Dynamic Panics: 
Theory and Application to the Eurozone*

Zachary R. Stangebye†
Job Market Paper‡

Abstract

This paper characterizes a new type of dynamic lender coordination problem in sovereign debt markets that I call a dynamic panic. During a dynamic panic, expectations of future negative investor sentiments reduce the willingness of the sovereign to repay in the future and thus translate to negative investor sentiments today. I find conditions under which such sentiment dynamics can be active and document their presence in standard models. When the debt is of longer maturity I show that such panics resemble the recent Eurozone crisis, and so I explore policy implications in this environment. I find that interest rate ceilings are an ineffective policy tool but that liquidity provision by the ECB could be welfare-improving. Motivated by this result, I perform a structural estimation exercise to determine investors’ ex ante forecast of such panics and the concomitant welfare consequences of liquidity provision. Using Bayesian methods and Spanish CDS spreads, I find that investors’ forecast of such a crisis ex-ante was once every 7.37 years, which is in close accordance with the realized frequency of 7.5 years. I also find that liquidity provision by the ECB was likely welfare-improving.

JEL: E44, F34, G01, H63

Keywords: Sovereign Debt Crises, Confidence-Driven Crises, Long-Term Debt

*First Draft: 07/31/2014. This Draft: 1/9/2015. I am deeply grateful to my advisor, Jesús Fernández-Villaverde, for his constant support and guidance throughout the course of this project. I would also like to thank the other members of my dissertation committee, Ufuk Ackigit, Guillermo Ordoñez, and Satyajit Chatterjee, for their support, as well as Leonardo Martínez, Burcu Eyigungor, Manuel Amador, Javier Bianchi and seminar participants of Penn Macro Club, the Federal Reserve Bank of Philadelphia, the Federal Reserve Bank of Minneapolis, and Princeton EconCon for useful discussions and comments. I also acknowledge the Becker-Friedman Institute and the Macroeconomic Modeling and Systemic Risk Research Initiative for financial support. Any errors or omissions are my own.

†Department of Economics, University of Pennsylvania, 160 McNeil Building, 3718 Locust Walk, Philadelphia, PA 19104. E-mail: zs-tang@sas.upenn.edu

‡Most recent version available at http://economics.sas.upenn.edu/graduate-program/candidates/zachary-stangebye
1. Introduction

Since the seminal contribution of Eaton and Gersovitz (1981), there has been a folk intuition in the sovereign debt and default literature that market sentiments can have real effects. The intuition is as follows: When investors expect the sovereign to default they demand a high spread, which makes repayment costly and raises the default frequency; on the other hand, when lenders do not expect a default they demand a low spread, which reduces the burden of repayment and with it the default frequency.¹

However, the recent quantitative literature in this field lacks these sentiment dynamics. For instance, in the seminal models of Aguiar and Gopinath (2006) and Arellano (2008), which have become the workhorse models for a vast quantity of applied work, default episodes always result from an unfortunate sequence of fundamental shocks. Furthermore, the underlying fundamental Markov-Perfect equilibrium in these models tends to be unique², which has motivated the application of method of moments in quantitative analysis.

Although this determinacy result is appealing from a tractability and econometric perspective, the lack of sentiment dynamics could be problematic since many empirical crises seem to resemble some sort of a market panic. Two noteworthy examples are Mexico’s 1994 Tequila Crisis or the recent crisis in the Periphery Eurozone countries. This paper reconciles this apparent inconsistency by showing that such confidence-driven crises can still arise in this canonical sovereign debt framework.

In particular, I outline a new type of dynamic lender coordination problem that I call a dynamic panic. During a dynamic panic, lenders today anticipate that, for no fundamental reason, lenders tomorrow will demand a very high spread on debt issued by the sovereign. In the face of these high spreads, the sovereign’s value of repaying existing debt obligations tomorrow will fall, which will induce him to default more often as a consequence. As a result, lenders today panic and demand higher spreads. Therefore a dynamic panic arises when expectations of lender panics tomorrow cause such panics today.

A simple illustration is provided in Figure 1. The red and green circles denote non-fundamental con-

---

¹This intuition was formalized in some form in the models of Calvo (1988) and Cole and Kehoe (1996).
²The term ‘Markov-Perfect Equilibrium’ has taken different meanings in the literature. Some, such as Arellano (2008), have taken it to mean an equilibrium dependent only on contemporaneous fundamentals, while others such as Conesa and Kehoe (2012) allow the equilibrium to depend on additional contemporaneous states that encode relevant information, such as a sunspot. In this paper, I will distinguish between these types by calling the former a fundamental Markov-Perfect Equilibrium.
fidence, which exhibits some persistence governed by the thickness of the transition lines. Suppose that when the confidence light is green tomorrow, lenders offer the sovereign a high price, $q$, on his debt. This expands his consumption possibilities set under repayment, which reduces his propensity to default, $p_D$. The opposite is true when the light is red.

![Figure 1: Dynamic Panics: A Simple Illustration](image)

If the confidence light is green today it is likely to remain green tomorrow, which implies that the sovereign is more likely to repay debt issued today. The opposite is true when the light is red in that lenders expect him to default more often. Lenders, internalizing this in the price, offer the sovereign a high $q$ when the light is green and a low $q$ when it is red.

Therefore, it is possible to have two stable, non-fundamental regimes: A high-confidence regime with high debt prices and repayment frequencies; and a low-confidence regime with low debt prices and repayment frequencies. A dynamic panic is a transition from the high confidence regime to the low one.

All that is needed to induce such crises is some persistent, non-fundamental object on which lenders can coordinate such as market sentiment or confidence. A persistent notion of confidence gives lenders
across time\textsuperscript{3} a way to communicate with each other and thus to coordinate on malignant spread dynamics.\textsuperscript{4} These dynamics are sustained in turn by optimal sovereign default behavior.

In this paper, I provide conditions under which non-fundamental confidence fluctuations have real effects and characterize their basic properties. I also explore how these crises differ with maturity structure of the debt. Unlike a rollover crisis in the tradition of Cole and Kehoe (1996), dynamic panics can exist even when the debt is of very long maturity and consequently there is almost nothing to roll over. When the debt is of longer maturity, instead of inducing outright higher default frequencies, a lender panic can instead generate a persistent period of costly and excessive borrowing. This occurs because the marginal cost to the sovereign of issuing new debt in the face of a panic is lower when the debt is of longer maturity because only a small fraction of the debt stock needs to be rolled over at these high spreads. Thus, the sovereign is much more willing to borrow excessively rather than default or reduce consumption, and this excessive borrowing raises the default frequency and justifies the lender panic.

In a sense, such long-term dynamic panics are the opposite of investor ‘runs’ in the tradition of Diamond and Dybvig (1983) or Rodrik and Velasco (1999). Rather than fearing that future investors will pull their funds out of the country, investors today are afraid of precisely the reverse: They are afraid that lenders tomorrow will lend to the sovereign far too liberally, which will raise the future default frequency and destroy the future price of their debt, about which long-term bondholders care. In fear of this, lenders demand high spreads, which forces the sovereign to borrow more today and, in a Markovian framework, fulfills those negative expectations.

This unusual feature of borrowing into high spreads, which comes quite naturally out of my model, is a hallmark feature of the Eurozone crisis and the one that has attracted the most attention from the recent literature (see Lorenzoni and Werning (2013), Corsetti and Dedola (2013), or Conesa and Kehoe (2012)). In light of this similarity, I explore the policy implications of dynamic panics. I find that in this environment an interest rate ceiling, which has been proposed as a potential policy, would be ineffective.

To see why an interest rate ceiling is ineffective during a dynamic panic, it is helpful to understand

\textsuperscript{3}Static investor coordination problems have been studied in some depth in such models as Diamond and Dybvig (1983), Obstfeld (1996), and Cole and Kehoe (1996) but little attention has been paid to the possibility of coordination failures over time. A recent exception is Lorenzoni and Werning (2013), who highlight the potential for dynamic lender coordination failures when the debt is of longer maturity.

\textsuperscript{4}There is empirical evidence to suggest that confidence or beliefs are persistent in nature even when fundamentals are accounted for. See, for example, Barsky and Sims (2009) or Lubik and Schorfheide (2004).
why it would normally work. The justification for such a policy is grounded in Calvo’s (1988) framework in which there are two ways a sovereign can generate the same of revenue: Issue a small amount of debt at low spreads, which are low because the probability of default is low, or a large amount of debt at high spreads, which are high because the probability of default is high. Through the lens of this model, distressed Eurozone countries were ‘stuck’ in the latter situation and therefore a simple, credible cap on the market rate would be enough to rule out this sub-optimal equilibrium. However, during a dynamic panic, the sovereign is always borrowing on the left side of this ‘Laffer curve’, even during a crisis, since it can freely choose its debt level and would never place itself on the Pareto-dominated right side. Thus, an interest rate ceiling in this environment is isomorphic to a revenue cap on debt issuance and will only reduce government consumption and increase the likelihood of default.

Even though rate ceilings are ineffective, I do find that liquidity provision, which is a policy that the ECB undertook in its OMT program, was effective. In providing liquidity, the central bank credibly pledges to purchase sovereign debt at potentially sub-market rates. The model suggests that such a policy is effective at removing sentiment fluctuations, but that its welfare consequences are ambiguous, since some limited-commitment based default will remain after implementation. This trade-off can be understood in the context of the debate between the core and the periphery: The periphery wants the central bank to provide liquidity to protect them from malignant market sentiments, while the core fears that with such a backstop the periphery will rack up unsustainable debt levels and bring about another, more fundamental crisis. Both channels are active in the model and so the welfare consequences of such provision will depend on the underlying parameterization.

Motivated by the applicability of dynamic panics to the recent crisis in Peripheral Europe and the ambiguous nature of the key policy implications, I explore quantitatively the model’s empirical and policy implications in a structural estimation exercise. In particular I perform a structural estimation on a quantitative business cycle model specially designed to isolate empirically dynamic panics at longer maturities. While this model will be more complex and substantially different in nature than the theoretical model, dynamic panics in both models will affect observables through the same mechanism, and so I can meaningfully interpret parameter estimates from this latter model.

\footnote{For a graphical illustration of this, refer to Figure 5.}
I address the principal difficulties associated with the introduction of default into this class of models by implementing a new solution method outlined by Foerster et al. (2013). Further, I develop a new algorithm to expedite the estimation that I call the \textit{Capital-Motion Algorithm} and which applies to a wider class of models than the one I consider.

I take this model to the Spanish time-series data with two key questions: First, what was the ex-ante likelihood of a transition into a dynamic panic? And second, was liquidity provision by the ECB welfare-improving? My structural estimation on Spanish data suggests that these crises may in fact occur frequently. The probability of switching confidence regimes in any given quarter is estimated to be around 3.39\%, which is roughly once every 7.37 years. This figure is quite robust and is computed from spread data before the crisis. It does not rely whatsoever on the relative frequency of these events in the data. Given that the crisis took place roughly 7.5 years after the inception of the monetary union,\textsuperscript{6} this figure is in close accordance with realized events and tells us that if anything, investors anticipated such crises more often than they occurred.

This result also helps to solve the puzzle of low sovereign debt spreads throughout the early 2000’s as well. Lane (2012) articulates this puzzle as follows: “(T)he low spreads on sovereign debt...indicated that markets did not expect substantial default risk and certainly not a fiscal crisis of the scale that could engulf the euro system as a whole.” On the contrary, I argue such low spreads could be compatible with a rational long-term dynamic panic. This is because during normal times investors did not fear default, but the possibility that the economy would enter a regime in which default is more likely. Since such a regime-shift was unlikely and during one debt would still have substantial value, the implied spread in non-crisis times would have been very small. However, a simple drop in investor confidence was sufficient to tip the whole economy into a high-spread panic of the magnitude we observed.

To answer the second question, the empirical exercise suggests that so long as liquidity provision does not induce limited-commitment default more than once every 9.7 years that it will indeed improve welfare. This is substantially less than historical default trends outlined by Reinhart and Rogoff (2010), and so the provision of liquidity was likely a welfare-improving policy.

\textsuperscript{6}Inception’ is a vague term here, since the launch of the Euro took several years following its initial circulation in 1997. The external validity measure of 7.5 years assumes the initial date is the full launch, which occurred in 2002Q1, and that the crisis occurred in 2009Q3.
While in this paper I consider the special case of sovereign debt markets and in particular the case of the periphery Eurozone, the dynamic lender coordination problem I document is far more general. These coordination failures could plague markets for municipal or even corporate bonds and thus understanding them could provide new insights into the debt dynamics in these markets.

In summary, this paper makes the following points: First, it outlines a new dynamic lender coordination problem, which is the susceptibility of the standard sovereign debt environment to dynamic panics; second, it provides conditions for the real effects of non-fundamental dynamics in this environment; third, it generates borrowing into high spreads endogenously in the context of an already canonical model; fourth, it suggests that interest rate ceilings would be an ineffective policy in combating such crises and highlights the trade-off faced in the provision of liquidity; fifth, it develops a new type of Markov-switching dynamic equilibrium model and implements a new solution method to isolate empirical dynamic panics; sixth, it develops several widely applicable techniques that can be used to reduce the computational burden of approximating Markov-switching dynamic equilibrium model solutions; and finally, it uses empirical estimates of the frequency of dynamic panics to argue that the provision of liquidity during the recent crisis was likely welfare improving.

The rest of the paper is organized as follows. In Section 2, I review the relevant literature. In Section 3, I describe a simple model with only extrinsic uncertainty and completely characterize the set of sunspot equilibria, highlighting several necessary conditions for sunspot activity. In Section 4, I embellish the simple model by adding intrinsic uncertainty, borrowing choice, and potentially longer maturities. I then explore several necessary features of dynamic panics in this environment and their relationship to the Eurozone crisis. Lastly, I explore the impact of several plausible policies designed to counter such panics. Section 5 applies these results directly to the Eurozone by estimating structural equilibrium model specially designed to allow for dynamic panics at longer maturities. It also outlines in detail the solution technique and describes how the model is used to evaluate the welfare consequences of liquidity provision. Section 6 concludes.
2. Literature Review

This paper contributes to several different strands of the literature. First, it contributes to the quantitative literature outlining the dynamics of sovereign debt and default episodes. This literature takes the seminal framework of Eaton and Gersovitz (1981) and applies it quantitatively to primarily Latin American economies to match business cycle statistics and the empirical regularities of developing nations. Noteworthy papers in this vein include Aguiar and Gopinath (2006) and Arellano (2008). There is a nice summary of this tradition in Aguiar and Amador (2014).

This literature has also developed a branch that explicitly considers debt of longer maturities, of which prominent examples include Hatchondo and Martinez (2009), Chatterjee and Eyigungor (2012), and Arellano and Ramanarayanan (2012). The lesson from this branch is that, apart from expected default, the movements in the expected future price of the debt can have a significant impact on spreads today. This effect has been called ‘dilution risk’, and Chatterjee and Eyigungor (2012) show that it accounts for a substantial fraction of long-term spreads. Dilution risk will feature prominently in my exploration of dynamic panics at longer maturities.

The framework developed by this literature has also been the benchmark for a string of recent applied work studying default episodes. This is in large part because of the tractability of the assumption of Markov-Perfection. Some prominent examples include Mendoza and Yue (2012), Gornemann (2014), Salomao (2014), and Na et al. (2014), who study respectively the impact of default on international private lines of credit, long-term growth, credit default swaps, and optimal devaluation policy.

A common thread in this tradition is the lack of multiplicity or self-fulfilling dynamics. In this class of models, default is always driven by an unfortunate sequence of fundamental shocks and often the underlying equilibrium is unique. This uniqueness result was recently formalized by Auclert and Rognlie (2014). A recent exception is Passadore and Xandri (2014), who find sufficient conditions under which multiplicity of equilibria can exist for the case of short-term debt and use these conditions to bound the impact of sunspot activity. This paper will have a similar goal, but will focus primarily on the recursive dynamics of sunspot activity itself rather than its bounds. It also explores the interaction of sunspot activity with the unique features of long-term debt.
This paper also contributes to the recent literature on the Eurozone. As of yet, the academic literature has had little time to keep pace with developments that took place in the Eurozone over the past 6 years or so. However, several noteworthy pieces have emerged that have tried to deal seriously with the peculiar circumstances surrounding the sovereign debt crisis in the Eurozone. These papers have been both empirical and structural. On the empirical side, recent work has taken aim at demonstrating the confidence-driven nature of this crises by documenting an unusually weak correlation between economic fundamentals and CDS spreads. Some prominent examples include De Grauwe and Ji (2013) and Aizenman et al. (2013). This work will rely in some sense on these empirical findings in its placement of malignant market sentiments at the heart of the story in its theory of the crisis.

On the structural side, much emphasis has been placed on the unusual phenomenon of borrowing into high spreads and its concomitant drastic effect on debt-to-GDP ratios. Conesa and Kehoe (2012) have termed this phenomenon ‘gambling for redemption,’ and have argued that being mired in a deep recession is a necessary condition for such behavior. Broner et al. (2014) and Corsetti and Dedola (2013) have also built models featuring borrowing into high spreads. The former emphasizes the crowding out effect of sovereign debt issuance when there is domestic preference for debt and the latter argues that the access to liquidity that the central bank provides is more important for preventing such crises than the printing press.

The only other authors that have highlighted the role of dynamic coordination problems in the recent Eurozone crisis are Lorenzoni and Werning (2013). These authors also argue for a Laffer-curve type multiplicity in the spirit of Calvo (1988), but with the explicit inclusion of long-term debt. In their environment, as in mine, a dynamic lender coordination failure can place the economy on a malignant trajectory of high spreads and debt ratios. They term such a crisis a ‘slow-moving’ crisis. The key difference between their paper and mine is that I place more focus on the dynamic coordination failure and give it the form of a persistent sunspot. This allows me to explore the possibility of such crises in the standard Eaton-Gersovitz framework in which the government can commit to a level of debt as well as to a level of revenue. This small difference has important policy implications. For instance, in their environment, an interest rate ceiling would be an effective tool at alleviating slow-moving crises, while in mine such a policy will tend to induce more default.
On the theoretical frontier, this paper also contributes the literature on sunspots, since that is the tool I choose to model non-fundamental confidence. This literature started with the works of Azariadis (1981) and Cass and Shell (1983). Shell (2008) provides a nice summary of the prerequisite conditions for the existence of sunspot activity. Gottardi and Kajii (1999) demonstrate that multiplicity of equilibria is not necessary for the presence of sunspot activity and Hoelle (2014) discusses in depth the relationship between sunspot activity and multiplicity of equilibria when markets are incomplete. My work contributes to this literature by providing a complete set of conditions for sunspot dynamics in a simple sovereign debt environment and also characterizing features of sunspot equilibria in more complex environments.

The last relevant strand of literature that this work advances is that of Markov-switching dynamic equilibrium models. This literature, which started with Hamilton (1989), has made the case that parameter instability in the form of regime-switching is often key to understanding macroeconomic time-series. One recent work, Foerster et al. (2013), solves these Markov-switching models from the full initial model by perturbing the underlying parameters of the model governed by regime-switching. I apply their framework in my solution of the model that I estimate. But further, I provide a set of tools to broaden the applicability of their method and demonstrate how it could be applied to models of sovereign default, provided that default behavior is taken as exogenous.

3. Simple Model

3.1. Environment

In this section I construct a simple sovereign debt environment in the tradition of Eaton and Gersovitz (1981) and Arellano (2008) but with no intrinsic uncertainty and then characterize completely how the model reacts to non-fundamental confidence. In the subsequent section I will relax many of the restrictive assumptions and explore quantitatively how a richer, more standard model reacts to confidence shocks.

In particular, consider an infinitely-lived sovereign borrower that receives a constant endowment, $y$, in each period. This sovereign has an increasing flow utility, $u(\cdot)$, over consumption in each period and discounts the future at a rate $\beta < 1$. The only uncertainty in this model is extrinsic confidence, $\xi$, which can take one of two values, $\{\xi_L, \xi_H\}$. I assume that $\xi$ follows a symmetric Markov process with transition
probability \( \eta \).

He has a constant stock of debt, \( b \), and he can either choose to roll over that debt at an exogenously given price, \( q \), which may depend on the level of confidence. If he does not roll over this debt, then he defaults on it. He makes this default decision as soon as the extrinsic uncertainty is realized i.e. before he goes to the auction to roll over his debt. When he defaults, he is excluded from credit markets forever and pays a constant additive cost \( \phi(y) \) in every subsequent period. I will restrict attention to equilibria that are Markov-Perfect in confidence, and so we can write his Bellman equation, conditional on repayment, as follows:

\[
V(\xi) = u(y - b + q(\xi)b) + \beta E_{\tilde{\xi}|\xi}[\max\{V(\tilde{\xi}), X\}]
\]

where \( X \), which is the value of default, can be computed as follows:

\[
X = u(y - \phi(y)) + \beta X
\]

Notice that because there is no re-entry the value of \( X \) is independent of any particular equilibrium. Hence, it is not an equilibrium object. The sovereign will borrow from a unit mass of risk-neutral, deep-pocketed lenders with an outside option with return \( R \). These lenders price default risk according to a no-arbitrage condition, so in equilibrium the price must be given by:

\[
q(\xi) = \frac{1}{R} E_{\tilde{\xi}|\xi}[1\{V(\tilde{\xi}) \geq X\}]
\]

We are now ready to define an equilibrium in our simple environment. In particular, a Markov-Perfect Equilibrium will be a pair of functions \( \{V(\xi), q(\xi)\} \) such that

1. Given \( q(\xi) \), the value function \( V(\xi) \) solves Recursion 1
2. Given \( V(\xi) \), the pricing function \( q(\xi) \) solves Recursion 2

I will call a Markov-Perfect Equilibrium a Fundamental Markov-Perfect Equilibrium if in it confi-
dence has no real effects. If confidence does have real effects, I will call the Markov-Perfect Equilibrium a **Confidence-Waves Equilibrium**. In a Confidence-Waves Equilibrium, a shift from $\xi_H$ to $\xi_L$ will be a **Dynamic Panic**.

### 3.2. Characterizing Confidence-Waves Equilibria

In this simple environment, it is possible to make analytic statements regarding the entire set of Confidence-Waves Equilibria. In particular, the following theorem can be established:

**Theorem 3.1.** The set of parameters over which a Confidence-Waves Equilibrium exists is completely characterized by the following two conditions:

1. $R^{-1+\eta} b \leq \phi(y)$
2. $\frac{\beta^\eta}{1-\beta(1-\eta)} \left[ u \left( y - \frac{R^{-1+\eta} b}{R} \right) - u(y - \phi(y)) \right] < u(y - \phi(y)) - u \left( y - \frac{R^{-\eta} b}{R} \right)$

**Proof** See Appendix A

Theorem 3.1 provides a set of both necessary and sufficient conditions for the potential for sentiment dynamics in this simple model of sovereign default. The first condition will ensure that repayment is optimal for some value of $\xi = \xi_H$ without loss of generality. The second condition ensures that, in addition, default is optimal in $\xi_L$. Since there is no other source of uncertainty, this is the only way in which confidence can have real effects in a Markov-Perfect and thus these conditions completely characterize the conditions necessary for sunspot activity.

From these conditions, we can derive several useful corollaries that highlight key model elements required for confidence to have a real impact. The first is that confidence must be persistent.

**Corollary 3.2.** In any Confidence-Waves Equilibrium, $\eta < 1/2$.

**Proof** See Appendix A

---

8Note that we cannot say that such conditions are absolutely necessary for sunspot activity since, as I will show, non-sunspot equilibria will also exist for these same parameterizations.
Why must confidence be persistent? When default is driven by market sentiment, it must be that high spreads themselves cause the default and low spreads themselves cause repayment. However, spreads reflect anticipated default \emph{in the future}, not contemporaneous default.

Suppose that we were trying to establish an equilibrium in which default only occurred in the face of low confidence. If confidence was transient, then high confidence today would imply that quite likely default would occur tomorrow. But this would drive down the price of debt today relative to the low confidence state. Since the price discrepancy can be the sole driver of default discrepancies, this would induce default in the high-confidence state instead of the low confidence state, which is a contradiction. Thus, if confidence has real effects, it must be persistent.

So confidence must be persistent. But there is another important characteristic about the sovereign debt environment that is not immediately obvious. It is summarized in the following corollary:

**Corollary 3.3.** Let $\Theta(u)$ be the set of parameters for which a Confidence-Waves Equilibrium exists given a utility function, $u$. If $\hat{u}$ is more concave than $u$, then $\Theta(u) \subseteq \Theta(\hat{u})$.

**Proof** See Appendix A

In words, any parameterization for which a Confidence-Waves Equilibrium exists will continue to do so as you increase the degree of risk-aversion of the sovereign. Further, the set of parameters over which confidence fluctuations can occur expands with the degree of risk-aversion. Why is this? It is because a more concave utility functions will punish a repaying sovereign in utility terms more severely when debt service costs are expensive and thus when consumption is low. This will occur when confidence is low.

### 3.2.1. Relationship to Rollover Crises and Multiplicity

An easy misinterpretation of Theorem 3.1 is that I am simply finding conditions for a ‘rollover crisis’ as in Cole and Kehoe (1996). This is not at all what I am describing here. During a rollover crisis, an individual investor fears that other investors will not show up to the auction to roll over the sovereign’s short-term debt. If the sovereign defaults in response, then the investors’ fears are justified and they do not show up.
This is not what is going on with my dynamic panics. First, this is because the timing is different: The default decision occurs prior to the sovereign’s debt auction. Thus, lenders cannot experience such a contemporaneous coordination failure in my set-up. Second, lenders in my set-up are not panicking about the behavior of other lenders today; rather, they are panicking about the behavior of lenders tomorrow. Because the sentiment shock is persistent, when it is shocked today, lenders anticipate that lenders tomorrow will offer a low price, which will induce default tomorrow. This fear induces them to offer a low price today, which in turn induces default today. It is for this reason that I call my confidence crises dynamic panics, since the intertemporal dimension is crucial.

This is formalized in the following corollary. During a rollover crisis, the equilibrium price of debt, \( q \), equals zero i.e. the sovereign cannot raise any revenue at the auction. However, during a dynamic panic the price of debt falls, but it is not zero.

**Corollary 3.4.** *During a dynamic panic, debt can be auctioned at \( q(\xi_L) > 0 \).*

Thus, unlike a rollover crisis, it is possible to raise revenues at auctions; it is simply sub-optimal to do so.

The last necessary condition I will discuss is the relationship of Confidence-Waves Equilibria to multiplicity. In particular, we can claim the following:

**Corollary 3.5.** *If a Confidence-Waves Equilibrium exist, then the full-default and full-repayment equilibrium both exist as well.***

**Proof** See Appendix A

This finding accords with the recent findings of Passadore and Xandri (2014), who find that multiplicity is a necessary pre-condition to sunspot activity. In other words, there must be some room for strategic complementarities of the sort that would induce multiple equilibria in order for a Confidence-Waves Equilibrium to exist.

This is not to say that Confidence-Waves Equilibria are simply randomizing over existing multiplicity. While this is one way that they can be generated, it is also possible that the sunspot is randomizing over pricing schedules that are not themselves equilibria. In Appendix A I provide an analytic example of such a case when there is some intrinsic uncertainty as well.
This result is similar in kind to that of Gottardi and Kajii (1999), who argue that sunspots need not randomize over existing multiplicity, but only over ‘potential multiplicity’. They define potential multiplicity to be the existence of multiple equilibria for a reallocation of endowments. In my set-up, such ‘potential multiplicity’ arises if there exist pricing schedules and default strategies that are close to satisfying the equilibrium conditions. Confidence fluctuations can randomize over these schedules even though they themselves are not equilibria.

4. Full Theoretical Model

In the last section I highlighted several features of the sovereign default environment that are necessary to generate confidence-driven fluctuations. In particular, I argued that the persistence of the confidence process is necessary and that the steepness of the flow utility function can expand the parameterizations that are subject to sentiment shocks. As it turns out, these two features will be crucial for finding these equilibria computationally using the tools developed by the literature.

I now embellish the simple model until it subsumes several of the standard sovereign default models in the literature. I then explore quantitatively the potential for Confidence-Waves Equilibria in these already standard models and characterize several necessary features of dynamic panics.

This full theoretical model will be similar to Chatterjee and Eyigungor (2012). In particular, there will be intrinsic as well as extrinsic uncertainty. There will also be an endogenous borrowing choice and debt of longer maturities. In this more sophisticated environment, I will no longer be able to characterize fully the set of Confidence-Waves Equilibria, but I will be able to find them computationally and derive several illuminative necessary conditions.

4.1. Augmented Environment

The full theoretical model will allow for three stochastic processes: A fundamental endowment shock, \( y \in \mathcal{Y} \), a continuous, fundamental consumption preference shock, \( \tilde{m} \in [\bar{m}, \bar{m}] \), and a non-fundamental confidence shock, \( \xi \in \Xi \). Both the endowment and the confidence shocks are assumed to be persistent and \( \mathcal{Y} \) and \( \Xi \) are assumed to be discrete sets. In particular, I continue to assume that confidence is binary i.e. \( \xi \in \{\xi_L, \xi_H\} \). The preference shock, \( \tilde{m} \), is assumed to be iid over time and will therefore not impact the
price in equilibrium. Its continuous nature, however, will smooth over the discrete nature of sovereign’s
decisions in expectation, which helps both with existence and computation. Its range is assumed to be
fairly small.

The sovereign borrower chooses a level of consumption, $c$, how much to borrow from abroad, $b' \in B$, and
whether or not to default. He receives a flow utility, $u(\cdot)$ from consumption, which I assume is increasing
and strictly concave. Debt is long-term as in Chatterjee and Eyigungor (2012) i.e. debt pays a coupon $\kappa$
for every period in which it does not mature and matures stochastically with a probability $\lambda$.

I will focus on Markov-Perfect Equilibria and so I can write the sovereign’s problem recursively. Taking
as given the demand schedule for its debt from foreign investors, $q(y, \xi, b')$, the government solves the
following Bellman, which is conditional on repayment this period:

$$
V(y, \xi, m, b) = \max_{c \geq 0, b' \in B} u(c) + \beta V(y, \xi, b') \\
s.t. c - m \leq y - [\lambda + (1 - \lambda)\kappa]b + q(y, \xi, b')\left[b' - (1 - \lambda)b\right]
$$

I assume that if the sovereign faces an empty budget set, he must default. The continuation value allows
for default is ex-post optimal, and is thus given by

$$
V(y, \xi, b') = E_{(\tilde{y}, \tilde{\xi}, \tilde{m})|(y, \xi)} \left[ \max \{ V(\tilde{y}, \tilde{\xi}, \tilde{m}, b'), X(\tilde{y}) \} \right]
$$

I will denote the sovereign’s borrowing function to be $a(y, \xi, m, b)$.

Unlike the simple model, I assume that when the country defaults, it is excluded from credit markets
only temporarily. It will continue so suffer some additive output loss, $\phi(y)$, in each period of this exclusion.
It re-enters stochastically at a rate $\pi_{RE}$ and it does so with a high level of confidence $\xi_H$, after which
confidence follows its typical Markov-process. This simple assumption will allow the default value to be
independent of $\xi$ while retaining its non-fundamental nature, which will be important both theoretically
and computationally. Also upon re-entry, it has no debt obligations.\(^9\)

\(^9\)This last assumption can be relaxed to allow for haircuts provided we restrict attention to the set of equilibria for which demand curves for
debt are always downward sloping.
Under these assumptions, we can express the value of default as

\[ X(y) = u(y + m - \phi(y)) + \beta E_{\bar{y},\bar{m}|y}[(1 - \pi_{RE})X(\bar{y} - [m - \bar{m}]) + \pi_{RE}V(\bar{y},\xi_H,\bar{m},0)] \]  (4)

Notice that the value of default is no longer independent of the equilibrium, since the value of default will depend on the equilibrium pricing function through its continuation value. Thus, it is an equilibrium object. Notice further that, as in Chatterjee and Eyigungor (2012), during the first period of default the sovereign faces the worst $\bar{m}$ shock but experiences it as its normal stochastic process thereafter.

The final augmentation that needs to be made is to the foreign lenders’ no-arbitrage condition. These lenders now care about the persistent part of the endowment today, $y$, and level of borrowing today, $b'$, since both of these determine provide information regarding the default frequency tomorrow. Since the debt is long-term, they also care about the expected future price of their debt and thus about the degree to which the sovereign borrows tomorrow. The pricing recursion thus becomes

\[ q(y,\xi,b') = \frac{1}{R} E_{(\bar{y},\bar{m})|(y,\xi)}\left[ 1\{V(\bar{y},\bar{\xi},\bar{m},b') \geq X(\bar{y})\} \times \left[ \lambda + (1 - \lambda)(\kappa + q(\bar{y},\bar{\xi},a(\bar{y},\bar{\xi},\bar{m},b'))) \right] \right] \]  (5)

4.1.1. Equilibrium Definition

A Markov-Perfect Equilibrium is a set of functions $V(y,\xi,m,b)$, $a(y,\xi,m,b)$, $X(y)$, and $q(y,\xi,b')$ such that

1. $V(y,\xi,m,b)$ satisfies Recursion 3 when given $X(y)$ and $q(y,\xi,b')$ and implies the borrowing policy function $a(y,\xi,m,b)$
2. $X(y)$ satisfies Recursion 4 when given $V(y,\xi,m,b)$
3. $q(y,\xi,b')$ solves Recursion 5 given $V(y,\xi,m,b)$, $X(y)$, and $a(y,\xi,m,b)$

A Confidence-Waves Equilibrium, Fundamental Markov-Perfect Equilibrium, and Dynamic Panic are defined relative to the Markov-Perfect Equilibrium as in the previous section.

4.2. Theoretical Results

In what follows I characterize the theoretical properties of Confidence-Waves Equilibria. In particular, I will show that dynamic panics at longer maturities exhibit several features peculiar to the Eurozone crisis, including excessive persistence and borrowing into high spreads. Because of this, I explore several policy implications of long-term dynamic panics and discuss briefly their applicability to the Eurozone crisis.
The first result tells us that confidence shocks are indeed an equilibrium phenomenon: Both lenders and the sovereign must actively respond to the shock in order for them to have any real effects:

**Proposition 4.1.** *In any Confidence-Waves Equilibrium, both the lenders and the sovereign must react to the confidence shock at some point in their respective state spaces.*

**Proof** This can be shown by contradiction. Suppose that we had a Confidence-Waves Equilibrium with a pricing schedule that never depended on $\xi$. Once given the pricing schedule, the sovereign’s Bellman equation becomes a contraction on $V$ and $X$. Since the only channel through which $\xi$ can affect the sovereign’s payoff is through the price, the resulting, unique fixed point will not depend on fluctuations in $\xi$. Thus, $\xi$ would have no effect in equilibrium, which contradicts the fact that the equilibrium is a Confidence-Waves Equilibrium.

The same argument holds if we suppose that sovereign behavior never depended on $\xi$, since the lenders’ pricing recursion is also a contraction on $q$. Thus, any Confidence-Waves Equilibrium must feature an active change of behavior on both sides of the market in response to sentiment shocks.

Having established the equilibrium nature of these sentiment shocks, I now begin to characterize them by defining a new term:

**Definition** A Confidence-Waves Equilibrium is **Default-Relevant** if the realization of $\xi$ matters for the default decision of the sovereign at some point in the fundamental state space.

In other words, if the equilibrium is default-relevant then there is a fundamental state for which the sovereign defaults when confidence is low and repays when confidence is high. It is possible to have an equilibrium that is non-default-relevant even though $\tilde{m}$ is continuously distributed provided that the range of $\tilde{m}$ is fairly small. I will provide an example momentarily.

With this definition in hand, we can distinguish how maturity will influence the underlying impact of confidence.
Proposition 4.2. If the debt is short-term i.e. $\lambda = 1, \kappa = 0$, then any Confidence-Waves Equilibrium must be default-relevant.

Proof See Appendix A.

Proposition 4.2 tells us that if the sunspot has any real effects, it must at some point make the difference between the sovereign defaulting and repaying. This proposition disappears when we extend the maturity of the debt. It will still be the case that default-relevance is sufficient to have an active sunspot, but it will no longer be necessary, since the sunspot can affect the future price of the debt if it affects borrowing behavior.

All of the results in the simple model outlined before were driven by default-relevance, which accords with the proposition. Investors today feared that investors tomorrow would panic and that this panic would induce more frequent default, which gave investors today a reason to panic. This proposition tells us that, for debt of longer maturities, this is not the only way that dynamic panics can unfold.

So how can we have a non-default-relevant dynamic panic? Figure 2 demonstrates. With longer term debt lenders care not only about whether the sovereign defaults tomorrow, but about the future price of the debt. Thus, if lenders in period $t$ anticipate lenders in $t+1$ to panic, it need not be the case that default probabilities actually rise in $t+1$, since lenders in $t$ already care about the price of debt in $t+1$. But the sovereign must respond somehow to this shock, even though he does not change his default behavior. Thus, he must change his borrowing behavior, and in particular he must borrow in the face of a panic to justify the panic occurring in the first place. He is willing to do this because the marginal cost of borrowing into high spreads is lower at longer maturities, since less debt is actually issued at these low prices.

In a sense, a non-default-relevant dynamic panic is the opposite of an investor ‘run’ as in Diamond and Dybvig (1983) or Rodrik and Velasco (1999). Investors in such a panic do not fear that investors tomorrow will withdraw lending from the sovereign. They are afraid of quite the reverse i.e. that lenders tomorrow will lend excessively to the sovereign, which will drive down the expected future price of the debt. Since long-term bondholders care about this future price, they too will panic and demand higher spreads.

Although the causal chain of a long-term dynamic panic is driven by borrowing, in equilibrium we will simply have two different regimes: One in which spreads, default probabilities, and borrowing are all low.
and another in which all of these objects are high.\textsuperscript{10} This can be seen in the numerical example provided in Figures 3 and 4.\textsuperscript{11} This example shows how a non-default-relevant dynamic panic can affect borrowing and pricing behavior. In particular, we get two distinct pricing and borrowing regimes: One in which low borrowing occurs at low spreads and one in which high borrowing occurs at high spreads.

Further, we can see from a simulation path that a regime change is associated with higher levels of borrowing.\textsuperscript{12} Figure 4 compares the spreads and debt-to-GDP paths of two sample economies which face the same endowment shocks. The first economy does not respond to non-fundamental activity and the second economy is in a confidence-waves equilibrium, experiencing a dynamic panic. One can see that when the confidence falls, its effect is to simultaneously increase both the spread and the debt position of the sovereign. While the debt position does not increase substantially, the spreads experience a massive spike when confidence falls: From roughly 6\% to nearly 12\%. They stay consistently higher as the panic persists.

\textsuperscript{10}The only time this will not be true is in the period of the initial panic itself, since here default probabilities will not increase, though spreads and borrowing will.

\textsuperscript{11}To calibrate these examples, I simply use the parameterization of Chatterjee and Eyigungor (2012) and add a large but non-binding subsistence level of consumption to induce steepness in the flow utility function.

\textsuperscript{12}Notice that both spreads and debt ratios are contemporaneous. High confidence-driven spreads induce excessive borrowing in the next period while high fundamental-driven spreads induce deleveraging in the following period.
Before I characterize theoretically such non-default-relevant panics, I will develop one more term to describe the sort of panic outlined in this quantitative model.

**Definition** A dynamic panic is **monotone** if \( q(y, \xi_H, b') \geq q(y, \xi_L, b') \) for every \((y, b') \in Y \times B\).

There are numerous necessary features of monotone, non-default-relevant dynamics panics that formalize the intuition just outlined. I will now outline each of them in turn.

**Proposition 4.3.** *During a monotone, non-default-relevant dynamic panic, it must be the case that borrowing increases in some states of the world relative to high confidence.*

**Proof** To see why borrowing must necessarily increase in some states of the world, suppose that it did not. Suppose that, faced with a confidence shock that uniformly lowered debt prices, the sovereign delevered
in every state relative to its behavior in that state with high confidence. Fixing this behavior, we turn to
the pricing function, which is a conditional contraction on $q$. Note that when confidence shifts down, the
sovereign delevers in every state. But this would necessarily increase the debt price relative to its high
confidence counterpart, since the continuation value of the debt is decreasing in $b'$, the new debt taken on
tomorrow,\textsuperscript{13} and confidence is persistent. But this contradicts the fact that $q$ drops with confidence. There-
therefore, in some states of the world, the sovereign must react to the investor panic with higher borrowing.

This result is a necessary condition of non-default-relevant long-term dynamic panics. The sovereign
has three options in the face of an adverse shock: Default, delever, or borrow into the spreads. This
third option, in which little to no domestic fiscal adjustment takes place, is a requirement to generate

\textsuperscript{13}See Chatterjee and Eyigungor (2012) for a proof of this claim. Their results generalize to my environment. This claim would also hold true if we allowed for recovery provided we restrict attention to those equilibria with downsloping demand schedules.

Figure 4: Simulation of a Long-Term Dynamic Panic (Non-Default-Relevant)
non-default-relevant dynamic panics. This is one of the key features of such panics that I will utilize in my structural estimation.

In practice, this result is far stronger than the fairly weak claim of this proposition: Such borrowing into high spreads tends to happen in every state of the world in previously calibrated models. In fact, in the face of a panic the sovereign tends to not change his consumption or default behavior at all and instead borrows additionally to fill the same primary deficit.

**Proposition 4.4.** Suppose that we are in a region of the state space in which the sovereign increases its borrowing in the face of a monotone, non-default-relevant dynamic panic. Then the subsequent probability of default increases in every state.

**Proof** This proof is simple. The value of repayment for the sovereign is decreasing in $b$ while the value of default is flat in $b$. Therefore, the default policy is weakly increasing in $b$. Therefore, increasing $b$ also increases the probability of default in the next period. This occurs even if confidence never directly impacts the default decision.

This proposition most aptly describes how deteriorating confidence has affected the Eurozone. In most countries it has not caused a panic-driven default. However, it has increased default risk because it has caused sovereign governments to borrow excessively in the face of this confidence.

This result is quite striking upon reflection. It tells us that during a dynamic panic, the sovereign will willfully increase his debt position and default probability. This stands in stark contrast to the behavior sovereigns in such models as Cole and Kehoe (1996), in which case the sovereign either delevers or defaults in response to the negative shock. The intuition behind this result is that the debt here is of longer maturity and therefore the marginal cost of new debt issuance, which is what is directly affected by the price, is not nearly as high as it is for short-term debt, the stock of which the sovereign must roll over every period. It is therefore much more willing to increase its debt position in the hopes of recovery.

The last useful property of dynamic panics is that they are in fact rationally anticipated. Because of this, they will be priced into the spreads. I can exploit this in the estimation.
Corollary 4.5 (Rationally Priced). Consider a Confidence-Waves Equilibrium in which the sovereign always borrows additionally in the face of a monotone, non-default-relevant dynamic panic. Then, fixing the value and policy functions of the sovereign, \( q(\cdot, \xi_L, \cdot) \) is decreasing in \( \eta \) and \( q(\cdot, \xi_H, \cdot) \) is increasing in \( \eta \).

This last result will be invoked heavily during the empirical exercise, in which I will assume the presence of such a panic and use spread data to identify \( \eta \).

4.2.1. Policy Implications

I now explore two new policy implications in this environment. The first is the efficacy of a rate ceiling. Many authors, including Corsetti and Dedola (2013) and Lorenzoni and Werning (2013) have argued that an interest rate ceiling could have been an effective tool in combating malignant market sentiments. The reason is the following: A graph of revenue versus debt at any debt-auction ought to be parabolic since low levels of debt with have high prices and thus raise revenue but high levels of debt will have lower prices due to increased default probabilities and thus actually lower revenue. Some examples of such Laffer-curves can be seen in Figure 5.

These authors follow Calvo (1988) in asserting that the confidence crisis experienced by the Eurozone was a result of the sovereign winding up on the right-hand side of this Laffer-curve i.e. raising the same amount of revenue but with higher debt and worse prices. In the presence of such a crisis, an interest rate ceiling can be an effective tool since it forces the investors to coordinate on the good equilibrium on the left-hand side of the Laffer curve.

This policy implication is lost when the crisis at hand is a dynamic panic and not a Calvo-style crisis, as is made clear by the following proposition.

Proposition 4.6. During a dynamic panic, a binding interest rate ceiling is equivalent to a revenue cap on debt issuance. Thus a binding, temporary interest rate ceiling will increase the probability of default.

Proof First, note that whether the economy is in a crisis or not the sovereign optimally borrows on the left-hand side of the ‘Laffer curve’, which plots revenue against debt issuance. This is because the sovereign can commit to not only to revenue raised at debt auctions, but also to the amount of debt issued. Thus, given to issuance options yielding the same revenue, the sovereign will always choose the one with less debt, since the value function is decreasing the level of debt.
During a dynamic panic, the entire Laffer curve shifts but the sovereign continues to remain on the left-hand side of it. Therefore, if we implement a binding interest-rate ceiling, it will necessarily lower the quantity of debt that can be issued. This is because the demand curve for debt is downsloping\(^{14}\), so a price floor (rate ceiling) translates directly to a ceiling on debt issuance. A ceiling on debt issuance also places a ceiling on the revenue that can be raised, since the sovereign is located on an upsloping portion of the Laffer curve. Since the ceiling is temporary, tomorrow the sovereign can expect to resume with the equilibrium dynamics.

Denote the value of the sovereign who faces the original equilibrium demand functions with an interest rate ceiling as \(\hat{V}(y, \xi, m, b')\). Note that since \(\hat{V}(y, \xi, m, b')\) is the objective function of the same maximization as \(V(y, \xi, m, b')\) but with an additional constraint, specifically one on revenue, we will necessarily have \(\hat{V}(y, \xi, m, b') \leq V(y, \xi, m, b')\). Therefore, the probability of default has risen.

There is an intuitive graphical exposition of Proposition 4.6 in Figure 5. The black line represents the debt cap imposed by the rate ceiling. Consider the level of revenue raised by the horizontal dashed line. The typical Calvo-style multiplicity dictates that the sovereign is on the far-right intersection with the blue curve, and thus a rate ceiling such as this forces investors to coordinate back on the good equilibrium on the left side of the curve.

However, during a dynamic panic, we are not on the right side of the blue curve; we are in fact on the left side of a new red curve that implies less revenue raised for any given level of debt. The debt cap imposed by the black line then simply implies a revenue cap. Given this revenue cap, consumption must drop in the case of repayment and so repayment becomes less attractive and default frequencies rise.

It is important to note that an interest rate ceiling is different than the arguably successful measures that the ECB took to avert the crisis such as the Outright Monetary Transactions (OMT) bond-buying program. As noted by Corsetti and Dedola (2013), these programs were note rate ceilings but guarantees that the ECB would purchase government debt at sub-market interest rates. I follow De Grauwe (2011) in calling such a policy *liquidity provision*. The next proposition demonstrates that in fact such a policy

---

\(^{14}\)See Chatterjee and Eyigungor (2012) for a proof of this.
Proposition 4.7. Liquidity provision can eliminate the impact of confidence fluctuations without the need to actually purchase any assets. The resulting economy will still suffer from a weakly positive probability of default driven by the problem of limited commitment.

Proof See Appendix A.

Proposition 4.7 tells us that the ECB can in fact judiciously apply provision of liquidity to eliminate the impact of malignant market sentiments, as they effectively did with the OMT Program. Such a policy does not actually require a purchasing of the debt, so long as the implied demand schedule is consistent with an equilibrium not subject to confidence fluctuations.

The proposition also tells us that under the resulting economy will continue to suffer from a weakly positive probability of default. This is actually a likely description of the OMT program, which did not provide unconditional liquidity, but required that certain sustainability measures be met. Wolf (2014) highlights that this aspect of the program drew criticism from its opponents, since it would presumably not be there to provide liquidity precisely when member countries needed it most: During a crisis. My
modelling choice for liquidity provision allows for precisely such a crisis to occur while simultaneously eliminating the direct impact of sentiment fluctuations.

It is clear from this proposition then that the central bank has the capacity to eliminate confidence fluctuations. What is less clear from this proposition alone are the welfare implications of such a policy. It is not clear that the ECB would want to remove confidence fluctuations in the first place, since it is not clear that the equilibrium with confidence fluctuations is better than the one without. This trade-off can be phrased more colloquially in terms of the position of the core countries relative to the periphery with regards to provision of liquidity. Periphery countries argue that such provision is necessary to protect them from malignant market sentiments, while core countries argue that if such provision were in place periphery countries would build up unsustainable debt-to-GDP ratios and ultimately threaten the stability of the currency union. This trade-off is exactly highlighted by Proposition 4.7. Thus, to determine whether the provision of liquidity was effective or not, we will need to take a stand on the model’s parameters, especially with regards to the stochastic structure of the confidence shocks. In the next section, I estimate a model on Spanish data to do just that.

5. Structural Estimation

In the previous sections, I outlined the theoretical properties of Confidence-Waves Equilibria and showed that confidence shocks manifest themselves as dynamic panics. Now, I seek to understand the recent Eurozone crisis in the context of these dynamics. After the theoretical analysis I am left primarily with two lingering questions with regard to the Eurozone: What was the anticipated frequency of a long-term dynamic panic? And, given this frequency, was the provision of liquidity through programs such as OMT actually welfare-improving? To answer these questions, I will construct a stylized business cycle model that incorporates monotone, non-default-relevant dynamic panics in a way that can be taken directly to Eurozone data.

My approach is motivated by a careful consideration of the impact of long-term dynamic panics on the key observable variables, debt levels and spreads. In the theoretical section, I first established that in the face of a non-default-relevant dynamic panic, the sovereign must maintain roughly the same primary surplus and borrows additionally to fill the gap; in other words, he follows a roughly constant domestic
fiscal rule and borrows additionally from abroad in the face of a lender panic to do so. Next, I demonstrated that as a consequence of this additional borrowing the benefits from default increase, which increases the future default frequency.

Rather than solve the fully endogenous model with confidence, when I go to the data I will take these two mechanisms as primitives and estimate the parameters governing them. In particular, I will assume that default occurs at some exogenously given frequency that varies across confidence regimes and that the fiscal rule governing domestic consumption and taxation is constant across these regimes. Doing so will allow me to nest dynamic panics in a much richer and more flexible business cycle model that will be much more conducive to empirical inference. Further, there will tend to be a one-to-one welfare map between the fully endogenous model and the empirical one, since in both certainty equivalent consumption will be decreasing in expected default frequencies. This will allow for plausible policy experimentation and welfare analysis.

It can be shown at great difficulty that all of the theoretical results outlined in the previous section continue to go through in a modification of this empirical environment in which the government and households make decisions separately, provided the households value government spending separably and the government maximizes only this portion of household utility.

My identification of the stochastic structure of the confidence regimes is motivated by Corollary 4.5, which tells us that the probabilities of transitions are priced into the spreads. Thus, I can use spread data to estimate the probabilities of regime switching. Since I am interested in only a few novel parameters of an otherwise standard model, I will employ full-information techniques to derive my estimates. Several authors interested in full-information estimation have taken the approach of estimating models with sovereign default by specifying an exogenous rule for fiscal policy and default, instead of actually allowing for endogenous default choice, as I have done in the theoretical section. Some noteworthy examples include Bi and Traum (2012) and Bocola (2014). With an exogenous fiscal rule, the equilibrium is determinate and the likelihood function is well-defined.

However, just because the equilibrium is determinate does not mean that the model exhibits steady-state dynamics. These authors address this problem by applying the particle filter method of Fernandez-Villaverde and Rubio-Ramirez (2007), which uses simulated ‘particles’ to approximate the likelihood of a
given parameterization from the full non-linear specification. Rather than taking this approach, I modify
the model such that it can be linearized and adopt the method of Foerster et al. (2013).

With the perturbation method described in Foerster et al. (2013), I can capture all of the essential
dynamics of confidence-waves equilibria while taking confidence and default as exogenous regime shifts.
This approach will result in determinacy and substantially faster computation time. This simple model
will produce spreads that explicitly price not only the probability of a sovereign default, but also of a
dynamic panic.

In employing this technique, I make a handful of technical contributions to speed up the computation
as well. In particular, they show in their paper that the primary bottleneck to solving a standard dynamic
equilibrium model with parameter instability involves solving a quadratic system. I demonstrate that,
for a large class of models, many of the unknowns in this quadratic system can be determined before
explicitly solving the system. Specifically, I argue that the response of investment to changes in the capital
stock is the only object that needs to be solved for in the quadratic system. Once this rule is known,
the rest of the equilibrium decision rules can be derived from a simple linear system. This exponentially
reduces computational time while retaining the useful property that one still derives all possible first-order
approximations.

5.1. Specification

In this section I present a standard business cycle model such that it retains the intuition of dynamic
panics while simultaneously being suited to estimation methods. First, I will unpack the endowment
fluctuations assumed in the theoretical model and have a unit mass of standard, neoclassical growth
households with endogenous labor supply and preferences as in Greenwood et al. (1988). They have a
constant degree of relative risk aversion, \( \sigma \), and a Frisch elasticity of labor supply, \( \chi \). These households
save in capital and can only trade in domestic markets.

There is a unit mass of competitive final goods firms with a Cobb-Douglas technology that experience
an aggregate productivity shock, \( z_t \). There is also a competitive investment goods sector that produces
subject to convex adjustment costs \( \Phi ( \frac{i_t}{k_{t-1}} ) \).

The government’s budget constraint remains the same, but it will follow a simple fiscal rule instead of
maximizing household utility.\textsuperscript{15} In particular, government expenditures will follow an exogenous AR(1) process in its log:

\[
\log(g_t) = (1 - \rho_g) \log(g^*) + \rho_g \log(g_{t-1}) + \sigma_g \epsilon_t
\]

To generate non-trivial borrowing behavior, I also specify the tax policy rule. What the government does not raise in domestic taxes it borrows from abroad in defaultable debt. In the event of default, it is assumed that the government sets \( \tau_t = g_t \) i.e. it must use current taxes to finance all expenditures. When it is not in default, it sets taxes in response to its current debt level and the exogenous interest rate it faces from the outside investors. In particular, I assume that the government chooses a lump-sum tax policy of the following form:

\[
\tau_t - \tau^* = \gamma_b (b_{t-1} - \hat{\gamma}_b^* b^*) + \gamma_R (R_t - R^*) + \gamma_g (g_t - g^*)
\]

\( \tau^*, \ b^*, \ R^*, \text{ and } g^* \) are target levels that will, in equilibrium, reflect the steady state values. The term \( \hat{\gamma}_b \) simply adjusts the fiscal rule for the maturity of the debt and possibility of default. It is given by

\[
\hat{\gamma}_b = \gamma_b - \lambda - (1 - \lambda) \kappa + \lambda q_{ss}
\]

where \( q_{ss} \) is the steady state price of foreign government debt. This rule is similar to those in Schmitt-Grohé and Uribe (2007) and Leeper (1991), but with the addition of explicit consideration of the foreign interest rate that will induce a downward sloping demand curve for foreign assets in response to exogenous fluctuations in the interest rate.

The parameter \( \gamma_R \) governs the response of the government to exogenous changes in the interest rate that it faces. When \( \gamma_R > 0 \), then the government raises taxes and thus lowers its debt issuance in response to interest rate shocks, generating a downsloping demand curve for foreign assets as a function of the exogenous interest rate.

\( \gamma_g \) determines the extent to which the government responds to shocks in spending with taxes and \( \gamma_b \) governs how aggressively the government responds its debt: If \( \gamma_b \) is high, then high debt levels are quickly

\textsuperscript{15}It is not hard to show that all of the theoretical results in the previous section go through in this environment if households value government spending separably and the government is quasi-benevolent in the sense that it only maximizes that separable portion of household utility via endogenous default and borrowing decisions.
adjusted; if $\gamma_b$ is low, then large debt levels will linger longer.\textsuperscript{16} Taken together, the magnitude of $(\gamma_b, \gamma_R, \gamma_g)$ can be interpreted as a measure of fiscal discipline, since they determine the extent to which adverse shocks are funded by painful domestic taxes relative to defaultable foreign debt.

The price of debt, $q_t$, continues to reflect the probability of default, but to generate more plausible debt price dynamics I allow for haircuts after the stochastic re-entry after default. All of the theoretical results will continue to go through in this environment provided we restrict our attention to the plausible case in which the demand for debt is downward-sloping. In particular, I assume that in each period of default a fraction $1 - \hat{\delta}$ of the face value of the bond is destroyed. This implies a pricing recursion as follows:

$$q_t = \frac{1}{R_t} E \left[ 1 - d_{t+1} \right] \left[ \lambda + (1 - \lambda)(\kappa + q_{t+1}) \right] + d_{t+1} \hat{\delta} q_{t+1}$$

Allowing for I allow for the outside option, $R_t$ to fluctuate over time. Notice that this recursion is valid when the sovereign is in default as well.

Lastly, I assume that non-default-relevant dynamic panics enter the model exogenously as a regime shift. However, I invoke their properties from the theoretical model. In particular, we know the following from Propositions 4.3 and 4.4:

1. During a non-default-relevant dynamic panic, the government maintains a roughly constant primary deficit and borrows more to fill it
2. During a long-term dynamic panic, the probability of default rises.

To generate exogenously a long-term dynamic panic, I take this last characteristic to be fundamental i.e. I assume that default occurs stochastically but that it has a greater likelihood during such a panic.\textsuperscript{17}

This simple assumption will generate a long-term dynamic panic along the model observables. First, it will induce higher spreads since investors demand compensation for the higher possibility of default, and thus both parties will change their behavior during a crisis. Second, it will dictate increased borrowing on the part of the government, since it must fill the same primary deficit with lower-priced debt. These trends can be seen in the simulation averages of the modified model in Figure 6.\textsuperscript{18}

Not only will a long-term dynamic panic will be associated with higher spreads, higher external bor-

\textsuperscript{16}The theoretical model will imply that $\gamma_b < 1$, since borrowing increases in response to higher debt levels. This is proven in Chatterjee and Eyigungor (2012).

\textsuperscript{17}In Appendix C, I present several empirical measures of the robustness of this specification and show that the data strongly favor it over several
rowing, and greater default probabilities, but we will also see a slump in investment and a concomitant contraction in output coming from the private sector. This happens for two complementary reasons: First, expected productivity falls during such a panic, since default productivity costs are more likely in the future; second, consumption may actually be higher during a default, since foreign debt obligations are repudiated. Provided the household is patient enough to internalize these expected changes, both of these effects create a strong disincentive to save that is reflected in low investment. This can be seen in Figure 7.

Lastly, for the purposes of the estimation, I assume that foreign interest rates and labor productivity follow AR(1) processes as well and that productivity drops during a default.

\[
\log(z_t) = (1 - \rho_z) \log(z^*(s_t)) + \rho_z \log(z_{t-1}) + \sigma_z \epsilon_{z,t}
\]

\[
\log(R_t) = (1 - \rho_R) \log(R^*) + \rho_R \log(R_{t-1}) + \sigma_R \epsilon_{R,t}
\]

where \(s_t\) denotes the current regime, of which default is a possibility.

---

The trajectories in Figure 6 are computed using the estimated parameters I later obtain, but the pattern looks the same for any standard parameter values.
Note that a lender shock will increase the spread today even though the risk-free rate is differenced out. This is because the lender shocks are persistent i.e. \( \rho \in (0, 1) \). While higher rates today are differenced out of the spread, higher anticipated future rates are not. Rather, they bring down the price of debt tomorrow and thus drive down the price of debt today.

### 5.2. Parameter Instability

I consider an equilibrium in which four key parameters are subject to switching: \((z^*, rr, d, p_D)\), where \(rr\) is the recovery rate on bonds in default. I denote these parameters by the vector \(\theta(s_t)\), where \(s_t \in \{1, 2, 3\}\) i.e. there are three distinct regimes. The parameters take the following values in the three different regimes:

\[
\begin{pmatrix}
z^*(s_t) \\
rr(s_t) \\
d(s_t) \\
p_D(s_t)
\end{pmatrix}
\in
\begin{pmatrix}
\begin{pmatrix}
\mu \\
\delta_u \\
0 \\
p_H
\end{pmatrix} & , & 
\begin{pmatrix}
\mu \\
\delta_u \\
0 \\
p_L
\end{pmatrix} & , & 
\begin{pmatrix}
\mu_D \\
\hat{\delta} \\
1 \\
1 - \pi_{RE}
\end{pmatrix}
\end{pmatrix}
\]

(8)

where \(\mu_d < \mu\). \(\hat{\delta}\) governs the recovery rate of bonds in default; \(\delta_u\) is never observed on the equilibrium path and so can be judiciously chosen to ensure a well-defined steady state. The change from \(s_t = 1\) to \(s_t = 2\) will behave as the completely endogenous confidence-switching regimes described in the general theoretical
model. Estimating the transition between these regimes and the implications for policy of this switch is
the primary goal of this exercise, since a default has yet to occur in the Spanish data. In particular, the
transition matrix $P = (p_i')_{i=1,2,3}$ will be given by:

\[
P = \begin{bmatrix}
1 - \eta - p_H & \eta & p_H \\
\eta & 1 - \eta - p_L & p_L \\
\pi_{RE} & 0 & 1 - \pi_{RE}
\end{bmatrix}
\]  

It is assumed that $p_L > p_H \geq 0$ i.e. default is more likely in the low-confidence regime. I will also assume
that $\eta < .5$ i.e. the regimes are persistent. Notice that I allow for stochastic re-entry with probability $\xi$ in
the event of a default and that it is assumed, as is the case in the general model, that the sovereign re-enters credit markets with high confidence.

5.3. Model Solution

The equilibrium conditions can be written in the following form:

$$E_t[f(y_{t+1}, y_t, x_t, x_{t-1}, \chi_{t+1}, \epsilon_t, \theta_{t+1}, \theta_t)] = 0_{n_x + n_y}$$

where $y_t = (c_t, i_t, Q_t)$ are the primary control variables, $x_t = (k_t, b_t, R_t, z_t, g_t)$ are the endogenous state variables, and $\theta_t = (p_D(s_t), z^*(s_t), d(s_t))$ are those parameters that are subject to regime switching, and $s_t$ denotes the current regime. $\bar{\chi}$ is the perturbation parameter. It is convenient to write the equilibrium conditions in this way because I can then apply the method of Foerster et al. (2013). Note that the function
f looks as follows:

\[
\begin{align*}
(1) & \quad c_t - \kappa_1 (q_t^{1-\alpha} k_{t-1}^{1+\alpha})^{-\gamma} - \beta [c_{t+1} - \kappa_1 ([z^*(s_{t+1})]^{1-\rho_z} z_t^{\rho_z} e^\delta z_{t+1})^{1-\alpha} k_t^{\alpha}]^{1+\alpha} \\
& \quad \times \left( \kappa_2 \left[ [z^*(s_{t+1})]^{1-\rho_z} z_t^{\rho_z} e^\delta z_{t+1} \right]^{1-\alpha} k_t^{\alpha} \right)^{\frac{1+\alpha}{1+\gamma}} + (1 - \delta) \left( 1 + \Phi \left( \frac{t}{k} \right) + \Phi' \left( \frac{t}{k} \right) \right) \\
(2) & \quad c_t + \left[ 1 + \Phi \left( \frac{t}{k} \right) \right] i_t + q_t - [1 - d(s_t)][\lambda + (1 - \lambda)(\kappa + q_t)] b_{t-1} + q_t b_t - \kappa_0 \left[ z_t^{1-\alpha} k_{t-1}^{1+\alpha} \right]^{1+\alpha} \\
(3) & \quad - q_t b_t + dq_t r r(s_t) b_{t-1} + [1 - d(s_t)] [\lambda + (1 - \lambda)(\kappa + q_t) - \gamma_b] b_{t-1} + \gamma_R R_t + (\gamma_g - 1) g_t + (\tau^* + \gamma_b^* - \gamma_R R^* - \gamma_g g^*) \\
(4) & \quad k_t - (1 - \delta) k_{t-1} - i_t \\
(5) & \quad \log(R_t) - (1 - \rho_c) \log(R^*) - \rho_R \log(R_{t-1}) - \sigma_R e_{R,t} \\
(6) & \quad \log(g_t) - (1 - \rho_g) \log(g^*) - \rho_g \log(g_{t-1}) - \sigma_g e_{g,t} \\
(7) & \quad \log(\hat{z}_t) - (1 - \rho_z) \log(\hat{z}(s_t)) - \rho_z \log(\hat{z}_{t-1}) - \sigma_z e_{z,t} \\
(8) & \quad q_t - \frac{\rho}{\rho} \left[ 1 - d(s_{t+1})[\lambda + (1 - \lambda)(\kappa + q_{t+1})] + d(s_{t+1}) r r(s_{t+1}) q_{t+1} \right]
\end{align*}
\]

(11)

I seek a solution to this model of the following form:

\[ y_t = g(x_{t-1}, \epsilon_t, \bar{\chi}, s_t), \quad y_{t+1} = g(x_t, \bar{\chi}\epsilon_t, \chi, s_{t+1}), \quad x_t = h(x_{t-1}, \epsilon_t, \bar{\chi}, s_t) \]

(12)

An exact solution to this model is computationally burdensome and, given the model’s design, unnecessary. Instead, I will find a linear approximation to the model around a non-stochastic steady state. I will search for a set of matrices \( \{g_{ss}(s_t), h_{ss}(s_t)\}_{s_t=1,n_s} \), where \( g_{ss}(s_t) \) has dimension \( n_y \times (n_x + n_\epsilon + 1) \) and \( h_{ss}(s_t) \) has dimension \( n_x \times (n_x + n_\epsilon + 1) \). When in a regime \( s_t \), \( g_{ss}(s_t) \) will map deviations in \( (x_{t-1}, \epsilon_t, \bar{\chi}) \) from their non-stochastic steady state into deviations of \( y_t \) from its non-stochastic steady state. Thus, if \( \hat{z}_t \) is the steady-state deviation of an equilibrium object, \( z_t \), then \( \hat{y}_t = g_{ss}(s_t)[\hat{z}_t'] + \hat{z}_t = h_{ss}(s_t)[\hat{z}_t'] \) when in a regime \( s_t \).

In order to perturb this model, I must have a well-defined notion of a steady state that is independent of the Markov-switching regimes. To do so, I follow Foerster et al. (2013) and perturb the parameters

\[ 19\text{The non-stochastic steady state of } \epsilon_t \text{ and } \bar{\chi} \text{ are 0. } \bar{\chi} \text{ is 1 in the perturbation solution.} \]
where \( \hat{z}(s_t) = z^*(s_t) - \bar{z} \), \( \hat{r}\hat{r}(s_t) = rr(s_t) - \bar{r}r \), and \( \hat{d}(s_t) = d(s_t) - \bar{d} \) and \((\bar{z}, \bar{r}r, \bar{d})\) are taken to be the ergodic mean of \((z^*(s_t), rr(s_t), d(s_t))\). I construct the steady state of the dynamic system in terms of the \((\bar{z}, \bar{r}r, \bar{d})\), and thus the steady state is independent of the current regime. I calibrate \( \hat{\delta} \) and choose \( \delta_u(\hat{\delta}) \) to ensure that \( \bar{r}r = 1 \), which guarantees that the following expression holds:

\[
f(\bar{y}, \bar{y}, \bar{x}, \bar{x}, 0, 0, \bar{\theta}, \bar{\theta}) = 0_{(n_x+n_y) \times 1}
\]
i.e. the equilibrium conditions equate to zero at the non-stochastic steady state.

In order to solve this system, I must take a series of derivatives of Equation 11 with respect to all endogenous objects and the Markov-switching parameters and evaluate them at the steady state. Foerster et al. (2013) demonstrate that a first-order approximation to the solutions \( g \) and \( f \) can then be obtained in two steps. The first step entails solving the following quadratic system for \( \{D_{1,n_x}g_{ss}(s_t), D_{1,n_x}h_{ss}(s_t)\}_{s_t=1}^{n_s} \), which are the first \( n_x \) columns of the approximated policy rules and laws of motion, respectively, for each state. The relevant quadratic system is given below:

\[
A(s_t) \begin{bmatrix} I_{n_x} \\ D_{1,n_x}g_{ss}(1) \\ \vdots \\ D_{1,n_x}g_{ss}(n_s) \end{bmatrix} D_{1,n_x}h_{ss}(s_t) = B(s_t) \begin{bmatrix} I_{n_x} \\ D_{1,n_x}g_{ss}(s_t) \end{bmatrix}
\]

for all \( s_t \). Where \( A(s_t) \) is an \((n_x + n_y) \times (n_x + n_xn_y)\) matrix and \( B(s_t) \) is an \((n_x + n_y) \times (n_x + n_y)\) matrix. Both are functions of the derivatives of Equation 11 and their full specification can be found in Foerster et al. (2013).

Once a solution to Equation 14 has been found, the remaining elements of the matrices \( h_{ss} \) and \( g_{ss} \) can be found by solving a simple linear system that is provided in the computational appendix. If there are multiple mean-square stable approximations, I denote the one that provides the higher likelihood to be
the true one.\footnote{Note that a multiplicity of first-order approximations does not imply a multiplicity of equilibria. The equilibrium of the model is demonstrably determinant.}

I validate the accuracy of the first-order approximation by checking the unconditional Euler Equation errors, as suggested by Foerster et al. (2013). I find that at the posterior mean, \(\log_{10}(EE\ Error) = -3.6078\).\footnote{This error is much smaller than comparable models described by Foerster et al. (2013) that also have some form of adjustment costs.} To put this figure in perspective, note that when this object is \(-3\ (\sim -4)\), there is a $1 error for every $1,000 ($10,000) of consumption determined by the Euler Equation.

**5.3.1. Reducing the Dimensionality**

Equation 14 is the matrix representation of a quadratic system with \(n_s n_x (n_x + n_y)\) equations and the same number of unknowns. Foerster et al. (2013) suggest the use of Grobner bases to solve for all possible solutions to this system. While this method is satisfyingly exhaustive, its full implementation can be burdensome, as computational time is for most algorithms is exponential or even doubly exponential in the number of potential solutions. In their expository examples, the number of unknowns i.e. \(n_s n_x (n_x + n_y)\) is never more than 8.

The solution I seek has a substantially larger dimensionality. In particular, \(n_s n_x (n_x + n_y) = 3 \times 5 \times 8 = 120\). Given an exponential rate, even one iteration could take hundreds of thousands of years to compute, which is clearly impractical. However, there is much that we know about how the equilibrium operates that can be imposed on the solution before we even begin that allow me to reduce the dimensionality of the system. In particular, I develop a new method called the **Capital-Motion Algorithm** that reduces the quadratic system to 3 equations in 3 unknowns, which can be solved in hundredths of a second. The Capital-Motion Algorithm can be implemented in a wide class of models beyond the one at hand, and thus render this solution method more applicable in general.

The **Capital-Motion Algorithm** proceeds in 4 steps:

1. Fix the coefficients governing exogenous laws of motion.
2. Use the resource constraint to express consumption in terms of investment.
3. Solve a smaller quadratic system to determine the derivative \(i_k(s_t)\) in each state i.e. how investment responds to a shock to capital.
4. For each solution \( i_k(s_t) \), solve a linear system to determine all other model derivatives.

We can demonstrate a very useful property of the Capital-Motion Algorithm as it pertains to our case:

**Theorem 5.1.** The Capital-Motion Algorithm reduces the dimensionality of the quadratic system governing the model solution from 120 equations/unknowns to 3 equations/unknowns and still delivers all solutions to the original system.

**Proof** See Appendix B.

Theorem 5.1 is tailored to the model at hand for simplicity, but it can be generalized to a wider class of models: Essentially any model for which the crux of the intertemporal dynamics is the Euler equation. It relies on the fact that the key unknowns in the quadratic system are the coefficients governing the future capital choice in each state with respect to shocks to current capital, of which there are \( n_s \) i.e. one for each state. The decision of the agent to consume or to save today will reflect his propensity to consume or save tomorrow *in response to the same shock*; hence, when the rule is linear the relevant system is quadratic. Given the information imposed on the system so far, knowledge of saving behavior tomorrow in response to a capital shock in each of the different states is sufficient for determining saving behavior today in response to the same shock, and thus we can solve for these objects.

Once the saving response to a capital shock has been determined, the other equilibrium objects can be solved for linearly, since none of the others require solving an intertemporal problem, which is the source of the problem’s quadratic nature. A shock to any other object in the model will only be affected by future objects insofar as it has adjusted investment *today*.

Theorem Appendix B.4 reduces the dimensionality of the problem drastically, to the point where there are only \( n_s \) coefficients that must be solved for, which is 3 in this model. Upon reaching this point, I can easily solve the model for all possible solutions in fractions of a second and proceed with the rest of the solution as described in the previous section.

---

22 A general form of Proposition Appendix B.4 would entail solving for the number of endogenous equilibrium objects that jointly affect the intertemporal decision. For instance, if we were to introduce cyclical fiscal policy into the model, then we would need to solve a quadratic system of \( 2n_s \) equations and unknowns, since the dynamics of investment and debt be interdependent.
5.4. Estimation Procedure

5.4.1. Data

I use three quarterly time series data from Spain from 2001 until 2012: GNP, 5-year CDS spreads, and the public current account as a fraction of GNP. I take gross external government debt and GNP from the ECB Statistical Data Warehouse and I take spread data on 5-year debt from the MARKIT database. The first two objects require detrending, for which I use a Hodrick-Prescott filter (Hodrick and Prescott (1997)) to remain agnostic. The spread data requires no de-trending, though I reduce the frequency from daily to quarterly by means of an average. The filtered data can be found in Figure B.9 in Appendix B.23

I use Spanish data because De Grauwe (2011) and others have noted that Spain’s crisis seems most confidence-driven i.e. spreads tend to co-move the least with its fundamentals relative to other countries in the Eurozone periphery, although they all exhibit these trends. Since it is this shift in confidence that I seek to estimate, I find Spanish data to be the most appropriate tool available.

5.4.2. Identification

The shocks are all identified because they have orthogonal impacts on the observable variables: An interest rate shock today will impact positively debt and spreads but leave contemporaneous output untouched; a government policy shock will have a negative impact on debt but leave spreads and output untouched; and a productivity shock will increase contemporaneous output without impacting either the debt level or the spreads.

There are two potential sources of identification of the probability of regime switching: First, there is the length of time that the economy spends in one period versus another; and second, there is the fact that the probability of a regime switch is priced into the spreads. Given the relatively short span of the data, I follow the second approach for identification.

To identify this probability from spread data, I make a couple of identifying assumptions. First, I assume that the probability of regime switching is symmetric. This condition, although perhaps not necessary, is the condition under which I can ensure existence in the theoretical model. The results change only negligibly if I assume that confidence regime shifts are asymmetric for reasons I will discuss below. Second, I assume that there is no probability of default in normal times i.e. $p_H = 0$. Thus, any positive

---

23In this figure, I mark the start date of the crisis as 36 quarters, which corresponds to 2009Q4.
spread that we see in normal times reflects the possibility of a regime shift into a panic state in which a
default is possible. The divergence in the spreads in the panic state plus the symmetry assumption will
allow me to jointly pin down the probability of default and the probability of a confidence shift. Note that
the probability of a default in a panic is not uniquely determined by the spread, however. Private sector
expectations, through investment and consumption, help provide additional identification regarding this
probability.

Last, I denote the start of the crisis i.e. the regime shift as occurring in 2009Q4, which roughly dates
the start of the sovereign debt crisis by most accounts. Since there is little dispute regarding the start date
of the crisis, I prefer this route to estimating the start date, an approach taken by, for instance, Bianchi
(2013).

5.4.3. Fixed Parameters

I fix many of the model’s key parameters and estimate only 5: $\eta$, $p_L$, $\gamma_b$, $\gamma_g$, and $\gamma_r$. The parameterization
is given in Table B.3, which can be found in Appendix B. A few require discussion.

First, I choose $z_L$ based on Mendoza and Yue (2012) i.e. 5% less than normal times. I choose the mean
risk-free rate to be 0.01. To match the volatility, I choose to set the volatility of this shock to the estimated
volatility of a T-bill to an emerging market from Neumeyer and Perri (2005). As for the defaultable debt, I
assume a 4% coupon, which is in line with the average for Spanish long-term debt, and choose $\lambda$ to match
the average maturity of Spanish debt, which was 6.5 years at the time of the crisis.

The specification of the government spending and TFP shocks is taken from Arias et al. (2007) who
calibrate these parameters in a simple RBC model. Though they calibrate to the US, several authors have
noted that the Spanish economy before the crisis was not terribly different from the US in its cyclical
properties (see Puch and Licandro (1997)).

I calibrate $\pi_{RE}$ such that the average default lasts 2 years, which is roughly the length of Grecian
exclusion from international credit markets following its default in 2012. The results do not substantially
change even when this parameter varies widely. I also calibrate the haircut following default to match that
of Greece on average. In particular, note that the face value of a bond in default is given by
\[
\tilde{b}_t = \sum_{\tau=1}^{\infty} \pi_{RE}(1 - \pi_{RE})^{\tau-1} \left( \frac{\hat{\delta}}{R^*} \right)^{\tau} b_t \\
\rightarrow \frac{\tilde{b}_t}{b_t} = \sum_{\tau=1}^{\infty} \pi_{RE}(1 - \pi_{RE})^{\tau-1} \left( \frac{\hat{\delta}}{R^*} \right)^{\tau}
\]
I equate this expression to the average recovery rate of the face value of Greek debt, which was 29.5%, to determine the value of \(\hat{\delta}\).

I calibrate impatience,\(^{24}\) adjustment costs, the Frisch elasticity of labor supply, labor disutility, capital share of income, intertemporal elasticity of substitution, and capital depreciation to standard values in the RBC literature.

I take the fraction of average foreign debt to GNP as well as government spending to GNP direction from Spanish data.

### 5.4.4. Estimation Procedure

The remaining 5 parameters, which are those of most interest, are estimated using a Bayesian approach. I specify the priors in Table 1. I take a fairly agnostic stance with regard to the prior distributions, assuming that they all fall in the range \([0, 1]\)\(^{25}\) and that the government does not respond to adverse shocks 1 to 1 with taxes i.e. it smooths such shocks with debt. Changing these priors change the results only negligibly.

Their distribution is attained via a Random-Walk Metropolis Algorithm as outlined in Schorfheide and An (2007). To derive the likelihood of a particular parameterization, I solve the model to a first-order approximation using the algorithm described previously. I then place this approximation into state-space

\(^{24}\)This parameter is on the high end of standard discount rates. This simply helps to ensure that household cares enough about the future to disinvest when a default is likely. A low depreciation rate serves this same purpose.

\(^{25}\)The domain restriction imposed by the priors helps deliver comparable estimates across different models.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Median (CW)</th>
<th>Mean (CW)</th>
<th>Credible Set (CW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_b$</td>
<td>0.0584</td>
<td>0.0616</td>
<td>[0.0267, 0.1109]</td>
</tr>
<tr>
<td>$\gamma_g$</td>
<td>0.5095</td>
<td>0.5064</td>
<td>[0.4037, 0.5987]</td>
</tr>
<tr>
<td>$\gamma_r$</td>
<td>0.6441</td>
<td>0.6327</td>
<td>[0.3402, 0.8816]</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.0339</td>
<td>0.0472</td>
<td>[0.0080, 0.1394]</td>
</tr>
<tr>
<td>$p_L$</td>
<td>0.2204</td>
<td>0.2710</td>
<td>[0.0744, 0.6336]</td>
</tr>
</tbody>
</table>

**Table 2**: Posterior Statistics and 90% Credible Sets

form and evaluate the likelihood with the Kalman filter, taking the series on output, the public current account, and spreads to be my observables.\(^{26}\)

After specifying the prior, I follow the RWM outlined in Schorfheide and An (2007) to obtain a modal estimate and simulate draws from the posterior distribution. Although the entire distribution is of interest for constructing credible sets and understanding how the data operate, I am most interested in the point estimates given by the mode, since they will provide my estimates of probability of regime switches.

### 5.5. Results

In this section I present the results of the estimation. The mean and the median are similar in all regards, but for the purposes of inference I will take the posterior median to be my primary estimate. The posterior distributions can be found in Figure B.13 in Appendix B.

First, notice that the estimate of the probability of regime-switching is actually not that unlikely: The data suggest a median estimate of 3.39%, which corresponds to roughly 7.37 years. Recall that this figure is identified from spread data prior to the crisis and spread levels during the crisis. It does not use the relative frequency of the regimes. If we take 2002 to be the initial date at which investors fully anticipated monetary union for the foreseeable future, this figure seems quite reasonable: Investors expected such a crisis to occur on average every 7.37 years and it took about 7.5 years for one to occur. The closeness of this non-targeted moment speaks strongly to the validity of the model.

The mean estimate of $\eta$ is actually slightly higher than the median estimate: 4.72%. If we took this to be our estimate, it would imply that investors anticipated a crisis once every 5.3 years. This suggests that, if anything, investors anticipated such crises to occur more frequently than they actually did, not less frequently. Even though the credible set reaches all the way to 13.94%, Figure B.13 shows that most of

---

\(^{26}\)I initialize the Kalman filter mean at the non-stochastic steady state and the variance according to the non-crisis regime.
the mass is concentrated around the mean and median and that there is a long, thin tail on the right-hand side.

This high-frequency estimate helps us to reconcile the ‘low spreads puzzle’ of the Eurozone in the mid-2000s. Lane (2012) articulates this puzzle as follows: “(T)he low spreads on sovereign debt...indicated that markets did not expect substantial default risk and certainly not a fiscal crisis of the scale that could engulf the euro system as a whole.” My empirical analysis suggests quite the contrary: The Eurozone crisis may have been rationally anticipated and in fact priced. To see why, consider the position of an investor in the early 2000s. Such an investor did not fear an outright default, but rather a shift into a low-confidence regime next period. In such a regime, spreads, borrowing, and default probabilities will be significantly higher, but debt will still hold substantial value. Thus, it is the anticipation of this regime shift and not of a default that drove the spreads in this early period.

And these spreads were quite low for two complementary reasons. First, confidence processes are by necessity persistent, and so such a shift was an unlikely event. Second, even when confidence drops, debt still has value. Thus, the return on debt falls, but the value of the debt is not completely obliterated. These two forces can generate a very small but significant spread that is completely consistent with a panic of massive proportions unfolding quickly afterward.

I provide the model-implied shocks at the median estimate in Figure B.11 in Appendix B. Two interesting results can be gleaned from this figure. First, notice that the main driver of activity in this model is the government spending shock. This is because government spending in the model, as in the data, accounts for a large fraction of output: 40% for Spain. Also, in the baseline model government spending shocks directly translate to higher debt levels while productivity shocks do not since there is no cyclical policy. Since the model is matching the public current account, which moves cyclically, it will naturally place more weight on these shocks to explain this variation.

Second, the model suggests that the crisis was accompanied by a period of excessively low risk-free rates. It has generally been thought that the low risk-free rates during this period were associated with a monetary policy response. While this response did in fact lower borrowing costs, the model suggests that it may have raised spreads. This is because a negative shock to the risk-free rate will reduce the investors’ outside option today and tomorrow, but it will reduce it by more today than it will tomorrow because of
the mean-reverting nature of these shocks. Thus, the price of debt today will rise by more today than it is expected to rise tomorrow, which raises the dilution spread.

Last, let us turn to the estimated probability of default. Notice first that it is quite high in the model with confidence waves, with default expected to occur with probability 22.04% in any given quarter. This is in accordance with Proposition ??.

However, this default probability is not terribly well-identified, as can be seen by the broad span of the credible set. This is simply because the transition probabilities are pinned down by the mean of the spreads in each regime, and there are only 9 quarters in the data used to pin down the default transition. The credible set for the confidence-wave estimate is smaller because there are 36 quarters over which to average, implying greater accuracy in the face of unrelated lender shocks.

Notice, however, that the large credible set on $p_L$ does not translate to $\eta$, even though they are jointly identified.\(^{27}\) This is because information travels from $p_L$ to $\eta$ through the price, and there are several forces in the model that dampen the impact of $p_L$ on the price of debt. First, the assumption of long-term debt implies that the spread is partially dictated by the expectation of default and partially dictated by the expected future price of debt. Thus, changes in the default frequency may not impact the spread as much for long-term debt.\(^{28}\) Second, the inclusion of re-entry and haircuts mitigates the impact of default on investors’ return, since investors know that a default will not destroy all of their wealth, but only part of it.

In Appendix C I consider a slew of robustness exercises and tests of model fit. First, I examine the fit of the model along several relevant dimensions with posterior predictive distributions; next, I use posterior odds ratios to gauge the adequacy of my model against other candidate models, including a model without regime shifts and one driven by sudden loss of bailout expectations. By these metrics, I find that the data strongly favor my model.

5.6. Policy Experiment: Liquidity Provision

Given a set of estimated parameters, we can now begin to think seriously about the model’s implications for policy. The key policy question tends to revolve around the provision of liquidity by the European Central Bank: Should the ECB act as the lender of last resort in sovereign debt markets for the constituent

\(^{27}\)This is the reason why the specification of the exit probability from a crisis regime has little to no impact on ex-ante forecasts.

\(^{28}\)This is the typical reason given for why yield curves invert during crises.
countries of the Eurozone? We know that it did with the Outright Monetary Transactions bond-buying program, but was this optimal? The theoretical model of dynamic panics is ambiguous on this point, and so we need the data to provide the answer.

So-called fundamentalists, such as Issing (2011) have argued no, citing both the problem of unsustainable fiscal policy in the periphery and cross-subsidization from fiscally responsible countries to fiscally irresponsible ones. Others, such as De Grauwe (2011), take a multiple-equilibrium view, arguing that panic in financial markets caused self-fulfilling increases in the likelihood of default, since such panic raises the cost of debt repayment.

We can analyze the provision of liquidity in the context of this model with a simple exercise. Suppose that if the ECB begins providing liquidity, two things happen: First, confidence-waves are eliminated, since the ECB is not affected by market sentiments; and second, the country defaults more often because of greater limited commitment issues, since such provision encourages fiscal irresponsibility since member countries can rely on the ECB to purchase the debt and fill revenue gaps if times turn bad.

We can approximate the value function of the household in each regime, compute its certainty-equivalent consumption, and then ask ourselves whether provision of liquidity could improve on this in each regime. We certainly do not know by exactly how much the limited commitment problem would increase the default probability, but we can ask ourselves how bad it would have to be for liquidity provision to become sub-optimal. The estimated model suggests that liquidity provision can improve welfare so long as the limited-commitment-induced likelihood of default does not exceed 2.59% in any given quarter.

5.6.1. Procedure

In computing the welfare statistics, I need to ensure that my approximation of the value function is accurate for the task at hand. This is not an easy task, given that the regime-specific ‘steady states’ may be far away from the ergodic distribution of model objects.

I overcome this problem via simulation. In particular, I approximate the unconditional value of being

---

29 It is not difficult to demonstrate that an equilibrium always exists in the absence of confidence fluctuations. Chatterjee and Eyigungor (2012) provide a proof of this when the state space is discrete. An equilibrium of this type would come into action if confidence-waves were eliminated.

30 Since there is no default in the baseline model when confidence is high, it is assumed here that the limited commitment problem would be worse in non-crisis times, though it may be better than non-crisis times.

31 Given the GHH preferences, I take certainty-equivalent consumption to be the constant stream of consumption the household must receive if it provides \( l = 0 \) forever to be indifferent with its situation in the recursive equilibrium.

32 These should not be confused with the model’s non-stochastic steady state, which is regime-independent.
in one regime or another as follows:

1. Simulate the model for a long time e.g. $N_{inner} = 10,000,000$ quarters
2. Compute the average household value conditional on being in a given regime for each regime.
3. Invert this value to obtain the certainty equivalent consumption implied by the simulation.
4. Repeat Steps 1-3 many times e.g. $N_{outer} = 1000$
5. Take as an estimate of the certainty-equivalent consumption in each regime its average over the $N_{outer}$ simulations.

Given the stationarity of the model, this estimate will converge to the true certainty-equivalent consumption as $N_{inner} \to \infty$. The use of an outer loop $N_{outer}$ helps to speed up the process by providing independent estimates instead of relying on ergodicity. In practice, this generates estimates for CEC which vary from each other by less than $1e-4$, which I take to be the desired tolerance when comparing the efficacy of policy.

To determine the threshold default frequency under liquidity provision, I repeat the above procedure several times in the context of an interval bisection, since welfare will be decreasing in the likelihood of default.\textsuperscript{33}

\textbf{5.6.2. Welfare Results}

Figure 8 shows the difference in certainty-equivalent consumption between the model with liquidity provision and the baseline model for different degrees of the limited commitment problem. The threshold default frequency will equate the certainty-equivalent consumption under the liquidity policy to that in the baseline model. The graph shows this to be 2.59%, which translates to once every 9.65 years.

This figure is the same for crisis times and non-crisis times, so there is no time-inconsistency. The reason for this is because of the endogenous labor supply. When the economy enters a crisis, consumption and investment fall as taxes rise to service mounting debt burdens; however, labor supply also falls. The net effect of these two on welfare is essentially zero, though they have large implications for other model objects.

\textsuperscript{33}Though I have no guarantee that welfare is decreasing in the default probability, in every numerical application it is. This is because the costs of default together with the higher ex-ante spreads tend to drain welfare much more than expected debt repudiation raises it.
The default frequency in the absence of confidence-waves is lower than the frequency of confidence-waves in the baseline model. This does not speak to any particular welfare implications of the model, but simply reflects the fact that with confidence-waves it takes at least two periods to default instead of one. To match in welfare terms this default structure, defaults must occur less frequently if they are not preceded by a confidence-waves crisis.

So can we expect liquidity provision to be welfare-improving? The model puts no rigor on the frequency of such default as it pertains to the Eurozone, but we can compare it to the historical experience of other countries. For instance, Reinhart and Rogoff (2010) show that the average external default rates for Brazil and Greece are once every 20 and 31.3 years, respectively over the past two centuries. Since the welfare threshold for our estimated Spanish data is once every 9.65 years, the provision of liquidity by the central bank in the Eurozone is quite likely to be welfare-improving for its member countries.

It is important to note that the exercise here does not explicitly incorporate the welfare of all member countries.
of a monetary union, but only those in distress. There is a fear, express by Issing (2011) and others, regarding the cross-subsidization of member countries implied by liquidity provision facilities. However, as history has shown and as the model predicts, it is sufficient for the ECB to declare credibly that it is willing to purchase the debt of distressed member countries. They need not actually do it.\footnote{Some distressed countries, such as Greece and Ireland did receive rescue packages from the ECB's Emergency Lending Facility. However, it is not clear that at the time these amounted to liquidity provision, since the sovereign did not purchase the debt in secondary markets but rather lent directly to the country when that country faced exclusion from capital markets.}

6. Conclusion

In this paper, I characterized a new type of dynamic lender coordination problem, which I call dynamic panics. I demonstrated their existence in the standard quantitative sovereign debt model as well as characterized their basic properties. In particular, I showed that they appear as true panics in the sense of monotone price shifts, and that their existence implies that the uniqueness result associated with fundamental Markov-perfection is a fragile one. I also show that such panics can affect both long-term and short-term debt, but if they affect short-term debt it must at some point act through the default channel. However, with long-term debt we can have crises driven solely by borrowing behavior, which I argue occurred in the Eurozone periphery. I further demonstrated that in this environment interest rate ceilings are ineffective but that liquidity provision can eliminate the impact of market sentiments.

I then performed a structural estimation on a standard business cycle model with time-varying default probabilities Spanish data to estimate the ex-ante frequency of such panics as well as fiscal parameters. I outlined a new algorithm to speed up the computation of this class of models that can be generalized to increase their applicability. The estimation of this model told us that the median estimate of the probability of a confidence-wave crisis, as determined by spreads, is 3.39\%, which is validated externally by the actual frequency of such crises relative to the inception of the Euro and helps to resolve the low-spreads puzzle. Further, I demonstrated that the provision of liquidity by the ECB is welfare-improving provided it does not induce limited commitment based default more than once every 9.65 years on average.

This paper lays the groundwork for much potential future research. For instance, I have only just begun to outline the theoretical properties of these confidence-waves and have only been able to prove their existence for short-term debt, though computational examples with long-term debt can be found. An
existence theorem for the case of long-term debt would likely be quite enlightening.

Further, the dynamic panics may have significantly broader implications than simply in sovereign debt markets. There is no reason why we would not expect such panics in markets for, say, municipal debt or commercial paper. A more in-depth exploration of the potential for dynamic panics in these markets would also prove illuminating.

Last, the structural model I've developed here that incorporates exogenous default into a standard business cycle model can be expanded and used for forecasting. For instance, the existence of a non-stochastic steady state in a model with default can allow for a well-specified Taylor rule and would allow for a tractable exploration of the joint dynamics of monetary and default policy in more developed economies.

7. References


**Appendix A. Theoretical Proofs**

**Appendix A.1. Proof of Theorem 3.1**

To characterize the set of Confidence-Waves Equilibria, note first that there is only one way that \( \xi \) can have real effects: It must induce the sovereign to default in one state and repay in the other. If the sovereign defaults in both states, then confidence has no effects; the same is true if the sovereign repays in both states. Without loss of generality, let us search for an equilibrium in which the sovereign repays in \( \xi_H \) and defaults in \( \xi_L \).

If this is the case, then the equilibrium pricing function must be given as follows:

\[
q(\xi_L) = \frac{\eta}{R}, \quad q(\xi_H) = \frac{1 - \eta}{R}
\]
If we impose the default strategy in the continuation value of the sovereign, then we can write the Bellman of the sovereign conditional on repayment in $\xi_H$ as follows:

$$V(\xi_H) = u\left(y - b + \frac{1 - \eta}{R}b\right) + \beta [\eta X + (1 - \eta)V(\xi_H)]$$

Notice that if we difference this expression with the value of default and call this object $M(\xi_H) = V(\xi_H) - X$, then we have

$$M(\xi_H) = u\left(y - \frac{R - 1 + \eta}{R}b\right) - u(y - \phi(y)) + \beta(1 - \eta)M(\xi_H)$$

$$\rightarrow M(\xi_H) = \frac{u\left(y - \frac{R - 1 + \eta}{R}b\right) - u(y - \phi(y))}{1 - \beta(1 - \eta)}$$

In order for default to be the optimal response, it will be both necessary and sufficient that $M(\xi_H) \geq 0$. This condition is precisely the first assumption under the assumption of an increasing utility function.

We will also require that $V(\xi_L) - X = M(\xi_L) < 0$ i.e. default is optimal in the low-confidence state. This Bellman can be written as

$$V(\xi_L) = u\left(y - b + \frac{\eta}{R}b\right) + \beta [(1 - \eta)X + \eta V(\xi_H)]$$

We can again take the difference with $X$ to define $M(\xi_L)$:

$$M(\xi_L) = u\left(y - \frac{R - \eta}{R}b\right) - u(y - \phi(y)) + \beta \eta M(\xi_H)$$

$$\rightarrow M(\xi_L) = u\left(y - \frac{R - \eta}{R}b\right) - u(y - \phi(y)) + \frac{\beta \eta}{1 - \beta(1 - \eta)} \left[u\left(y - \frac{R - 1 + \eta}{R}b\right) - u(y - \phi(y))\right]$$

This last expression will be will be strictly less than zero if and only if the second assumption holds. In other words, the flow difference must be largely negative; enough so to compensate for the smaller but positive continuation value difference.

The intuition here is that the cost of debt service is greater than the default costs when confidence is low since debt prices are also very low. As such, it is no longer worthwhile to service the debt and default becomes optimal. However, when confidence is high so are debt prices and so the cost of debt service is now lower than default costs.

Thus, the two conditions in the theorem are both necessary and sufficient for the existence of Confidence-Waves Equilibrium.
Appendix A.1.1. Proof of Corollary 3.2

To see why this persistence holds, note that the second assumption requires the following to be true

\[ u(y - \phi(y)) - u \left( y - \frac{R - \eta b}{R} \right) > 0 \]

\[ \rightarrow \frac{R - \eta}{R} b - \phi(y) > 0 \]

i.e. the cost of debt service in the low confidence state is strictly greater than the default costs. If we take the difference of this expression with the first assumption, we arrive at the result:

\[ \frac{R - 1 + \eta}{R} b - \frac{R - \eta}{R} b < 0 \]

\[ R - 1 + \eta - R + \eta < 0 \]

\[ \eta < 1/2 \]

Appendix A.1.2. Proof of Corollary 3.3

Notice first that in any Confidence-Waves Equilibrium, it must be that

\[ \frac{R - 1 + \eta}{R} b \leq \phi(y) < \frac{R - \eta}{R} b. \]

If we increase the concavity, then the utility difference between the first and second of these terms will increase more than the utility difference between the utility difference between the second and third terms. But this will imply that the second condition of Theorem 3.1 will continue to hold.

Notice, however, that this result does not hold if we make \( u \) more convex. In fact, some Confidence-Waves Equilibria can disappear when this happens.

Appendix A.1.3. Proof of Corollary 3.5

First, let us find the conditions that govern the two non-sunspot equilibria: Full default and full repayment. Under the assumption of full-default, the price must be 0 in equilibrium. If we insert default as the optimal strategy in the continuation value, we derive the following Bellman:

\[ V = u(y - b) + \beta X \]

If we take the difference of this value with \( X \) we arrive at

\[ M = u(y - b) - u(y - \phi(y)) \]
We require that $M < 0$ in order for this to be an equilibrium, which will be true provided $b > \phi(y)$. Notice that this is implied by our second assumption, which requires that

$$\frac{R - \eta}{R} b - \phi(y) > 0$$

$$\Rightarrow b - \phi(y) > 0$$

Thus, if a Confidence-Waves Equilibrium exists, so too does the full-default equilibrium.

To verify the full-repayment equilibrium, the procedure is the same. Notice that the price here is $\frac{1}{R}$:

$$V = u\left(y - b + \frac{1}{R}b\right) + \beta V$$

$$\Rightarrow M = u\left(y - \frac{R - 1}{R}b\right) - u(y - \phi(y)) + \beta M$$

$$\Rightarrow M = \frac{u\left(y - \frac{R-1}{R}b\right) - u(y - \phi(y))}{1 - \beta}$$

We require that $M \geq 0$ in order for this to be an equilibrium, which is true provided $\frac{R-1}{R}b \leq \phi(y)$. But this follows direction from the first assumption which states that $\frac{R-1+\eta}{R} b \leq \phi(y)$. Thus, if a Confidence-Waves Equilibrium exists, so too does the full-repayment equilibrium.

Appendix A.1.4. Sunspots Randomizing over Non-Equilibrium Pricing Schedules: An Example

I provide a simple example here of a case in which sunspot activity can randomize over non-equilibrium pricing schedules. This example demonstrates that the regimes to which sunspots transition may not themselves be sustainable, which has important policy implications.

Consider again the simple model, but now suppose that $u(c) = c$ and that there is some simple intrinsic uncertainty as well. In particular, $y \in \{y_1, y_2\}$ and changes regimes with probability $p$. Continue to suppose that default costs can depend on $y$.

I now outline the conditions that define several non-sunspots equilibria. Using the techniques outlined in the proof of Theorem 3.1, it can be shown that a full repayment equilibrium exists if and only if the following condition holds:
Assumption FR: \( (\phi(y_i) - \frac{r}{1+r}b) + \frac{\beta p}{1 - \beta(1-p)} (\phi(y_{-i}) - \frac{r}{1+r}b) \geq 0 \) for both states, \( i \).

We can also find conditions under which an alternative equilibrium exists: One in which repayment occurs in \( y_2 \) but default occurs in \( y_1 \). This equilibrium can exist if and only if the following conditions hold:

Assumption INT1: \( \phi(y_2) \geq \frac{p + \eta}{1+r}b \)

Assumption INT2: \( \left( \frac{1+r-p}{1+r} + \frac{\beta p}{1 - \beta(1-p)} \frac{p + \eta}{1+r} \right)b > \phi(y_1) + \frac{\beta p}{1 - \beta(1-p)} \phi(y_2) \)

Now that we have outlined tightly how two non-sunspots equilibria can arise (or not arise), I will show how a sunspots equilibrium can randomize over these pricing schedules even if one of them is not an equilibrium. In particular, we will search for a Confidence-Waves Equilibrium in which the following holds:

\[
V(\xi_1, y_1) < X(y_1) \leq V(\xi_2, y_1) \\
X(y_2) \leq V(\xi_1, y_2) \leq V(\xi_2, y_2)
\]

Thus, we are searching for a sunspots equilibrium in which the sunspot randomizes over the RF default strategy and the INT default strategy. This pattern of default and repayment will suggest the following pricing schedule:

\[
q(\xi_1, y_1) = \frac{p + \eta - \eta p}{1 + r} \\
q(\xi_2, y_1) = \frac{1 - \eta + \eta p}{1 + r} \\
q(\xi_1, y_2) = \frac{1 - p + \eta p}{1 + r} \\
q(\xi_2, y_2) = \frac{1 - \eta p}{1 + r}
\]
Notice that, under sunspot persistence, the following is true:

\[
q(y_1) < q(\xi_1, y_1) < q(\xi_2, y_1) < \frac{1}{1 + r}
\]

\[
q(y_2) < q(\xi_1, y_2) < q(\xi_2, y_2) < \frac{1}{1 + r}
\]

where \(q(y_i)\) is the price of an INT equilibrium (if it exists, which it may not). Thus the sunspot is randomizing over these two regimes in which the sovereign follows either FR or INT.

In words, if this pricing schedule is an equilibrium, then it implies a welfare improvement over the interior solution without confidence waves. This is because the overall probability of default has fallen, since the sovereign does not always default in fundamental state 1. Even though it appears as if this might be a convexification over the risk-free equilibrium and the interior one, we will show that it is not in a moment.

To determine whether an sunspots equilibrium exists, define \(M_{ij}^k = V(\xi_i, y_j) - X(y_k)\). It is sufficient for the above equilibrium to exist provided that \(M_{11}^1 < 0\) and \(M_{21}^1, M_{12}^2, M_{22}^2 \geq 0\). Assuming the appropriate default/repayment scheme in the continuation value, we can difference the Bellmans of the value and default functions to derive these objects as functions of themselves as follows:

\[
M_{11}^1 = \phi(y_1) - \frac{1 + r - p - \eta + \eta p}{1 + r} b + \beta \eta (1 - p) M_{12}^2 + \beta p [(1 - \eta) M_{22}^2 + \eta M_{22}^2]
\]

\[
M_{21}^1 = \phi(y_1) - \frac{r + \eta - \eta p}{1 + r} b + \beta (1 - \eta) (1 - p) M_{11}^1 + \beta p [\eta M_{12}^1 + (1 - \eta) M_{22}^2]
\]

\[
M_{12}^2 = \phi(y_2) - \frac{r + p - \eta p}{1 + r} b + \beta p M_{21}^1 + \beta (1 - p) [(1 - \eta) M_{22}^2 + \eta M_{22}^2]
\]

\[
M_{22}^2 = \phi(y_2) - \frac{r + \eta p}{1 + r} b + \beta (1 - \eta) p M_{21}^1 + \beta (1 - p) [\eta M_{12}^1 + (1 - \eta) M_{22}^2]
\]

The above is a linear system of four equations in four unknowns. The analytic solution to this system is quite complicated and difficult to characterize (though feasible to find), but it is quite easy to determine computationally the solution for simple parameter values.

Consider the following parameterization, but which satisfies Assumptions FR and INT2, but which violates INT1. Thus, for these parameters, the full-repayment scheme is an equilibrium but the interior
For varying values of $\eta$ (non-fundamental persistence), we get the following differences. Recall that an equilibrium exists if $M_{11} < 0$ and the rest are positive.

<table>
<thead>
<tr>
<th>Value of $\eta$</th>
<th>0.0</th>
<th>0.002</th>
<th>0.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{11}^{11}$</td>
<td>-0.9703</td>
<td>-0.9681</td>
<td>-0.9507</td>
</tr>
<tr>
<td>$M_{11}^{21}$</td>
<td>0.1310</td>
<td>0.0468</td>
<td>-0.6068</td>
</tr>
<tr>
<td>$M_{12}^{12}$</td>
<td>-0.0138</td>
<td>0.0066</td>
<td>0.0372</td>
</tr>
<tr>
<td>$M_{22}^{22}$</td>
<td>0.3582</td>
<td>0.3101</td>
<td>0.0646</td>
</tr>
</tbody>
</table>

We can see that, as assumed, when $\eta = 0$ and there is no switching, there is no equilibrium. Also, if $\eta$ is too large, we lose the equilibrium as well. However, for some persistent process e.g. $\eta = 0.002$, a CW equilibrium does exist. Thus, in a sunspots equilibrium we can sustain a temporary pattern of default in state 1 and repayment in state 2 even when this pattern of default is not itself an equilibrium.

This is possible because INT is close enough to satisfying the equilibrium conditions i.e. it is a ‘potential’ equilibrium. In particular, the value of repayment in the high state is just a bit too small to justify repayment on its own. However, once we insert a sunspot that gives us the opportunity to jump back into a more beneficial regime, the price of repayment in $y_2$ increases enough to suddenly make repayment worthwhile. Thus the non-equilibrium INT regime is a potential equilibrium and thus can be randomized over in a sunspots equilibrium.
Appendix A.2. Proof of Proposition 4.2

When the equilibrium is not default relevant, then for any \((y, m, b)\), we will have either
\[ V(y, \xi, m, b) \geq X(y) \] or
\[ X(y) > V(y, \xi, m, b) \] for all \(\xi \in \Xi\). Further, if the debt is short-term, then we will have
\[ q(y, \xi_1, b') = 1 \]
\[ \rightarrow q(y, \xi_1, b') = q(y, \xi_0, b') = q(y, b') \]
and so the sunspot does not affect the price.

Appendix A.3. Proof of Proposition 4.7

This result follows because, ignoring the non-fundamental \(\xi\), the model becomes isomorphic to the model of Chatterjee and Eyigungor (2012). Thus, the existence result they provide for long-term debt without confidence fluctuations still holds. Denote this equilibrium price of debt to be \(q(y, b')\).

The European Central Bank can pledge liquidity by guaranteeing to purchase debt at a schedule \(q(y, b')\) for the foreseeable future. If it does so, it will induce the sovereign to adopt the policy rules from the equilibrium free of confidence shifts. When this happens, investors will lend to the sovereign at the price \(q(y, b')\), since it is in fact an equilibrium price, and the ECB never actually has to purchase the debt.

Appendix B. Computation and Estimation

Appendix B.1. Model Solution

We seek a solution of the form:
\[ y_t = g(x_{t-1}, \epsilon_t, \chi, s_t), \quad y_{t+1} = g(x_t, \chi \epsilon_t, \chi, s_{t+1}), \quad x_t = h(x_{t-1}, \epsilon_t, \chi, s_t) \] (B.1)

Foerster et al. (2013) demonstrate that a first-order approximation to the solutions \(g\) and \(f\) can be obtained in two steps. The first step entails solving the following quadratic system for \(\{D_{1,n_x}g_{ss}(s_t), D_{1,n_x}h_{ss}(s_t)\}^n_{s_t=1}\):

\[
A(s_t) \begin{bmatrix} I_{n_x} \\ D_{1,n_x}g_{ss}(1) \\ \vdots \\ D_{1,n_x}g_{ss}(n_s) \end{bmatrix} = B(s_t) \begin{bmatrix} I_{n_x} \\ D_{1,n_x}h_{ss}(s_t) \end{bmatrix}
\] (B.2)
for all \( s_t \). Where \( A(s_t) \) is an \((n_x + n_y) \times (n_x + n_s n_y)\) matrix given by

\[
A(s_t) = \left[ \sum_{s' = 1}^{n_s} p_{s_t, s'} D_{2n_y + 1, 2n_y + n_x f_{ss}(s', s_t), \ p_{s_t, 1} D_{1, n_y} f_{ss}(1, s_t), \ ..., \ p_{s_t, n_s} D_{1, n_y} f_{ss}(n_s, s_t) } \right] \tag{B.3}
\]

And \( B(s_t) \) is an \((n_x + n_y) \times (n_x + n_y)\) matrix given by:

\[
B(s_t) = - \sum_{s' = 1}^{n_s} p_{s_t, s'} \left[ D_{2n_y + n_x + 1, 2(n_y + n_x) f_{ss}(s', s_t)} \right] \tag{B.4}
\]

The second step uses the result from the first step and involves solving two linear systems, the first being

\[
\begin{bmatrix}
D_{n_x + 1, n_x + n_s g_{ss}(1)} \\
\vdots \\
D_{n_x + 1, n_x + n_s g_{ss}(n_s)} \\
D_{n_x + 1, n_x + n_s h_{ss}(1)} \\
\vdots \\
D_{n_x + 1, n_x + n_s h_{ss}(n_s)}
\end{bmatrix} = [\Theta_\epsilon, \Phi_\epsilon]^{-1} \Psi_\epsilon \tag{B.5}
\]

and the second being

\[
\begin{bmatrix}
D_{n_x + n_x + 1, n_x + 1 g_{ss}(1)} \\
\vdots \\
D_{n_x + n_x + 1, n_x + 1 g_{ss}(n_s)} \\
D_{n_x + n_x + 1, n_x + 1 h_{ss}(1)} \\
\vdots \\
D_{n_x + n_x + 1, n_x + 1 h_{ss}(n_s)}
\end{bmatrix} = [\Theta_\chi, \Phi_\chi]^{-1} \Psi_\chi \tag{B.6}
\]

The matrices \( \{\Theta_\epsilon, \Phi_\epsilon, \Psi_\epsilon, \Theta_\chi, \Phi_\chi, \Psi_\chi\} \) can be constructed from the solutions to the quadratic system and selected derivatives of the function \( f \). See Foerster et al. (2013) for more details.

The objects from the model that I match to the data are as follows:

- Output: \( y_t \)
- Public Current Account: \(-\frac{b_t - (1-\lambda)b_{t-1}}{y_t}\)
• Spread: $\frac{\lambda+(1-\lambda)(\kappa+\delta)}{q_t} - R_t$

Note that I compute the spread in the same way as Chatterjee and Eyigungor (2012), by assuming that the price tomorrow is expected to be the same as the price today.

**Appendix B.2. Proof of Theorem 5.1**

For the purposes of the following results, I denote the $i$th row and the $j$th column of $h_{ss}(k)$ to be $h_{ij}$. The same notation applies to the coefficients $g_{ij}$. I consider all rows of these matrices but only the first $n_x$ columns, since this is all that is required in the quadratic system. We can reduce the dimensionality of the quadratic problem over a series of three propositions.

(Proposition) **Appendix B.1.** The coefficients governing the stochastic process are predetermined. If row $i$ is an exogenous stochastic process, then we will have that $h_{ii}^k = \rho(i)$ for all states $k$, where $\rho(i)$ is the degree of persistence of the process in row $i$. All other elements in row $i$ must be zero.

**Proof** This follows mechanically since exogenous stochastic processes are, by definition, not affected by any of the other equilibrium objects. Further, an AR(1) process itself is already linear in its past values, with a coefficient equal to the persistence, so any valid approximation must reflect this. 

This simple and intuitive step reduces the dimensionality of the problem by $n_s n_x n_{exo}$, where $n_{exo}$ is the number of exogenous stochastic processes. In our case, the dimensionality of the problem drops by 45, which is a substantial improvement but nowhere near large enough yet. We can continue the reduction, though, with another result, which follows from exogeneity of the fiscal rule:

(Proposition) **Appendix B.2.** The coefficients governing the evolution of government debt process are predetermined.

**Proof** This follows from the exogeneity of the fiscal rule. The impact of interest rates, productivity, and policy shocks as well as the impact of past debt can be determined via the relevant derivatives of this fiscal rule.
This step reduces the dimensionality by \( n_x n_s = 15 \). With the exogenous processes predetermined, I now turn to the consumption-saving decision, which is at the crux of the model solution:

(Proposition) Appendix B.3. Given coefficients on investment movements, the coefficients on capital and consumption movements are uniquely determined. Coefficients on capital movements will be the same as the coefficients on investment movements with the exception of capital itself, which requires an additional \( 1 - \delta \). If row \( i' \) corresponds to consumption and row \( i \) corresponds to capital, then \( g^k_{i'j} = f^k_j - h^k_{ij} \). \( f^k_j \) will either be a known constant or some linear combination of other unknowns of the matrices \( h^k \) and \( g^k \).

Proof This result comes from the fact that the problem faced by the household is the marginal allocation of additional income to capital. Thus, given a positive shock to the budget set of the household, if we know how much of that additional income was allocated to capital, then we simply subtract that amount from the size of the shock to determine how much was allocated to consumption. Therefore, the constants \( f^k_j \) are simply the impact on the budget constraint in state \( k \) from a unit shock to the state variable in column \( j \).

In my case, the set \( f^k_j \) are known constants. This result, upon implementation, reduces the dimensionality by \( 2 \times n_x n_s \), which in our case reduces the dimensionality by 30. This proposition is useful because it is extremely applicable. The consumption-saving decision is the cornerstone of modern macroeconomics and nearly every recursive problem entails this decision at some level. Thus, this technique could have near universal applicability for those seeking to implement the method of Foerster et al. (2013) to derive an approximation to a given equilibrium.

We can reduce the dimensionality once more before we actually solve for the approximation with the following proposition:

(Proposition) Appendix B.4. Suppose that the consumption coefficients have been removed from the system using the techniques described thus far. If \( i \) is the row containing the investment coefficient, a solution to a quadratic system of size \( k \) entailing only \( \{h^k_{ii}\}_{k=1,n_x} \) is both necessary and sufficient to solve the entire quadratic system.

To see this result, note that after the requirements thus far that have been imposed will imply a quadratic system of 15 equations in 15 unknowns: These unknowns are the linear response of investment
to the 5 states variables in each of the 3 states. This system can be stated as follows after some tedious
algebra (or, more easily, by use of symbolic engine) for a set of parameter-determined constants \( \{c_{i,j}\} \):

\[
0 = c_{1,0} + c_{1,1} \hat{i}_{k,1} + c_{1,2} \hat{i}_{k,2} + c_{1,3} \hat{i}_{k,3} + \sum_{i=1}^{3} c_{1,i} \hat{i}_{k,i}
\]

\[
0 = c_{2,0} + c_{2,1} \hat{i}_{k,2} + c_{2,2} \hat{i}_{k,2} + c_{2,3} \hat{i}_{k,3} + \sum_{i=1}^{3} c_{2,i} \hat{i}_{k,i}
\]

\[
0 = c_{3,0} + c_{3,1} \hat{i}_{k,3} + c_{3,2} \hat{i}_{k,3} + c_{3,3} \hat{i}_{k,3} + \sum_{i=1}^{3} c_{3,i} \hat{i}_{k,i}
\]

\[
0 = c_{4,0} + \sum_{i=1}^{3} \sum_{j=1}^{3} c_{4,(i-1)*3+j} \hat{i}_{k,j} + \sum_{i=1}^{3} c_{4,i} \hat{i}_{k,i} + \sum_{i=1}^{3} c_{4,i+12} \hat{i}_{k,i}
\]

\[
0 = c_{5,0} + \sum_{i=1}^{3} \sum_{j=1}^{3} c_{5,(i-1)*3+j} \hat{i}_{k,j} + \sum_{i=1}^{3} c_{5,i} \hat{i}_{k,i} + \sum_{i=1}^{3} c_{5,i+12} \hat{i}_{k,i}
\]

\[
0 = c_{6,0} + \sum_{i=1}^{3} \sum_{j=1}^{3} c_{6,(i-1)*3+j} \hat{i}_{k,j} + \sum_{i=1}^{3} c_{6,i} \hat{i}_{k,i} + \sum_{i=1}^{3} c_{6,i+12} \hat{i}_{k,i}
\]

\[
0 = c_{7,0} + \sum_{i=1}^{3} \sum_{j=1}^{3} c_{7,(i-1)*3+j} \hat{R}_{i} \hat{i}_{k,j} + \sum_{i=1}^{3} c_{7,i} \hat{R}_{i} \hat{i}_{k,i} + \sum_{i=1}^{3} c_{7,i+12} \hat{R}_{i} \hat{i}_{k,i}
\]

\[
0 = c_{8,0} + \sum_{i=1}^{3} \sum_{j=1}^{3} c_{8,(i-1)*3+j} \hat{R}_{i} \hat{i}_{k,j} + \sum_{i=1}^{3} c_{8,i} \hat{R}_{i} \hat{i}_{k,i} + \sum_{i=1}^{3} c_{8,i+12} \hat{R}_{i} \hat{i}_{k,i}
\]

\[
0 = c_{9,0} + \sum_{i=1}^{3} \sum_{j=1}^{3} c_{9,(i-1)*3+j} \hat{R}_{i} \hat{i}_{k,j} + \sum_{i=1}^{3} c_{9,i} \hat{R}_{i} \hat{i}_{k,i} + \sum_{i=1}^{3} c_{9,i+12} \hat{R}_{i} \hat{i}_{k,i}
\]

\[
0 = c_{10,0} + \sum_{i=1}^{3} \sum_{j=1}^{3} c_{10,(i-1)*3+j} \hat{g}_{i} \hat{i}_{k,j} + \sum_{i=1}^{3} c_{10,i} \hat{g}_{i} \hat{i}_{k,i} + \sum_{i=1}^{3} c_{10,i+12} \hat{g}_{i} \hat{i}_{k,i}
\]

\[
0 = c_{11,0} + \sum_{i=1}^{3} \sum_{j=1}^{3} c_{11,(i-1)*3+j} \hat{g}_{i} \hat{i}_{k,j} + \sum_{i=1}^{3} c_{11,i} \hat{g}_{i} \hat{i}_{k,i} + \sum_{i=1}^{3} c_{11,i+12} \hat{g}_{i} \hat{i}_{k,i}
\]

\[
0 = c_{12,0} + \sum_{i=1}^{3} \sum_{j=1}^{3} c_{12,(i-1)*3+j} \hat{g}_{i} \hat{i}_{k,j} + \sum_{i=1}^{3} c_{12,i} \hat{g}_{i} \hat{i}_{k,i} + \sum_{i=1}^{3} c_{12,i+12} \hat{g}_{i} \hat{i}_{k,i}
\]

\[
0 = c_{13,0} + \sum_{i=1}^{3} \sum_{j=1}^{3} c_{13,(i-1)*3+j} \hat{z}_{i} \hat{i}_{k,j} + \sum_{i=1}^{3} c_{13,i} \hat{z}_{i} \hat{i}_{k,i}
\]

\[
0 = c_{14,0} + \sum_{i=1}^{3} \sum_{j=1}^{3} c_{14,(i-1)*3+j} \hat{z}_{i} \hat{i}_{k,j} + \sum_{i=1}^{3} c_{14,i} \hat{z}_{i} \hat{i}_{k,i}
\]

\[
0 = c_{15,0} + \sum_{i=1}^{3} \sum_{j=1}^{3} c_{15,(i-1)*3+j} \hat{z}_{i} \hat{i}_{k,j} + \sum_{i=1}^{3} c_{15,i} \hat{z}_{i} \hat{i}_{k,i}
\]
Let us denote a solution to this system by \( I^n = \{i^n_{j,i}\}_{j \in \{k,b,R,g,z\}, i=1,3} \), where \( n \) indexes the solution from a possible set of many solutions. We can glean from this system that the first the equations are in fact an isolated quadratic system i.e. they contain 3 equations in 3 unknowns: \( i_{k,1}, i_{k,2}, i_{k,3} \). None of the other coefficients enter into this system. Further, conditional on having a solution to this system, the remaining 12 equations are linear in their 12 unknowns.

Thus, we can find a solution to this large system as follows:

1. Solve the subsystem given by the first three equations for all solutions, \( \{i^n_{k,1}, i^n_{k,2}, i^n_{k,3}\}_{n=1,N} \), where there are \( N \) determinate solutions.
2. For each solution, \( n \leq N \), fix \( \{i^n_{k,1}, i^n_{k,2}, i^n_{k,3}\} \) as constants and solve Equations 4-15 as a linear system.

This will, of course, yield either no solution, a unique solution, or a continuum of solutions for each \( n \).

Let \( \hat{N} \leq N \) be number of total solutions for which the linear system for which Step 2 yields a unique solution.\(^{35}\) If at \( n \) the linear system had no solution, then \( \{i^n_{k,1}, i^n_{k,2}, i^n_{k,3}\} \) could not have formed the basis of a solution in the first place, and if at \( n \) the linear system had a continuum of solutions, then the approximation (not the equilibrium) would be indeterminate and thus of no use.

Now, I argue that this procedure yields all determinate approximations to the entire system and only those determinate approximations. The latter claim is easy to understand: By construction, any solution constructed with this procedure must be a determinate equilibrium.

The former is also fairly trivial: Suppose that there was another solution, \( \hat{I}^n \), to the entire system that was not found via this procedure. Then we could isolate the terms \( \{i^n_{k,1}, i^n_{k,2}, i^n_{k,3}\} \) from this solution and apply them to the first three equations. Because the procedure did not find them, this subsystem will not be satisfied. But this contradicts the fact that \( \hat{n} \) was indeed a solution to the system, since all conditions do not hold with equality. Thus, our procedure must find all valid solutions and only valid solutions. \( \Box \)

\(^{35}\)In practice, both \( N \) and \( \hat{N} \) almost invariably equal 8.
### Appendix B.3. Fixed Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Rationale</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>2</td>
<td>RBC Standard</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.33</td>
<td>RBC Standard</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.999</td>
<td>RBC Standard</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.005</td>
<td>RBC Standard</td>
</tr>
<tr>
<td>$z_H$</td>
<td>1.0</td>
<td>RBC Standard</td>
</tr>
<tr>
<td>$\Phi''$</td>
<td>0.455</td>
<td>RBC Standard</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.4</td>
<td>RBC Standard</td>
</tr>
<tr>
<td>$\psi$</td>
<td>1.5</td>
<td>RBC Standard</td>
</tr>
<tr>
<td>$R^*$</td>
<td>1.01</td>
<td>RBC Standard</td>
</tr>
<tr>
<td>$z_L$</td>
<td>0.95</td>
<td>Mendoza and Yue (2012)</td>
</tr>
<tr>
<td>$b^*$</td>
<td>$-0.19653 \times y_{ss}$</td>
<td>Spanish Data (ECB SWD)</td>
</tr>
<tr>
<td>$g^*$</td>
<td>$0.375289 \times y_{ss}$</td>
<td>Spanish Data (ECB SWD)</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.95</td>
<td>Arias et al. (2007)</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.0031</td>
<td>Arias et al. (2007)</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>0.0077</td>
<td>Arias et al. (2007)</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>0.98</td>
<td>Arias et al. (2007)</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>0.0063</td>
<td>Neumeyer and Perri (2005)</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>0.81</td>
<td>Neumeyer and Perri (2005)</td>
</tr>
<tr>
<td>$\pi_{RE}$</td>
<td>0.125</td>
<td>Match Grecian Exclusion</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.0385</td>
<td>Match Average Maturity</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.04</td>
<td>Match Average Coupon</td>
</tr>
<tr>
<td>$\hat{\delta}$</td>
<td>0.7861</td>
<td>Equation 15</td>
</tr>
<tr>
<td>$p_{IH}$</td>
<td>0.0</td>
<td>Calibrated (Identification)</td>
</tr>
</tbody>
</table>

**Table B.3:** Fixed Parameters
Appendix B.4. Kalman Filter: Observables

Figure B.9: Model-Implied Observables: Data

Figure B.10: Model-Implied Observables: Baseline Kalman Filter Predictions
Appendix B.5. Kalman Filter: Model Components

Figure B.11: Exogenous Shocks: Deviations from Steady State

Figure B.12: Endogenous State Variables: Deviations from Steady State
Appendix B.6. Posterior Distribution

Figure B.13: Posterior Distribution of Baseline Model

Appendix C. Robustness and Model Fit

I present in this section several tests of robustness and model fit for my empirical specification. I find that along all relevant dimensions, the data largely favor my specification.

Appendix C.1. Posterior Predictive Distributions

I explore here the predictions of my estimated model against the data itself with regards to several key moments. To do this, I follow techniques outlined in Geweke (2005) and discussed in Geweke (2007). In particular, I do the following:
1. For every draw from the posterior distribution $\theta^{(i)}$, simulate the model for a long time i.e. $T = 100000$ and record key moments in a vector $z^{(i)}$

2. Compare the distribution of $z^{(i)}$ against the same moments computed from the data itself, $z$

The results of this experiment can be found in Figure C.14. The moments I choose are the mean and volatility of the public current account deficit (PCA) as well as the spread. I also look at the cyclicality of the public current account as well as the correlation between the public current account and the spread. I compute all of these statistics both conditional on a non-crisis state and conditional on a crisis state. The 90% credible sets are outlined in blue, the red line gives the median, and the black lines denote the quartiles. The green dots represent these statistics computed from the data itself.

![Figure C.14: Posterior Predictive Distributions of Baseline Model](image)

Before I discuss the results, note first that the moments computed from the data, especially during the crisis, are computed from relatively few observations. I only have 9 quarters of crisis observations. Thus, in a frequentist sense, the moments taken from the data are likely not ‘accurate’. Nevertheless, we can see that the model and the data follow the same trends along each dimension and in many cases the model accurately predicts these moments.
First, note that in all cases the model gets the direction correct. For example, during a crisis the public current account deficit is in expectation higher but its volatility is lower; spreads, on the other hand, are higher and more volatile during a crisis. Both of these are true in the estimated model as well. We can see further that the model does a very good job of matching spreads before the crisis, which is precisely the parameter used to identify $\eta$. Thus, we can rest assured that our estimate of $\eta$ is likely quite consistent with the actual data, even if the estimated default frequency during a crisis is not.

We also see that virulent spreads tend to be more correlated with the deficit during crisis times and that the model predicts that the public current account is more counter-cyclical during a crisis, which is true in the data as well. This last feature of the model is particularly desirable. It implies that during non-crisis times the economy behaves more like a developed economy, driven largely by TFP and government spending shocks and with the government tending toward consumption smoothing. During a crisis, however, it begins to look more like a developing economy: The risk-free rate shock begins to play a more crucial role since the higher spreads magnify these shocks. This in turn creates a strongly counter-cyclical current account, which accords with the data (see Aguiar and Gopinath (2006) or Neumeyer and Perri (2005)).

Appendix C.2. Alternative Model Specifications

To determine if my model captures the data accurately, I now estimate a pair of competing models. I will call the baseline model $M_A$. First, I consider a fairly trivial model, which is simply the baseline model without regime-shifts. In other words, I impose that $p_L = p_H$ and that $\eta = 0.0$. I will call this model $M_B$.

Next, I consider a variant of the model that allows for bailouts. I call this model $M_C$. In particular, I assume that there is a constant probability of default, $p_D$, over the entire horizon. However, during the mid-2000s investors anticipated a full bailout upon a default i.e. $\hat{\delta} = 0$ and only 1 period exclusion from credit markets. The crisis was then a period in which investors suddenly and unexpectedly ceased to expect a bailout. In other words, there is some new, non-zero $\hat{\delta}_c$ that I will estimate and which is followed by a longer period of credit market exclusion.

To solve model $C$, I consider 4 regimes instead of 3. There are two default regimes, one with a bailout and one without, and two non-default regimes, one that expects a bailout during a default and one that does not. I will interpret the crisis to be switch from anticipating bailouts to not.
The estimates and 90% credible sets for all three models are given in Table C.4. We can glean from casual inspection that none of the fiscal rules differ from one another statistically; they are all within one another’s credible sets. Nevertheless, the models differ starkly from one another in terms of their overall fit. The last row in the table contains the posterior odds ratio of the three models, which is the ratio of the model likelihoods integrating out the uncertainty with respect to the parameters in each case. I normalize these figures such that they sum to one. We can see that this figure is .9754 for my specification, which clearly and necessarily dominates both of the other two specifications. This implies that the data strongly favors my baseline specification to these plausible alternatives.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Median ($M_A$)</th>
<th>Credible Set ($M_A$)</th>
<th>Median ($M_B$)</th>
<th>Credible Set ($M_B$)</th>
<th>Median ($M_C$)</th>
<th>Credible Set ($M_C$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_b$</td>
<td>0.0584</td>
<td>[0.0267, 0.1109]</td>
<td>0.0646</td>
<td>[0.0280, 0.1330]</td>
<td>0.0418</td>
<td>[0.0198, 0.0846]</td>
</tr>
<tr>
<td>$\gamma_g$</td>
<td>0.5095</td>
<td>[0.4037, 0.5987]</td>
<td>0.4869</td>
<td>[0.3579, 0.5813]</td>
<td>0.5349</td>
<td>[0.4353, 0.6172]</td>
</tr>
<tr>
<td>$\gamma_r$</td>
<td>0.6441</td>
<td>[0.3402, 0.8816]</td>
<td>0.6319</td>
<td>[0.3050, 0.8626]</td>
<td>0.6115</td>
<td>[0.3073, 0.8534]</td>
</tr>
<tr>
<td>$\eta(\delta)$</td>
<td>0.0339</td>
<td>[0.0080, 0.1394]</td>
<td>N/A</td>
<td>N/A</td>
<td>0.6695</td>
<td>[0.3894, 0.8829]</td>
</tr>
<tr>
<td>$p(D</td>
<td>M_i</td>
<td>y)$</td>
<td>0.2204</td>
<td>[0.0744, 0.6336]</td>
<td>0.0297</td>
<td>[0.0072, 0.0556]</td>
</tr>
</tbody>
</table>

| $\sum_i p(M_i|y)$ | 0.9754 | 0.0228 | 0.0017 |

**Table C.4:** Posterior Statistics and 90% Credible Sets

It is not surprising that $M_B$ is rejected, since there are fewer parameters estimated and therefore the model imposes more restrictions on the empirical inference. What is more surprising is that $M_C$ is strongly rejected as well, since there are as many estimated parameters in this model as in $M_A$. The reason for the rejection is that, while $M_C$ is able to replicate well the spread dynamics of the data, the response of the real economy is required to be homogeneous before and during the crisis, since the real probability of default never changed. To the contrary, the data suggest a strong, negative response from the domestic economy in response to the crisis. This response can never be captured in $M_C$, which only features bailout dynamics.

Finally, notice that estimated probability of default is significantly higher in the model with dynamic panics. This result is the least surprising, since the dynamic panics model has the ability to fit two different default probabilities while the other models are required to fit only one. The model without such panics thus places the uniform default probability somewhere between the crisis and non-crisis estimates.