R = 0.167$, $\beta = 0.8$, $h(x) = x^\alpha$, $\alpha = 0.8$, $r = 1$, $c(x) = \frac{1}{2}x^2$, and $\hat{c} = 1.5$.

The shape of the objective function is not surprising. Intuitively, when firm B invests 0, firm A trades off the investment cost against the benefit of an earlier arrival of the continuation payoff, $\pi + NV$. When $x$ is small, firm A pays the investment cost and receives almost no benefit, since discoveries arrive far in the future. This is why the payoff decreases below zero for some small $x$. When $x$ is larger than some threshold, the time at which all the components are discovered is significantly reduced, and the continuation payoff becomes significant and not so heavily discounted, so the firm has incentives to invest. Finally, for relatively large values of $x$, increasing $x$ even more will not improve firm’s profits because the gains from discovering faster are relatively smaller than the cost of investment.

Next, consider the general problem when $y > 0$. Firm A now also faces a rent-seeking effect, as firms compete to get more components than their rivals. Moving in the opposite direction is the “free riding” benefit that firm A gets when firm B invests more, because the continuation payoff arrives earlier.

To understand how $x^*(y)$ changes with $y$, we can compute the cross partial derivative

$$\frac{\partial^2 u_A(x, y)}{\partial x \partial y} = G_{x,y}(x, y)\Pi(x, y) + G_x(x, y)\Pi_y(x, y) + G_y(x, y)\Pi_x(x, y) + G(x, y)\Pi_{x,y}(x, y)$$
Re-arranging terms, we can show that:

\[ G_{x,y}(x, y) = G(x, y) r \ln(N) \frac{h'(x)h'(y)}{(h(x) + h(y))^4} [r \ln(N) - 2h(x) - 2h(y)], \]

and

\[ G_x(x, y) \Pi_y(x, y) + G_y(x, y) \Pi_x(x, y) = G(x, y) r \ln(N) \frac{h'(x)h'(y)}{(h(x) + h(y))^2 h(x)h(y)} \sum_{k=0}^{\frac{N}{2}} \left( \frac{N}{k} \right) p^k(1-p)^{N-k}(k-Np)U(k)[h(y) - h(x)] \]

Next, we can combining these expressions to sign \( \frac{\partial^2 u_A(x, y)}{\partial x \partial y} \). In particular, since we are interested in a symmetric equilibrium, we can study the local behavior when \( x = y \). In a symmetric equilibrium, we have:

\[ G_{x,y}(x, x) = G(x, x) r \ln(N) \frac{h'(x)^2}{(2h(x))^4} [r \ln(N) - 4h(x)] < 0 \text{ iff } h(x) > \frac{r \ln(N)}{4} \]

\[ G_x(x, x) \Pi_y(x, x) + G_y(x, x) \Pi_x(x, x) = G(x, x) r \ln(N) \frac{h'(x)^2}{4h(x)^4} \Psi[h(x) - h(x)] = 0 \]

\[ \Pi_{x,y}(x, x) = - \frac{h'(x)h'(y)}{h(x)h(y)} \sum_{k=0}^{\frac{N}{2}} \left( \frac{N}{k} \right) p^k(1-p)^{N-k}[(k-Np)^2 - Np(1-p)]U(k) = 0 \text{ by Lemma 7} \]

Combining the 3 terms above, in a symmetric equilibrium we have

\[ \frac{\partial^2 u_A(x, x)}{\partial x \partial y} = G_{x,y}(x, x) \Pi(x, x) \]

and hence

\[ \text{sign} \frac{\partial^2 u_A(x, x)}{\partial x \partial y} = \text{sign} (r \ln(N) - 4h(x)) \]

Therefore, as long as the intersection of the best response and the 45 degree line occurs at \( x \) such that \( h(x) > \frac{r \ln(N)}{4} \), the local behavior in the symmetric equilibrium is such that \( \frac{\partial^2 u_A(x, x)}{\partial x \partial y} < 0 \). Since we have a symmetric game, this condition implies that locally around the region where \( x = y \) the investments \( x \) and \( y \) are strategic substitutes: when \( y \) increases, the best response \( x^*(y) \) decreases.

The figure below depicts the typical shape of the best responses when the parameter \( \pi \) is large enough so the condition \( \hat{\pi} > N > 3\hat{c} \) is satisfied.
Figure 11: Simulation of the best responses for parameter values $r = 1$, $N = 8$, $\pi = 5$, $R = 0.167$, $\beta = 0.8$, $h(x) = x^\alpha$, $\alpha = 0.8$, $c(x) = \frac{1}{2}x^2$, $\hat{c} = 1.5$.

Notice there is a unique symmetric equilibrium where firms exert a positive level of R&D. Next, we show that whenever the symmetric equilibrium involves a level of investment large enough, there is a unique symmetric equilibrium.

**Condition for uniqueness**

We show that when there exists a symmetric equilibrium with positive level of investment $x^*$ satisfying $h(x^*) \geq \frac{r\ln(N)}{2}$, then it is unique. We know that any interior symmetric equilibrium $x$ must satisfy the condition:

$$G_x(x, x)\pi + G(x, x)\Pi_x(x, x) = c'(x).$$

Define $foc(x) = G_x(x, x)\pi + G(x, x)\Pi_x(x, x) - c'(x)$. Using the approximation for $G(x, x)$ and the computation of $\Pi_x(x, x)$ (derived in Appendix XXX) we have

$$foc(x) = e^{-\frac{r\ln(N)}{2h(x)}} \left[ \frac{\pi r \ln(N)}{2h(x)} + \Psi \right] - c'(x).$$

Taking derivative we obtain:

$$foc'(x) = e^{-\frac{r\ln(N)}{2h(x)}} \frac{h'(x)}{h(x)} \left[ \frac{\pi r \ln(N)}{2h^2(x)} \right] - c''(x),$$

where $A(x) = \frac{\pi r \ln(N)}{2h(x)} + \Psi$. Rearranging terms we obtain:

$$foc'(x) = e^{-\frac{r\ln(N)}{2h(x)}} \frac{h'(x)}{h(x)} \left[ \left( \frac{\pi r \ln(N)}{2h(x)} - 1 \right) \left( \frac{r \ln(N)h'(x)}{2h^2(x)} + \Psi \right) - \frac{\pi r \ln(N)}{2h(x)} + \frac{h''(x)}{h'(x)} A(x) \right] - c''(x).$$
Notice that for any \( x \) such that \( \frac{r \ln(N)}{2h(x)} < 1 \) we have that \( f'(x) < 0 \). Let \( \bar{x} \) be such that
\[
h(\bar{x}) = \frac{r \ln N}{2}.
\]
Hence, there can be only one symmetric equilibrium with a level of investment such that
\[
h(\bar{x}) > \frac{r \ln N}{2}.
\]
Notice that \( foc(0) = 0 \) does not imply that \( x \) is an equilibrium. For example, \( foc(0) = 0 \) and we showed that \( x^* = 0 \) is not an equilibrium for \( \pi \) large enough. Depending on the parameters, there might be other candidates for equilibrium. For any \( x \) such that \( foc(x) = 0 \) and \( foc'(x) > 0 \), \( x \) cannot be an equilibrium, since locally around \( x \) there are profitable deviations, as the condition \( foc'(x) > 0 \) implies that \( x \) is a local minimum of \( G(x,y)\Pi(x,y) - c(x) \) at \( y = x \). Generally, there are at most two positive levels of investment such that \( foc(x) = 0 \), as depicted in the figure below: one where \( foc'(x) > 0 \), and one where \( foc'(x) < 0 \). In this case, the unique equilibrium with positive level of investment is where \( foc(x) = 0 \) and \( foc'(x) < 0 \).

![Equilibrium candidates](image)

**Figure 12:** Simulation of the best response functions for parameter values \( N = 6, \pi = 2, R = 1, \beta = 0.8, h(x) = x^\alpha, \alpha = 0.8, r = 1, c(x) = \frac{1}{2}x^2 \), and \( c = 1.5 \).

For a large set of parameter values the conditions presented in the previous discussion are satisfied. Moreover, for those parameters we find that there is a unique equilibrium and it is symmetric. In the figure below we show comparative statics on the parameters \( \pi \) and \( r \). When \( \pi \) increases, the symmetric equilibrium features larger levels of investment, as expected. When \( r \) changes, we obtain a non-monotonic behavior in the equilibrium level of R&D. The intuition is simple: When \( r \) is very small, firms are very patient and they do not heavily
discount profits. Hence, the effect that dominates is the rent-seeking behavior of firms. As $r$ starts to increase, discounting provides an extra incentive for firms to invest. However, when $r$ is very large, the present discounted value of entering the market is small, even when a firm wins the race for every component.

![Figure 13: Comparative statics on the parameters $\pi$ and $r$. Parameter values $N = 6$, $R = 1$, $\beta = 0.8$, $h(x) = x^\alpha$, $\alpha = 0.8$, $c(x) = \frac{1}{2}x^2$, $\hat{c} = 1.5$, with $r = 1$ (on the left) and $\pi = 10$ (on the right).]

**Non existence of equilibrium with positive investment**

As is standard in rent seeking games, we cannot guarantee that the investments are strategic substitutes for all parameter values. For example, when $r$ is high or when $\pi$ is low the best responses might be strategic complements everywhere, or in some region.

In the first example we have increased the value of the discount rate $r$. Firms discount so heavily the payoff after entry, that a firm finds it worth investing only when its rival has also invested sufficient resources in R&D. Moreover, in this case R&D investments are strategic complements, and when firm B increases its R&D investment, firm A will slowly increase its own R&D investment. In fact, the best response looks like the best response of a firm that simply maximizes $G(x,y)\pi - c(x)$. That is, firms are more concerned with their effect on bringing the payoff sooner than with competing for rents. In the existence section, we ruled out this case by assuming that $r$ is small and $\pi$ is large, since for those parameters we know the best response to $y = 0$ is $x^*(0) > 0$ and not $x^*(0) = 0$. When $r$ is large, so the payoff from entry is still heavily discounted, but the competition effect is significant, investments are both strategic substitutes and complements in different portions of the best response function. In these cases, there is no equilibrium with positive level of investment.
For intermediate values of the parameter $r$, we can find cases in which we have a discontinuity in the best response (since we cannot guarantee that the objective function is pseudo-concave with respect to the firm’s own decision variable). This is because the rent seeking effect becomes relevant all of a sudden, after the rival has invested a small amount in R&D. Thus, the other firm can still ‘catch up’ and compete against its rival.

Figure 14: Simulation of the best response functions for $r = 2.5$ (on the left) and $r = 5$ (on the right), with parameter values $N = 6$, $\pi = 10$, $R = 2$, $\beta = 0.8$, $h(x) = x^\alpha$, $\alpha = 0.8$, $c(x) = \frac{1}{2} x^2$, and $\hat{c} > 2$.

E Appendix E: Firms Lose Rents to the PAE

In this appendix we study how our result changes if, for some reason, firms are not allowed to bargain before engaging in bilateral bargain with the PAE. Our result is only modified for patent allocations in which one firm owns less than $\hat{c}$ patents, the equilibrium selection is such that the firm with the smallest portfolio keep some patents, and the PAE has positive bargaining power. When these conditions hold, the PAE is able to extract rents from the producing firm, lowering total industry profits.

Qualitatively, the continuation payoff looks similar to the one presented in figure 5. The differences are that the firm with the smallest portfolio gets a lower payoff, since the PAE is extracting rents, and the firm with the largest portfolio gets a premium that depends on the bargaining power of the PAE and the equilibrium selection.

The PAE rent extraction effect lowers total industry profits, which could potentially lower ex-ante incentives for R&D compared to the case without PAEs. In particular, the main change occurs in Proposition 4, because in a symmetric equilibrium the rent extraction effect is negative, as it takes into account what the PAE extracts. However, the main qualitatively
result holds, as we can replace Proposition [4] by the following:

**Proposition 6.** When \( N > 3\hat{c} \), for symmetric R&d investments \( x = y = x^*_{PAE} \) we have:

- The rent extraction effect (RE) is weakly negative and equal to
  \[
  RE(x^*_{PAE}) \equiv G(x^*_{PAE}, x^*_{PAE}) \frac{h'(x^*_{PAE})}{h^2(x^*_{PAE})} \frac{r \ln(N)}{2^{N+2}} \sum_{k=N-\hat{c}}^{N} \binom{N}{k} \eta(k; s) \leq 0,
  \]
  where \( \eta(k; s) = -(1-s)V\ell_k \), \( \ell_k \) is the amount of patents retained by the firm with the smallest portfolio, and \((1-s)\) is the PAE’s bargaining power.

- The winner premium (WP) effect is strictly positive and equal to
  \[
  WP(x^*_{PAE}) \equiv G(x^*_{PAE}, x^*_{PAE}) \frac{h'(x^*_{PAE})}{2^{N+1}h(x^*_{PAE})} \cdot \sum_{k=0}^{N} \binom{N}{k} (2k - N)\Delta(k) > 0.
  \]

- A sufficient condition for the winner premium effect to be larger than the rent extraction effect is
  \[
  h(x^*_{PAE}) > \frac{r \ln(N)}{2}.
  \]

**Proof.** Using symmetry of the payoff function we have \( U_{PAE}(k) = -U_{PAE}(N - k) \) for \( \hat{c} \leq k \leq N - \hat{c} \). For \( k \leq \hat{c} \) we have \( U_{PAE}(k) = -V(N - k) \), and for \( k > N - \hat{c} \) we have \( U_{PAE}(k) = Vk - (1-s)V\ell_k \), where \( \ell_k \) is the amount of patents retained by the firm with the smallest portfolio in equilibrium. Thus, for \( k \geq N - \hat{c} \) we have that \( U_{PAE}(k) + (1-s)V\ell_k = -U_{PAE}(N - k) \).

In a symmetric equilibrium \( p = \frac{1}{2} \) and by symmetry of the binomial coefficients we have that:
\[
\sum_{k=0}^{N} \binom{N}{k} p^k (1-p)^{N-k} U_{PAE}(k) = \frac{1}{2^N} \sum_{k=0}^{N} \binom{N}{k} U_{PAE}(k),
\]
and we can decompose the sum as:
\[
\sum_{k=0}^{N/2} \binom{N}{k} U_{PAE}(k) + \sum_{k=N/2}^{N} \binom{N}{k} U_{PAE}(N - k).
\]

Using the relation described above for different regions between \( U_{PAE}(k) \) and \( U_{PAE}(N - k) \) (for all \( k \)) we have:
\[
\sum_{k=0}^{N} \binom{N}{k} U_{PAE}(k) = -(1-s)V \sum_{k=N-\hat{c}}^{N} \binom{N}{k} \ell_k.
\]

Defining \( \eta(k; s) = -(1-s)V\ell_k \) and noticing that \( \eta \leq 0 \), we have the result.
• Same as in Proposition 4.

• To show the last part of the proposition, define

\[
\kappa(x_{PAE}^*) = \left[ WP(x_{PAE}^*) + RE(x_{PAE}^*) \right] \cdot \left[ G(x_{PAE}^*, x_{PAE}^*) \frac{VH'(x_{PAE}^*)}{2N + 1} \right]^{-1}.
\]

We have that \( \kappa(x_{PAE}^*) > 0 \) if and only if

\[
\sum_{k=0}^{N} \binom{N}{k} (2k - N) \Delta(k) + \frac{r \ln(N)}{2h(x_{PAE}^*)} \sum_{k=N-\hat{c}}^{N} \binom{N}{k} \eta(k; s) > 0,
\]

which implies that \( \kappa(x_{PAE}^*) > 0 \) if and only if

\[
h(x_{PAE}^*) > \frac{r \ln(N)(1 - s)V \sum_{k=N-\hat{c}}^{N} \binom{N}{k} \ell_k}{2 \sum_{k=0}^{N} \binom{N}{k} (2k - N) \Delta(k)}.
\]

It is easy to see that

\[
0 \leq \frac{\sum_{k=N-\hat{c}}^{N} \binom{N}{k} (1 - s)V \ell_k}{\sum_{k=0}^{N} \binom{N}{k} (2k - N) \Delta(k)} \leq \frac{\sum_{k=0}^{\hat{c}} \binom{N}{k} V k}{\sum_{k=0}^{N} \binom{N}{k} (2k - N) \Delta(k)} \leq 1.
\]

This implies the sufficient condition in the proposition.

The previous proposition characterizes the PAE effect in a symmetric equilibrium. First, it shows that the rent effect extraction is always negative, unless \( s = 1 \) or \( \ell = 0 \). Equivalently, the PAE is unable to extract rents if it does not have any bargaining power or the firm with the smallest portfolio sells everything to the PAE. In any other case, the PAE is able to extract a positive amount of rents from the market. This implies that the PAE has a negative impact on the marginal effect of bringing the future sooner. Second, the proposition shows that the PAE has a positive impact on the rent seeking incentive. Although the continuation payoff becomes “flatter” on the extremes, it becomes “steeper” in the middle (see figure 5). In a symmetric equilibrium, being in the middle region is more likely and this outweights the potential negative effect on incentives on the extremes. Finally, the last result in the proposition provides a sufficient condition to have an unambiguous result on the total effect of the PAE. It shows that \( PAE_x(x, x) > 0 \) whenever \( h(x) > \frac{r \ln N}{2} \). In other words, when firms invest the same amount in R&D, the winner premium dominates the rent extraction effect when the symmetric R&D investment is larger than some threshold.

The other result that needs to be modified is Proposition 5. The following proposition is the analogous result.
Proposition 7. If a symmetric equilibrium with and without PAEs exist, and the equilibrium values are such that \( h(x) > \frac{r \ln N}{2} \), then the equilibrium with PAEs is larger than the equilibrium without PAEs.

Proof. Let \( x^* \) be the equilibrium without PAEs, so \( \text{foc}(x^*) = 0 \). By proposition 4 we know that \( \text{PAE}_x(x) > 0 \) for all \( x \geq \tilde{x}_{PAE} = h^{-1}\left( \frac{r \ln N}{2} \right) \). Therefore, any \( x_{PAE}^* \) such that \( \text{foc}(x_{PAE}^*) + \text{PAE}_x(x_{PAE}^*) = 0 \), we have that \( \text{foc}(x_{PAE}^*) < 0 = \text{foc}(x^*) \). But in the region where \( h(x) > \frac{r \ln N}{2} \), we showed that \( \text{foc}(x) \) is strictly decreasing and therefore we must have \( x^* < x_{PAE}^* \). \( \square \)

This last proposition shows that PAEs can increase the equilibrium level of R&D, even in the case where they extract rents from the market.