Experimentation in Organizations

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January 5, 2014

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Abstract

I consider a dynamic moral hazard model in which a principal provides incentives to a team of agents who work on a risky project. The project involves several milestones of unknown feasibility. At each point in time agents exert private effort. While agents exert effort without achieving milestones, their private belief in the feasibility of the project declines. This learning gives rise to rents. Agents have incentives to delay effort and free-ride on other agents' discoveries when the principal attempts to extract full surplus. In the revenue maximizing contract the amount of experimentation is inefficiently low. Agents' contracts are highly sensitive to their performance in early stages. Agents who succeed are rewarded with bonuses, reduced competition, more leeway to experiment and higher bonuses conditional on success later in the project. The principal prefers to reward agents for early successes with better contract terms or promotions rather than with monetary bonuses. I provide conditions under which projects start small, with some workers sitting idle until a milestone is reached. Under these conditions identical agents face ex-ante asymmetric contracts. My results can be applied to the design of contests for innovation.

Keywords: principal-agent, moral hazard in teams, experimentation, two-armed bandit, contests.

JEL Codes: D82, D83, D86.

*I am grateful to Johannes Hörner, Larry Samuelson and Dirk Bergemann for their invaluable guidance and support. I would also like to thank Alessandro Bonatti, Joyee Deb, Rahul Deb, Yeon-Koo Che, Florian Ederer, Yingni Guo, Yuhta Ishii, Yuichiro Kamada, Adam Kapor, Chiara Margaria, Aniko Oery, Anne-Katrin Roesler, Ennio Stacchetti and the seminar audiences at Yale and the game theory conference at Stony Brook for their insightful comments.

1 Introduction

Motivation. Most innovative activity takes place in groups and organizations. Most potentially lucrative projects require a large amount of work, and one individual's labor will not suffice.[1] It is difficult, however, to design an environment that supports innovation. As people work on risky but potentially lucrative projects, they will learn from their own outcomes and from their coworkers' about the project's feasibility. This source of dynamic private information makes it difficult for a principal or manager to provide incentives.[2]

In this paper, I develop a model of experimentation in teams and solve for the optimal (profit-maximizing) contract. A manager (principal) contracts with a group of workers (agents) to complete a project. The project consists of multiple milestones of unknown feasibility, each of which has to be achieved for the project to yield a final payoff. I model this setting as a sequence of experiments. The agents experiment simultaneously and each agent has private information about his effort provision. As the agents experiment they privately learn about the feasibility of each stage. The principal chooses a history-contingent payoff scheme to incentivize agents to exert effort at each time. The principal has the ability to commit to a contract.

The literature on contracts for experimentation focuses mainly on principal-agent relationships with a *single* agent in which all uncertainty is resolved after a *single* success.[3] However, projects typically involve many milestones that need to be reached and have many possible points of failure. The workers in the organization interact through all these stages until a project is abandoned or completed. Workers' beliefs in the feasibility of the project will increase after they achieve milestones and decreases when time passes without progress. For example, a founder of a start-up hires a group of engineers to develop a new product. The start-up needs to get enough funding, produce a prototype, scale production and promote the product to the public. All of these steps are uncertain and crucial for the success of the new business.

The key features of the model are 1) there are multiple agents. 2) Innovations can involve multiple milestones that have to be completed for the project to yield a final payoff. Each milestone might be unachievable with some probability. 3) The agents are subject to limited liability.

[1] According to a recent Harvard Business Review article, "Today, innovation requires capabilities, experience, relationships, expertise, and resources of big organization". (S. Anthony, "The New Corporate Garage", Harvard Business Review [serial online]. September 2012;90(9):44-53. Available from: Business Source Complete, Ipswich, MA. Accessed October 31, 2014.)

[2] According to the CEO survey CEO Challenge 2004: perspectives and Analysis, The Conference Board, Report 1353, "stimulating innovation, creativity and enabling entrepreneurship" is "the greatest human resource challenge" facing organizations.

[3] See, for example, Bergemann and Hege (2005), Bergemann and Hege (1998), Hörner and Samuelson (2013) and Halac, Kartik, and Liu (2013).

In order to maximize profits, the principal implements inefficiently low levels of experimentation. Initially the principal and agents are optimistic about the project. The principal would like to offer low payments for success as the probability of a breakthrough is relatively high. As agents work without a success, however, their posterior belief about the feasibility of the project falls. As a result, in order to induce effort the principal must offer higher payments after time has passed without a breakthrough. If payments for success are sufficiently higher in the future, however, agents may gain from delaying their effort. The agents receive rents to prevent them from delaying effort. These rents are larger when agents have more leeway to experiment.

In early stages of the project, agents must also be given rents to not free ride on other agents' discoveries. Agents have the option of exerting no effort and waiting for their coworkers to achieve a milestone, after which they receive the rents that were needed to prevent them from delaying effort.

The optimal contract has three key features. First, when it is relatively costly for the principal to deter free-riding and the value of the project is relatively low, the optimal contract excludes some agents from participating in the early stages until a milestone is reached. Thus, the number of agents in the project grows. Even when agents are identical, the principal may assign ex-ante asymmetric levels of experimentation. Second, I find that the principal prefers to reward an agent for an early success with a better experimentation assignment in the future, rather than a monetary bonus. Early in the project, an agent does not receive bonuses unless the value of allocating more responsibility to him is negative. Because it is profitable for the principal to distort experimentation down, when the principal has to reward an agent, she prefers to do so by reducing the distortion in experimentation. Third, agents' contracts are sensitive to their performance in the early stages. Agents who succeed early are rewarded with reduced competition, more opportunities to succeed and higher bonuses conditional on success later in the project. Agents who fail in early stages are assigned less experimentation or are allocated to less valuable, low risk projects in later stages.

My results shed light on how a government might set up a contest for innovation. Suppose that a government is interested in the development of a vaccine.[4] If the development of the vaccine cannot be divided into multiple milestones, the profit maximizing contest involves setting a schedule of increasing prizes. As long as the vaccine is not discovered the prize for it increases. If a success is not achieved before a time threshold the prize ceases to increase and the contest is abandoned. The contest ends as soon as the first contestant solves the problem successfully, and successes are announced immediately to the remaining participants. If the development of the vaccine can be divided into steps, the contest designer gives prizes for the intermediate discoveries and rewards

[4] Kremer (2001) discusses the WHO/World Bank proposal on how to provide incentives for the development of vaccines for illnesses that affect poor countries.

the winners of the early stages with better terms and longer deadlines in the later stages.

I consider the case of a project consisting of a single experiment in which the principal learns about an agent's breakthrough and can choose whether to disclose it to the other agents. When an agent achieves the first breakthrough, an agent that has not learned about it may continue to work and receive a bonus if he obtains a breakthrough later. Non-disclosure may be beneficial for the principal because she can offer lower bonuses for breakthroughs. However, non-disclosure involves duplication of effort. I show that the optimal disclosure policy involves immediate disclosure to all agents and, thus, exhibits no duplication of effort.[5]

Finally, I show that the basic results are preserved in a more general setting in which agents learn about the project they are involved in as they work. The agents' work affects the rate at which a verifiable signal–say for instance, a breakthrough or a breakdown–arrives. I show that an agent receives rents as long the slope of the rate at which verifiable signals arrive is strictly decreasing in his effort in some time interval. In this case, when the principal attempts to extract full surplus, the agents have incentives to delay effort. As a result, any learning process in which, at any point during the project, the agent becomes more pessimistic about obtaining a verifiable signal as he exerts effort, will imply that the principal cannot extract full surplus. Thus, many conclusions of my model apply to more general settings. When agents receive rents, competition is useful to discipline them. The principal can use assignments of responsibility to reward agents and the agents have to be given rents to prevent them from free-riding on other agents' verifiable signals in early stages.

Conversely, if at every point in time verifiable signals become more likely when agents exert effort, the principal can extract full surplus. I apply my results to a model in which there are two verifiable signals: breakdowns and breakthroughs. There are two states of the world: either a project is "good" and gives a breakthroughs at some rate, or it is "bad" and gives breakdowns at some rate. The principal extracts full surplus by rewarding the event that becomes more likely as the agents exert effort.

Analysis. I analyze the dynamic relationship between a principal and a group of agents who are working on a new innovative product. I model innovation as a sequence of experiments with exponential bandits. The project only yields a positive profit once all the sequential experiments have been successful. Each experiment represents a milestone or a task that needs to be completed for the project to be profitable. Agents continuously choose unobservable and costly effort. If a given milestone is feasible each agent achieves a success at a rate proportional to his effort. As the

[5]This result is in contrast with Halac, Kartik, and Liu (2014). In their paper, because the contest designer has either a fixed budget or a fixed bonus it is sometimes optimal to share the prize among all the agents who succeed.

agents exert effort in each task they become more pessimistic about the feasibility of that task. The attainment of a milestone is publicly observed and once it occurs all agents proceed to experiment on the next task. The principal has to decide the level of effort that each agent should exert and design a contract such that the agents will find it optimal to exert the desired level of effort at each time. Agents have limited liability.

Notice that if the interaction between the principal and the agents were static, the principal could extract all surplus from the relationship. The principal would offer each agent a contract that pays an agent only when he obtains an innovation, with a bonus that in expectation exactly makes up for the agent's cost of effort. This contract satisfies limited liability and gives the agent zero payoff for every level of effort an agent may choose and therefore, in particular, it is optimal for the agent to exert the maximal effort at that time.

In contrast, in this dynamic setting the principal cannot extract full surplus. Consider a project that consists of a single risky task. Define a full-rent contract as one that gives in expectation the cost of effort at each time and therefore leaves the agent with zero expected payoff. There is no non-zero effort function that can be implemented by a full-rent contract. When an agent is offered the full-rent contract he has a profitable deviation from a strictly positive effort. By exerting zero effort for a time interval and exerting his allocated effort thereafter he can guarantee a strictly positive payoff. During the time interval in which no effort is exerted the payoff is zero, which is the same as he gets in the full-rent contract at the allocated effort. After the interval the agent is more optimistic about obtaining a success than he would have been if he had exerted the allocated effort. Since the full-rent contract makes an agent that has behaved as expected just indifferent between exerting effort or not, it must give an agent who is more optimistic than expected a strictly positive payoff. It follows that in the optimal contract the agents have to be given information rents because of their unobservable effort costs. These information rents are such that agents are just indifferent between exerting effort at any one time and delaying effort to the next instant. Since these rents arise because of the agents' incentives to shift effort to the future, I call these rents *procrastination rents*. The principal faces a trade-off between efficiency and information rents. As a result, experimentation is low, relative to the first best.

Consider now a project that consists of two tasks. From the discussion above the agents receive rents above their cost of effort in the final task. Thus, agents expect strictly positive rents after another agent reaches the milestone that solves the first task. If agents were just indifferent between exerting effort at two consecutive instants in the absence of positive payoffs following other agents' successes, they now have strict incentives to delay effort. By slacking, agents save the cost of effort and receive a strictly positive payoff in the event that another agent completes the first task. As a result, the optimal contract has to give agents information rents to not free-ride on the other agents' efforts earlier in the project, in addition to the no-procrastination rents.

Free-riding is so costly to the principal in some cases that she prefers to keep agents out of the early stages and add them later when the first hurdles are overcome and the reward from the project is closer at hand. That is, it is optimal for some projects to start small, with few workers, while other available workers sit idle.

When the costs of giving incentives in the first task are high relative to the costs in the second task, free riding-rents give rise to distortions. In particular, the principal distorts the second task experimentation of the agents who do not succeed in the early task. When the agents expect a high payoff after another agent reaches a milestone, they have to be given high rents to prevent them from free-riding. The principal lowers these rents, at her own cost, by lowering the amount of experimentation, and thus the information rents, of the losing players in the second task. As a result, the agents who do not succeed are assigned an amount of experimentation in the second task that is even more inefficient than the amount they are assigned in a one-milestone project.

At the same time, agents who succeed in early stages are assigned higher and more efficient levels of experimentation in later stages. Recall that it is optimal for the principal to distort each agent's experimentation in the second stage. If the principal needs to reward an agent for an early success, she can do so by reducing the distortion in the following stage. The agent is rewarded because an agent who is assigned more experimentation has to receive more information rents to prevent him from choosing the wrong actions. The principal faces the choice of rewarding an agent with just a bonus or with an assignment that involves more responsibility. She chooses the latter because it generates additional surplus arising from the successful agent's work. This observation can explain why firms use job assignments or promotions to reward workers instead of only bonuses.[6] Symmetric agents may end up with very different career paths, not because something has been learned about their abilities, but because the principal stands by her promise of rewarding agents who succeed.

The expected payoff of the agent in early stages has a very intuitive form. It can be decomposed as the bonus wage in the one task project plus the payoff an agent receives if he were to slack during the first task. Thus, the relative importance of procrastination and free-riding for incentives determines the shape of the optimal contract. When procrastination is more costly to the principal the expected payoff of the agents tends to increase with the timing of the first discovery. When free-riding is more costly the expected payoff of the agent tends to decrease.

The incentive to delay effort is reduced as the number of agents involved in the project in-

[6]Baker, Jensen, and Murphy (1988) pose the question of why promotions are so widely used to provide incentives in real world firms.

creases. In contrast, the free-riding incentive increases with the number of agents in the early stages of the project and decreases with the number of agents in the later stages of the project. Thus, it is always optimal to add more agents in the last stage of the project but the effect on profits of additional agents in early stages is ambiguous.

In the paper I develop techniques for solving sequential bandit problems. I write the incentive constraint of each agent as an optimal control problem. I obtain a differential equation for the bonus contract and the agent's co-state variable associated to the agent's belief at each time. I set up the principal's problem as an optimal control problem with the agents' differential equations as constraints and the agents' co-state variables as choice variables for the principal.[7] In order to solve the two stage problem I characterize the optimal contract that the principal offers for every continuation value and show that it can be summarized by a single variable for each agent: the experimentation threshold in the second stage. This result allows me to write the two stage problem as a standard optimal control problem. I then solve the two stage experimentation model by optimizing over first period contracts and experimentation thresholds in the second stage. A similar approach can be used to solve the model for any number of stages.

Related Literature. This work adds to the literature of experimentation with exponential bandits, (see for instance Bolton and Harris (1999) Keller, Rady, and Cripps (2005) and Klein and Rady (2011)), the literature on contests, and the literature of incentives for teams of agents under moral hazard.

The problem of moral hazard in teams was first explored by Holmstrom (1982) and Alchian and Demsetz (1972). In their main model each agent's contribution to output cannot be individually identified. Therefore, agents free-ride on other agents' efforts. As a result, agents exert inefficiently low effort. In my model, in contrast, there is a principal that serves as a budget breaker and perfectly observes agents' outcomes. Agents do not free-ride under the optimal contract. However, in order to induce full effort the principal must pay agents rents whenever there are multiple agents participating in an early stage. These rents arise endogenously because each agent expects to receive rents in a future period after his co-worker makes a discovery.

I depart from the recent literature by focusing on the case in which an agent is able to work without receiving a flow of funding from the principal. My model captures the key features of a firm that employs workers. In contrast, most of the literature has focused on situations in which the principal must provide a flow of funding that the agent can appropriate. These models are designed to capture the essential features of investor-entrepreneur relationships. See for example,

[7] Bonatti and Hörner (2009, 2011) also write the agent's problem as an optimal control problem. In their case it can be shown that the co-state variable is equal to zero. This simplification is not available with multiple stages.

Bergemann and Hege (1998), Bergemann and Hege (2005), and Hörner and Samuelson (2013). In these models, because the agent must receive a nonnegative payoff in every history, the more effort that the principal wants to implement the higher is the payoff to the agent from slacking.

Green and Taylor (2014) consider a two-stage project without uncertainty about the quality of the project under a "no divestment" constraint. They find interesting dynamics, and show that exploration stops inefficiently early. In contrast, in my model with the weaker limited liability constraint, when there is no uncertainty the principal would be able to implement efficient experimentation.

Bonatti and Hörner (2011) analyze a game in which agents who have private information about their efforts collaborate to obtain a success in a risky project. The equilibria of the game have inefficient delays in provision of effort. Bonatti and Hörner (2009) ask what contract a principal would optimally offer the agents to complete their project. The difference is that in their setting the principal cannot observe individual outcomes and therefore free-riding is a sufficiently large concern that the principal prefers to have only one agent to complete the project.

There is also a relationship between my paper and the literature on contests. Halac, Kartik, and Liu (2014) ask how to design a contest for experimentation for a group of symmetric agents. In their paper the principal maximizes the amount of experimentation subject to a fixed budget constraint which bounds the maximum prize. They find that it is sometimes optimal to not disclose breakthroughs to other participants.[8] In my paper, in contrast, I find the expected revenue maximizing contest without a budget constraint. To do so I characterize the cost-minimizing contract for a given level of experimentation. I find that in the single milestone project, the cost minimizing contest for a given amount of experimentation discloses breakthroughs immediately and features no duplicated effort.

Manso (2011) and Ederer (2013) consider a setting in which agents can privately choose between a safe and a risky action. The risky action represents an innovative, new method, whereas the safe action represents a known and tested method. The principal would like to incentivize the agent to take an innovative action but cannot observe whether successes arose from a tested or a new method. In my model the agents can produce a success only by investing in a risky arm. My model represents a better informed or more hands-on principal who knows what discovery needs to be made and understands how a breakthrough came to be once it is found.

Other papers consider incentives in teams. Campbell, Ederer, and Spinnewijn (2014) model a game with multiple agents and multiple breakthroughs which are privately observed, but without

[8] Note that the problem in Halac, Kartik, and Liu (2014) is not the dual of the problem I consider. It would be the dual if they were considering the maximization of experimentation subject to a constraint on the expected budget rather than a fixed budget.

uncertainty about the quality of the project. Georgiadis (2014) presents a model of project and team dynamics in which the commonly observed state of the project evolves according to a controlled stochastic process driven by a Brownian motion. He finds that the principal pays the agents only at the end of the project, and that the principal's optimal team size is larger when the expected length of the project is lower. Georgiadis, Lippman, and Tang (2014) consider the problem of a principal with limited commitment power managing a team of workers.

The paper is also related to the literature on efficiency wages (Shapiro and Stiglitz (1984); Acemoglu and F. Newman (2002)). Efficiency wages arise when the principal has an imperfect monitoring technology and cannot bring the agent's payment below zero when the agent is discovered to have shirked. This limited liability constraint together with the incentive compatibility constraint implies that the agent has to be given a strictly positive rent. In my model, the agents can never succeed when they exert zero effort. That is, if the principal wanted to give incentives for effort for just one instant she would extract full surplus. However, because of the dynamic nature of the model the principal gives rents to prevent agents from shifting effort over time in an uncertain environment.

This paper contributes to a literature on contracting with a single agent and unobserved states and private effort. He, Wei, and Yu (2012) consider a principal-agent problem with moral hazard in which there is uncertainty about the project's profitability. Because the principal does not observe the agent's effort, the agent can manipulate the principal's beliefs about the project's profitability, leading to informational rents. Prat and Jovanovic (2014) and Bhaskar (2014) consider other moral hazard settings in which an agent can manipulate a principal's beliefs by choice of effort, leading to informational rents.

Halac, Kartik, and Liu (2013) characterize optimal contracts between a single agent and a principal in discrete time without limited liability. In their model the agent privately observes his own effort and type. Adverse selection in conjunction with moral hazard gives rise to inefficiencies and information rents to the agents. In contrast, I do not model adverse selection, but my model allows for projects with multiple discoveries and multiple agents subject to limited liability constraints. Even in the absence of adverse selection, contracts are non-trivial and the principal cannot extract all rents.

Finally, this paper contributes to a literature on the role of promotions as incentive mechanisms. Baker, Jensen, and Murphy (1988) ask why firms use promotions to provide incentives. Fairburn and Malcomson (1994) show that promotions allow the manager to implement higher effort when it is possible for workers to bribe the manager. Prendergast (1993) models promotions as a way to provide incentives to make unobservable investments in specific human capital. Gibbons and

Waldman (1999) provide a survey of this literature. My paper shows that the presence of informational rents causes the principal to prefer promotions to bonuses.[9]

2 Model

2.1 Description

There are n agents attempting to complete a project and a principal who owns the production of the agents. The project consists of N stages or *tasks* which have to be completed sequentially in order to finish the project successfully. Each task is of uncertain feasibility. A task may be "good" or "bad" (or else "feasible" and "impossible"). Only good tasks can be completed. The probability that task j is good is $\bar{p}^j \in (0,1]$ which is commonly known by all participants. Once task j is completed all agents start working on the next task simultaneously. Most of the results in the paper are for projects with one or two tasks, that is, projects with $N \in \{1,2\}$.

Time is continuous with time $t \in [0,\infty)$. At each task j each agent exerts a privately observed and costly effort. Agent i exerts effort $a_{i,t}^j \in [0,\bar{a}_i]$ at time t on task j at cost $\kappa_i a_{i,t}^j$, where $\kappa_i > 0$.

If task j is good and agent i exerts effort $a_{i,t}^j$ on task j at time t, he completes the task with instantaneous probability $a_{i,t}^j$.

We refer to the completion of task j as a *breakthrough* on task j. When a breakthrough is achieved in task j, the principal receives a transfer π^j (not necessarily positive). A breakthrough in task N has a value of $\pi^N > 0$ for the principal. As long as no breakthrough has occurred the principal does not reap any benefit from the project. All players discount the future at common rate $r > 0$. We assume that the game ends after the Nth breakthrough.

All agents and the principal observe a breakthrough as soon as it occurs as well as the identity of the agent who attained it.

The set of public histories at time t is denoted $\mathscr{H}^t$ and it specifies which tasks have produced breakthroughs, the timings at which breakthroughs were attained, and which agent attained each breakthrough. Formally a history $h^t \in \mathscr{H}^t$ contains a sequence of time and agent pairs (τ^j, k^j) for $j \leq N$. $\tau^j \leq t$ is the time at which the j'th breakthrough was attained by agent k^j. We denote $\mathscr{H}$, the set of realized histories of breakthroughs until the end of the game, a history $h \in \mathscr{H}$ contains a sequence of time-agent pairs $\left((\tau^j, k^j)\right)_{j=1}^{J}$ that represent the breakthroughs that were attained until the end of the game. Let $\mathscr{H}^{J,t}$ denote the set of histories at time t in which the last breakthrough occurred in task $J-1$ and, thus, agents are working in task J at time t. A history $h^t \in \mathscr{H}^{J,t}$ has the

[9] Che, Iossa, and Rey (2014) find a similar result in the context of procurement auctions for innovations.

form $\left((\tau^j, k^j)\right)_{j=1}^{J-1}$.

The principal has full commitment. A contract offered to agent i is a wage schedule $\tilde{w}_i$, contingent on the public history. The wage schedule at time t consists of a flow payoff $\tilde{w}^f_{i,t} \in \mathbb{R}$ and lump-sum transfers $\tilde{w}^l_{i,t} \in \mathbb{R}$. That is, heuristically the revenue accruing to the agent over the time interval $[t, t+dt]$ is

$$\tilde{w}^f_{i,t} dt + \tilde{w}^l_{i,t}.$$

The wage schedule $(\tilde{w}^f_{i,t}, \tilde{w}^l_{i,t})$ is adapted to the $\sigma-$algebra induced by the public histories in set $\mathscr{H}^t$ and maps public histories to $\mathbb{R}$.

I assume that the contracts offered by the principal are publicly observed by all agents. Fix contracts $\tilde{w}_i$ for $i \in \{1, \ldots, n\}$ accepted by all agents. Given those contracts, the agents have strategies and realized payoffs. Let $\mathscr{H}_i^{j,t}$ be the private history of agent i at time t in stage j, consisting of the public history and the effort exerted by agent i up to time t. Agent i's strategy is a measurable function $a_i^j : \mathbb{R}_+ \times \mathscr{H}_i^{j,t} \to [0, \bar{a}_i]$ from times and private histories to actions. $a^j_{i,t}(h^t)$ is the instantaneous effort that agent i exerts at time t in task j, after history $h^t \in \mathscr{H}_i^{j,t}$, as long as no breakthrough has been achieved in that task.

I now describe the payoffs of the players after each history. Let history $h \in \mathscr{H}$ be such that J tasks were completed at times $\{\tau^1_, \tau^2, \ldots, \tau^J\}$ and let $\tau^0 = 0$. Let $\tilde{w}^f_{i,t}(h)$ denote the realized flow payoff to agent i at time t given terminal history h. Suppose that at history h lump sums $\tilde{w}^l_{i,t_k}(h)$ are paid to each agent i at times $\{t_k\}_{k \in I(h)}$ for some set $I(h) \subseteq \mathbb{N}$. The payoff to the principal is:[10]

$$r\left(\sum_{j \leq J} \pi^j e^{-r\tau^j} - \sum_{i=1}^{n}\left(\int_0^\infty \tilde{w}^f_{i,s}(h) e^{-rs} ds + \sum_{k \in I(h)} \tilde{w}^l_{i,t_k}(h) e^{-rt_k}\right)\right),$$

and agent i's payoff from exerting effort $(a^j_{i,t})_{t \geq 0}$ for each task j is:

$$r\left(\int_0^\infty e^{-rs}(\tilde{w}^f_{i,s}(h) - \kappa_i a^j_{i,s}(h))\, ds + \sum_{k \in I(h)} \tilde{w}^l_{i,t_k}(h) e^{-rt_k}\right).$$

The wages offered define a game between the agents. We will look at the Perfect Bayesian equilibria of that game. Namely, each agent i chooses $a_{i,t}$ to maximize his expected payoff. Among the equilibria induced by a given contract we will look for the one that maximizes the principal's payoff subject to the constraint of the agent getting a payoff of at least zero which is each agent's normalized outside option. The objective of the principal is to offer contracts to each agent so as to maximize her expected payoff.

[10] The factor r that multiplies the payoff is a normalization.

As agents exert effort on task j with $\bar{p}^j \in (0,1)$ they become more pessimistic about the feasibility of the task. Conditional on strategies $(a^j_{1,t}, \ldots, a^j_{n,t})$ on task j the common belief that j is good at time t, p^j_t, evolves according to the differential equation

$$\frac{dp^j_t}{dt} = \dot{p}^j_t = -p^j_t(1-p^j_t)a^j_t$$

where $a^j_t = \sum_i a^j_{i,t}$ and $p^j_0 = \bar{p}^j$.

2.2 Bonus contracts and limited liability

The space of possible contracts is large. In order to simplify the analysis I show that risk neutrality allows me to restrict attention to a small subset of contracts, which pay only lump-sum transfers when the project begins and when breakthroughs occur.

Let $\bar{\mathscr{H}}^t$ denote the set of histories at time t in which some breakthrough is attained at time t.

Definition 1. A *bonus contract* consists of a transfer $W_{i,0}$ at time zero and transfers $w_{i,t}(h^t)$ to each agent i at time t if $h^t \in \bar{\mathscr{H}}^t$. The agents do not receive transfers or flows after $h^t \notin \bar{\mathscr{H}}^t$.

I adapt the definition of a bonus contract from Halac, Kartik, and Liu (2013). I also assume throughout that the agents are subject to limited liability, that is, the principal cannot extract a negative sum of discounted transfers from the agents after any history. This assumption is reasonable for agents who are credit constrained, or cannot legally commit to the contract, as is the case in employment contracts.

Definition 2. A contract satisfies *limited liability* if after every history the discounted sum of all transfers and flows to each agent i is positive. Formally, the contract must satisfy the following condition after each history $h \in \mathscr{H}$

$$\int_0^\infty e^{-rs}\tilde{w}^f_{i,s}(h)\,ds + \sum_{k\in I(h)} \tilde{w}^l_{i,t_k}(h)e^{-rt_k} \geq 0.$$

Proposition 1 (bonus contracts)**.** *For every contract and equilibrium under that contract there exists a bonus contract and an equilibrium under the bonus contract that gives the same discounted payoff to all agents and the principal after every realized history $h \in \mathscr{H}$ as the original contract. If the original contract satisfies limited liability so does the associated bonus contract.*

From now on, we restrict attention to bonus contracts in which the principal offers a lump sum transfer $W_{i,0}$ at time zero and gives a transfer $w_{i,t}(h^t)$ to agent i at time t if a breakthrough occurs at time t in history h^t. This restriction is without loss in view of Proposition 1.

2.3 The first-best allocation

We begin with the social planner's problem that characterizes the efficient level of experimentation. The social planner maximizes the sum of payoffs of all players. The social planner solves for task N

$$\tilde{\Pi}^N = \max_{a_{i,t}^N} \sum_i r \int_0^\infty (p_t^N \pi^N - \kappa_i^N) a_{i,t}^N e^{-\int_0^t (p_s a_s^N + r) ds} dt,$$

where the belief evolves according to

$$\dot{p}_t^N = -p_t^N (1 - p_t^N) a_t^N, \quad p_0^N = \bar{p}^N.$$

The term

$$e^{-\int_0^t (p_s^N a_s^N + r) ds}$$

is the probability that no breakthrough has occurred yet and therefore

$$p_t^N a_{i,t}^N e^{-\int_0^t (p_s^N a_s^N + r) ds}$$

is the probability density that i obtains a breakthrough at time t. The belief that the arm is good p_t^N decreases over time as long as no breakthrough has occurred and its time derivative is proportional to the aggregate effort exerted by all agents.

Defining recursively $\tilde{\Pi}^{j-1}$ for $j \in \{1, 2, \ldots, N\}$, the social planner solves

$$\tilde{\Pi}^{j-1} = \max_{a_{i,t}^{j-1}} \sum_i r \int_0^\infty (p_t^{j-1} (\pi^{j-1} + \tilde{\Pi}^j) - \kappa_i^{j-1}) a_{i,t}^{j-1} e^{-\int_0^t (p_s a_s^{j-1} + r) ds} dt,$$

where the belief evolves according to

$$\dot{p}_t^{j-1} = -p_t^{j-1} (1 - p_t^{j-1}) a_t^{j-1}, \quad p_0^{j-1} = \bar{p}^{j-1}.$$

Note that the term in the integral is positive if and only if $p_t^j (\pi^j + \tilde{\Pi}^{j+1}) > \kappa_i^j$. Therefore, the solution to the planner's program is a threshold strategy for each agent: $a_{i,t}^j = \bar{a}_i$ when $p_t^j (\pi^j + \tilde{\Pi}^{j+1}) > \kappa_i^j$ and $a_{i,t}^j = 0$ when $p_t^j (\pi^j + \tilde{\Pi}^{j+1}) \leq \kappa_i^j$. Each agent exerts effort as long as the expected marginal gain from effort is above its marginal cost. The previous discussion allows us to conclude:

Lemma 1 (Social planner's solution)**.** *The unique social planner's solution is*

$$a^j_{i,t} = \begin{cases} \bar{a}_i & \text{if } p^j_t(\pi^j + \tilde{\Pi}^{j+1}) > \kappa^j_i \\ 0 & \text{if } p^j_t(\pi^j + \tilde{\Pi}^{j+1}) \leq \kappa^j_i \end{cases}$$

If the agents are symmetric with $\bar{a}_i = \bar{a}$ and $\kappa_i = \kappa^j$, the latest time at which the agents stop working in the last task j is given by

$$\bar{T}^j = \frac{-\ln\left(\frac{1-\bar{p}^j}{\bar{p}^j}\right) + \ln\left(\frac{\pi^j + \tilde{\Pi}^{j+1} - \kappa^j}{\kappa^j}\right)}{n\bar{a}}. \tag{1}$$

$\bar{T}^j$ is positive whenever $\bar{p}^j\left(\pi^j + \tilde{\Pi}^{j+1}\right) > \kappa^j$. The total amount of work exerted conditional on no breakthrough is given by $-\ln\left(\frac{1-\bar{p}^j}{\bar{p}^j}\right) + \ln\left(\frac{\pi^j+\tilde{\Pi}^{j+1}-\kappa^j}{\kappa^j}\right)$. This amount does not depend on the number of agents nor on their maximum effort $\bar{a}$. The total amount of work is also decreasing in the cost of effort κ^j and increasing in the initial belief $\bar{p}^j$.

3 Benchmark: project with a single task

In this subsection I characterize the optimal contract for teams for a discovery that consists of only one task. The principal offers a bonus contract $w = (w^k_{i,t}, W_{i,0})_{i,k}$ to the agents where $w^k_{i,t}$ is the transfer agent i receives when agent k achieves the breakthrough at time t.[11] In the optimal contract the principal does not pay agents for other agents' successes and therefore $w^k_{i,t} = 0$ for $k \neq i$. Maximizing over the schemes that set $w^k_{i,t} = 0$ is without loss when there is limited liability. These payments would not contribute to incentives and, since under (LL) they must be weakly positive, they would be wasteful from the principal's perspective. In what follows we denote $w_{i,t}$ for $w^i_{i,t}$.

When we restrict attention to bonus contracts in the one task project, the limited liability condition is equivalent to requiring that all transfers in the bonus contract be non-negative, as stated in the following Lemma.

Lemma 2 (Limited liability)**.** *In the one-task project the limited liability constraint can be replaced*

[11] Since there is only one task I have omitted the task superscripts in the notation in this section. In the one task project there is only one possible history preceding a breakthrough, the history in which no breakthrough has occurred yet. Thus, the principal can condition the contract on the timing of the breakthrough and the agent who attained it.

by the condition

$$w_{i,t} \geq 0, W_{i,0} \geq 0. \tag{2}$$

Constraint (2) is a priori a stronger condition than the limited liability requirement of Definition 2. It may not be satisfied by non bonus contracts that give a positive payoff after every history and a strictly positive payoff after no breakthrough is achieved (see proof of Proposition 1). However, these contracts cannot be optimal for the principal and, therefore, the constraint (2) is without loss of generality.

The principal seeks to maximize her payoff over bonus contracts and effort functions, solving the following program:

$$\max_{a_{i,t},w_{i,t},W_{i,0}} \sum_i r \int_0^\infty p_t a_{i,t}(\pi - w_{i,t}) e^{-\int_0^t (p_s a_s + r)\, ds} dt + W_{i,0}. \tag{OB}$$

subject to

$$a_{i,t} \in \text{argmax}_{\tilde{a}_{i,t} \in [0,\bar{a}_i]} r \int_0^\infty (p_t \tilde{a}_{i,t} w_{i,t} - \kappa \tilde{a}_{i,t}) e^{-\int_0^t (p_s(a_{-i,s}+\tilde{a}_{i,s})+r)\, ds} dt. \tag{IC}$$

$$r \int_0^\infty (p_t a_{i,t} w_{i,t} - \kappa a_{i,t}) e^{-\int_0^t (p_s a_s + r)\, ds} dt + W_{i,0} \geq 0 \tag{IR}$$

$$w_{i,t} \geq 0, W_{i,0} \geq 0, \tag{LL}$$

for $i \in \{1, \ldots, n\}$ and time t, where $a_s = \sum_j a_{j,s}$ and $a_{-i,s} = \sum_{j \neq i} a_{j,s}$.

The principal's objective function (OB) is the expected payoff of the principal if each agent i is paid $w_{i,t}$ if he obtains a breakthrough at time t and his effort function is given by $a_{i,t}$. Since the effort of the agents is unobserved, the (IC) constraint says that the agent has to find it optimal to exert the level of effort $a_{i,t}$ that the principal wants to induce. Finally, the (IR) constraint says that the agents' payments have to be greater than their outside option which is assumed to be zero.

3.1 Importance of the limited liability constraint

If the principal is allowed to offer contracts that do not satisfy the limited liability constraint, she can extract full surplus and implement the first best effort. In fact, because of risk neutrality, the principal can "sell each agent his own arm." That is, each agent makes a transfer to the principal at the beginning of the game equal to the expected value of his own arm and receives π if he is the first to complete the task. This scheme implements the first best effort. Each agent finds $a_{i,t}$ to maximize

$$\int_0^\infty (p_t\pi - \kappa)a_{i,t}e^{-\int_0^\infty (p_s a_s + r)ds}dt.$$

Thus, each agent will choose $a_{i,t} = \bar{a}_i$ as long as $p_t\pi > \kappa$ and $a_{i,t} = 0$ when $p_t\pi \leq \kappa$ and the principal obtains the first best payoff through the initial transfers.

Because of risk neutrality, there are many contracts that give the principal the first best payoffs. For example, the first best can be attained by a contract that makes a transfer to the agents at the beginning of the game and then charges them flow penalties as long as they do not complete the task.[12]

3.2 Procrastination rents

The principal will not be able to extract full surplus from the agents. In order to extract the surplus subject to limited liability the principal has to pay each agent i a bonus conditional on success that exactly offsets the cost of effort in expectation at each time. That is the principal has to offer agent i the *no-rent contract* $w_{i,t}^{NR}$ that satisfies $p_t w_{i,t}^{NR} - \kappa_i = 0$, and each agent has to exert maximum effort at every time t. The belief about the quality of the arm p_t decreases as agents exert effort and therefore, $w_{i,t}^{NR}$ must be non-decreasing.

However, under the no-rent contract agent i can guarantee a strictly positive payoff by exerting less than the maximum effort in some time interval before the efficient stopping time. To understand this result, it is useful to consider the dynamic programming problem of the agent. Let $V_{i,t}$ denote the expected payoff of agent i at time t. $V_{i,t}$ must satisfy

$$V_{i,t} = \left(p_t w_{i,t}^{NR} - \kappa_i\right) a_{i,t}dt + \left(1 - \left(r + p_t\left(a_{i,t} + a_{-i,t}\right)\right)dt\right) V_{i,t+dt} + o(dt).$$

Under $w_{i,t}^{NR}$ agent i gets zero payoff at every time when exerting maximal effort and therefore $V_{i,t+dt} = 0$ if i exerts the maximum effort as expected. If i were to stop working for an instant at t his private belief about the state of world would be strictly above $\kappa/w_{i,\tau}$ for every $\tau > t$ and later effort would give him a strictly positive payoff, obtaining $V_{i,t+dt} > 0$. At time t agent i obtains zero payoff by setting $a_{i,t} = 0$ which is the same he obtains by exerting effort $\bar{a}_i$. Thus, under $w_{i,t}^{NR}$ agent i has incentives to shift effort to the future knowing that he will be more optimistic about the state

[12] Halac, Kartik and Liu (2013) find a similar result in a model with one agent in discrete time.

of the world at that time. This decision to delay effort is what I call *procrastination.*[13] We will see that under the optimal contract agents receive bonuses that are strictly above $w_{i,t}^{NR}$ as shown in Figure 1. The agents receive rents because they would like to delay their effort, under the no-rent contract. For this reason I denote these information rents *procrastination rents*.

3.3 Symmetric agents

In this section we assume agents are symmetric, that is $\bar{a}_i = \bar{a}$ for all i and $\kappa_i = \kappa$.

We saw in the previous section that $w_{i,t}^{NR}$ does not provide incentives for maximum effort. I show that the optimal contract, in contrast, incentivizes the agents to exert maximum effort until a deadline. The principal designs the bonus contract that pays as little as possible to the agents without giving incentives to procrastinate. A crucial result is that this optimal contract is such that the agents are indifferent between exerting effort now and at the next instant. Intuitively, since the agents exert maximum effort in the optimal contract, if they had strict incentives to exert effort at some time the principal could lower the payment at the instant without affecting incentives for effort at other times. The result is not obvious though, because of the dynamic nature of the problem. Changing the contract at one instant can affect the incentives at all times, not just at the consecutive instant.[14] The following proposition characterizes the contracts that the principal offers to agent i. Define $x_t = \int_0^t (a_{i,s} + a_{-i,s})\, ds + \log\left(\frac{1-\bar{p}}{\bar{p}}\right)$.

Proposition 2 (Agent's contract)**.** *Suppose the principal wants to implement effort functions $(a_{i,s})_{i=1}^n$. Each agent i's bonus wage $w_{i,t}$ satisfies the following differential equation* [15]

$$\dot{w}_{i,t} = (a_{-i,t} + r)(w_{i,t} - \kappa) - r\kappa e^{x_t}. \tag{3}$$

[13] This effect is also present in the models found in Bergemann and Hege (1998); Bonatti and Hörner (2011); Hörner and Samuelson (2013).

[14] Halac, Kartik, and Liu (2013) find a similar result in a model in a discrete time model.

[15] A dynamic programming heuristic can be used to gain intuition about the equation for the wage schedule Consider the decision of the agent to shift effort ε from time interval $[t, t+dt]$ to time interval $[t+dt, t+2dt]$. The expected payoff of agent i at time t can be approximated as

$$V_{i,t} = \left(w_{i,t}(1 - e^{-a_{i,t}p_t dt}) - \kappa_i a_{i,t} dt\right) + e^{-(r + p_t(a_{i,t} + a_{-i,t}))dt} V_{i,t+dt},$$

where $e^{-a_{i,t}p_t dt}$ is the probability that player i does not get breakthrough in instant dt. Replacing $V_{i,t+dt}$, approximating the exponentials with a second order Taylor series and computing $\frac{\partial}{\partial (dt)^2}\left(\frac{\partial V_{i,t}}{\partial \varepsilon}\right)$ and setting it to zero one obtains equation (3).

This derivation is closely related to the one in Bonatti and Hörner (2011). In their model agents are also indifferent between exerting effort in two consecutive instants. However, the reason why agents are indifferent is different in the two models. In their model the indifference arises because of the agents' optimization problem, whereas in my model it is decided by the principal in order to minimize the cost of incentives for effort.

with boundary condition $w_{i,T} = \kappa(e^{x_T} + 1)$ *where* $T = \sup\{t | a_{i,t} > 0\}$.

To understand why the principal sets the effort at the maximum until a deadline let us consider the dynamic programming problem of the principal. Let $\Pi_{i,t}$ denote the expected payoff that the principal obtains from agent i and let $V_{i,t}$ denote the expected payoff of agent i at time t. Consider the principal's decision to shift effort ε from time interval $[t, t+dt]$ to time interval $[t+dt, t+2dt]$. To evaluate this trade-off we first write the value function of the principal as

$$\Pi_{i,t} = \Big((1 - e^{-p_t a_{i,t} dt})\pi - \kappa a_{i,t} dt - ((1 - e^{-p_t a_{i,t} dt}) w_t - \kappa a_{i,t} dt)\Big) + e^{-(r + p_t(a_{i,t} + a_{-i,t}))dt} \Pi_{i,t+dt}$$

Replacing $\Pi_{i,t+dt}$ recursively we obtain

$$\begin{aligned} \Pi_{i,t} &= \Big((1 - e^{-p_t a_{i,t} dt})\pi - \kappa a_{i,t} dt - ((1 - e^{-p_t a_{i,t} dt}) w_t - \kappa a_{i,t} dt)\Big) + e^{-(r + p_t(a_{i,t} + a_{-i,t}))dt} \times \\ & \Big((\pi - w_{t+dt})(1 - e^{-p_{t+dt} a_{i,t+dt} dt}) + e^{-(r + p_{t+dt}(a_{i,t+dt} + a_{-i,t+dt}))dt} \Pi_{i,t+2dt}\Big) \end{aligned}$$

Approximating the exponentials with first order Taylor expansion we obtain[16]

$$\frac{\partial \Pi_{i,t} / \partial \varepsilon}{\partial dt} = 0.$$

Thus, we need to look at the second order approximation to find the effect of shifting effort. Approximating the exponentials with a second order Taylor expansion we obtain

$$\frac{\partial \Pi_{i,t} / \partial \varepsilon}{\partial (dt)^2} = -(r + a_{-i,t})(p_t \pi - \kappa) - a_{-i,t} \kappa (1 - p_t) - \underbrace{\frac{\partial}{\partial (dt)^2}\left(\frac{\partial V_{i,t}}{\partial \varepsilon}\right)}_{=0} < 0.$$

The second term in the previous expression is zero because the agent is made indifferent between exerting effort in two consecutive instants under the optimal contract (see footnote 15). The first term is negative when $p_t \pi > \kappa$—which is true as long as experimentation is efficient. Thus, the principal does not want to delay effort from time interval $[t, t+dt]$ to time interval $[t+dt, t+2dt]$ and the optimal contract does not involve effort delays. Agents exert maximum effort until a deadline. That is, there is no procrastination by the agents, and the principal pays procrastination rents.

The following theorem characterizes the optimal contract that the principal offers to each agent. The optimal contract is symmetric and the agents stop working at a belief that is above the efficient one.

[16] These computations are made with more detail in the appendix, section A.2.

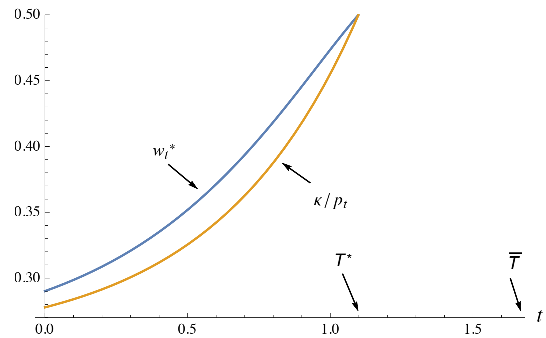

Figure 1: w_t^* : Optimal bonus wages for parameter values: $(\kappa, \bar{a}, \bar{p}, \pi, n) = (1/4, 1, 9/10, 1, 2)$. κ/p_t: no-rent bonus payment.

Define

$$w_t^*(T) = \kappa + \frac{1-\bar{p}}{\bar{p}} \frac{\kappa\left(-e^{nt\bar{a}}r + e^{r(t-T)+((-1+n)t+T)\bar{a}}\bar{a}\right)}{-r+\bar{a}}. \quad (4)$$

and

$$T^* = \frac{\ln\left(\frac{\pi-\kappa}{\kappa}\right) - \ln\left(\frac{1-\bar{p}}{\bar{p}}\right)}{(1+n)\bar{a}}.$$

The bonus wage $w_t^*(T^*)$ solves differential equation (3) when all agents exert maximum effort until time T^*.

Theorem 1 (Optimal contract)**.** *The unique optimal bonus contract $w_{i,t}$ is given by*

$$w_{i,t} = w_t^*(T^*) \text{ for } t \leq T^* \text{ and } w_t = 0 \text{ for } t > T^*$$

with $a_{i,t} = \bar{a}$ for $t \leq T^*$ and $a_{i,t} = 0$ thereafter for each agent i.

Theorem 1 states that for symmetric agents the optimal contract is symmetric and each agent works at maximum effort until a time threshold. Figure 1 shows that the optimal contract gives higher transfers to the agents and increases more slowly compared to the no-rent curve κ/p_t. Intuitively, the optimal bonus payment increases in order to compensate the agents as they become more pessimistic over time but it cannot increase so fast as to make agents want to delay their effort. $w_t^*(T^*)$ is the lowest bonus contract that provides incentives to exert maximal effort up to time T^*.

Under the optimal contract experimentation stops inefficiently early. Recall that efficiency requires that players experiment at their maximum effort until $\bar{T}$ (as seen in equation (1)) which

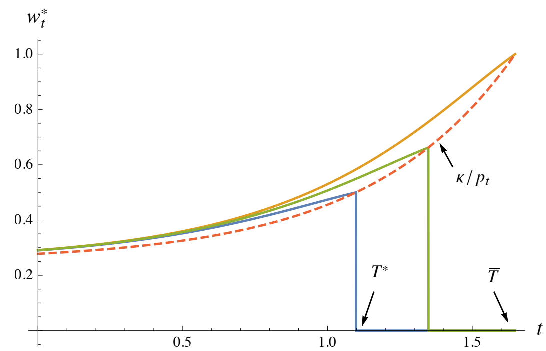

Figure 2: Solid curves: bonus contracts for different stopping times. Parameter values: $(\kappa, \bar{a}, \bar{p}, \pi, r) = (1/4, 1, 9/10, 1, 0.5)$. Dashed curve: no-rent bonus payment. The agents' bonus contracts increase in the experimentation threshold.

is greater than T^*. This inefficiency arises because agents have to be compensated with more rents if they are expected to experiment until a later time threshold. Thus, the principal trades off longer experimentation with increased rents and opts to stop experimentation at an inefficient level. Recall that, at the first best, experimentation stops when $p_t\pi = \kappa$. When the belief is such that $p_t\pi$ approaches κ the principal has to pay a wage that is close to π in order to induce effort. Thus, it cannot be optimal for the principal to have the agents work until time $\bar{T}$. By having the agents stop slightly earlier the principal incurs a loss in profits from experimentation of second order, since she is obtaining nearly no surplus from breakthroughs at times close to $\bar{T}$. At the same time, by stopping work slightly earlier the principal sees a first order drop on the wages paid in case of breakthrough since w_t^* is strictly increasing in T^* for all $t < T^*$ as illustrated in Figure 2.[17] Thus, the principal gains from having the agents stop earlier than time $\bar{T}$.

Corollary 1. *The bonus wage $w_t^*(T^*)$ is increasing in t.*

Corollary 1 says that the wage is increasing in t. The agents need to be given a higher bonus as they become more pessimistic because they expect the bonus to arrive with lower probability. However, the wage schedule grows slower than the no-rent bonus transfer κ/p_t in order to prevent procrastination.

3.4 Comparative statics

As the number of agents increases, holding the rate $\bar{a}$ fixed, the amount of work converges to the efficient level since $T^* = \frac{n}{n+1}\bar{T}$. Moreover, even keeping total capacity fixed, that is keeping $n\bar{a}$ constant, the principal prefers to hire more and more agents. Lemma 3 below shows that as

[17] Note that the derivative of w_t^* with respect to T^* is given by $\kappa\bar{a}e^{\bar{a}((n-1)t+T^*)+r(t-T^*)+x_0} > 0$

$n \to \infty$ the principal's payoff converges monotonically to the first best payoff. The reason why the principal prefers to split capacity into more and more agents is that agents have an externality on each other. First, if an agent stops working it is likely that another agent gets the reward. Second, agents procrastinate in order to manipulate their private belief and exert effort when it is most profitable. The smaller share of the total effort each agent represents, the less control each agent has over his own private belief and the less he stands to gain from procrastination. Thus, as the number of agents increases procrastination becomes less profitable. Figure 3 shows the optimal contract for different number of agents while keeping the total capacity $n\bar{a}$ fixed. As the number of agents increases the principal has the agents work longer and offers a wage closer to the κ/p_t curve. The comparative static on the number of agents relies partly on our assumption that the outside option is worth zero for all agents. If the agents' outside option were greater than zero, hiring more agents can only be profitable up to a point. For sufficiently many agents the sum of the outside options of all agents will surpasses the value of the breakthrough π. In section 5.5 I characterize the optimal contract when there is a positive outside option. I find that the principal's payoff is single-peaked in the number of agents and that there is an optimal number of agents to include in the project.

Lemma 3 (Number of agents)**.** *As the number of agents increases while keeping the total capacity $n\bar{a}$ fixed, the agents wages converge uniformly to κ/p_t and the principal's payoff converges to the first best.*

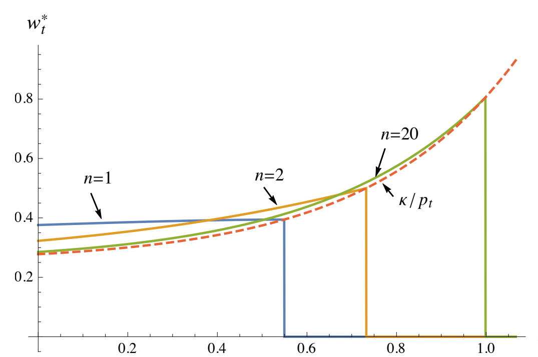

Figure 3: Optimal bonus contracts for different numbers of agents keeping the total capacity fixed. Parameter values: $(\kappa, \bar{a}, \bar{p}, \pi, r, n\bar{a}) = (1/4, 1, 9/10, 1, 0.5, 3)$. As the number of agents increases agents receive less rents.

The following comparative statics are derived from the expressions for the wage schedule in Theorem 1.

Corollary 2 (Comparative statics)**.** *The optimal payment scheme with symmetric players has the following properties:*

1. *w_t^* is decreasing in r and in $\bar{p}$ and increasing in $\bar{a}$.*

2. *The total experimentation conditional on no breakthrough and the terminal belief does not depend on $\bar{a}$ or r. The terminal belief increases in κ and decreases in $\bar{p}$.*

Corollary 2 says that the agents' bonuses increase in the riskiness of the project. That is, projects with a lower prior probability $\bar{p}$ give higher bonuses to the agents. Thus, if two projects differ in $\bar{p}$ and π such that they have the same experimentation threshold T^*, the expected bonus conditional on both projects being successful is higher in the riskier project. This result is in contrast with the usual risk-incentives trade-off derived from Holmstrom (1979). This trade-off is hard to find empirically (see Prendergast (2000) and Prendergast (2002)). Furthermore, in Corollary 3 below I show that, fixing all other variables, conditional on a breakthrough, the expected discounted bonuses are higher when $\bar{p}$ is smaller.

Bonus contracts are decreasing in r. As agents becomes more impatient they value future bonuses less and thus their temptation to procrastinate is diminished. Figure 4 shows how the agents' bonus transfers diminish and are closer to κ/p_t as the r increases.

The total experimentation conditional on no breakthrough is given by $T^*\bar{a}n$ and it does not depend on $\bar{a}$ nor r, nor does the terminal belief. Thus, the experimentation threshold takes a very simple form. The principal chooses a terminal belief that only depends on the benefits of the project, its prior probability of being good and the number of agents. In section 5.3, I show that when agents are asymmetric the total amount of work depends on the discount rate. As the agents become faster the bonus contracts give higher transfers.

Corollary 3 (Risk incentives trade-off)**.** *The expected bonus conditional on a bonus being paid is decreasing in $\bar{p}$.*

Consider two projects with different $\bar{p}$ and π such that they give the same expected payoff to the planner.[18] The project with lower $\bar{p}$ will give higher expected bonuses, conditional on a bonus being paid.

[18]The same argument applies for two projects with the same expected payoff under the optimal contract.

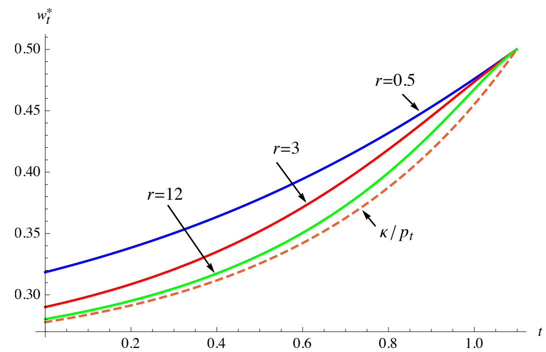

Figure 4: Optimal bonus contracts. Parameter values: $(\kappa, \bar{a}, \bar{p}, \pi, n) = (1/4, 1, 9/10, 1, 2)$.

4 Main results: project with two tasks

I now assume the completion of the project requires the completion of two risky tasks. Agents have to experiment and complete a task before they can move on to experiment in the second task. If they discover a second breakthrough they complete the project. For instance, engineers must first develop a product and then improve its performance to an acceptable level and solve any remaining issues. The exact issues that arise will not be known until a first prototype is completed. Agents working on developing medical drugs might first find a promising drug or compound to address an ailment and then proceed to test its efficacy and safety in several trials. In research contexts an important discovery may lead to new avenues of research that build on it.

When experimenting in the first task, agent i exerts private effort $a^1_{i,t} \in [0, \bar{a}^1_i]$ at flow cost $\kappa^1_i a^1_{i,t}$. The first task is good or feasible with probability $\bar{p}^1$. A breakthrough in the first task occurs at rate $a^1_{i,t}$ if the arm is good. When an agent obtains a breakthrough all players learn how to begin work on a second task.

I drop the superscripts for task 2. In the second task, agent i exerts effort $a_{i,t} \in [0, \bar{a}_i]$. The cost to agent i of experimenting in the second task is $\kappa_i a_{i,t}$. The second task is good with probability $\bar{p}$ and bad with probability $(1 - \bar{p})$. The task gives a breakthrough at rate $a_{i,t}$ only if it is good.

The principal receives transfer π^1 from a breakthrough in the first task and transfer $\pi > 0$ for a breakthrough in the second task.

When an agent obtains a breakthrough in the first task, all agents are able to begin the second task. No agent can work on the second task until some agent completes the first task. From Proposition 1 we can restrict attention to bonus contracts in which the principal pays each agent i a transfer a time zero $W_{i,0}$, a transfer $w^{1,k}_{i,t}$ if player k completes first task at time t and transfer

$w^{2,k}_{i,t}(k',\tau)$, if player k completes the second task at time t and agent k' completed the first task at time τ. If there are two contracts that give the same discounted payoffs after every history–and thus produce the same incentives for effort–I assume that the principal chooses the contract that pays each agent at the earliest possible time.

4.1 The second task

The key to solving the two stage model is characterizing the continuation contract after any history in the first stage. As is shown in the following proposition, the second task contract will have the same form as the contract in the one task project except that the experimentation deadline depends on the history in the first task.

The following proposition characterizes the wage schedule in the second stage. Each agent gets a positive transfer only if he finds the breakthrough that completes the first task. The transfer and the total amount of work conditional on no breakthrough depend on the identity of the agent who completed the first step and on the time at which the first task was completed. Suppose the first-task breakthrough is obtained at time τ.

Define

$$w^2_{i,t} = \begin{cases} \kappa_i + e^{\int_\tau^t a_{-i,s}\,ds + rt} \int_t^T e^{-rl} e^{\int_\tau^l a_{i,s}\,ds + x_0} r\kappa_i\,dl + \kappa_i e^{-\int_t^T (r-a_{i,s})\,ds + x_0} & \text{if } \tau \le t \le T, \\ 0 & \text{if } t > T. \end{cases} \tag{5}$$

$w^2_{i,t}$ is the least cost bonus contract that implements efforts function $a_{i,s}$ for each i and solves the differential equation of the one task project given by equation (3).

Proposition 3 (Second task contract)**.** *Suppose agent k' obtained the first-task breakthrough at time τ. There are experimentation thresholds $T_k(k',\tau)$ for $k \in \{1,\dots,n\}$ such that i's bonus payment for success at time $t \ge \tau$, $w^{2,i}_{i,t}(k',\tau)$, is given by $w^2_{i,t}$ as defined in equation (5) with $a_{k,t} = \bar{a}$ if $\tau \le t \le T_k(k',\tau)$ and $a_{k,t} = 0$ if $t > T_k(k',\tau)$ for $k \in \{1,\dots,n\}$. If agent $k \neq i$ succeeds in the second task at time t, agent i does not get a bonus, that is, $w^{2,k}_{i,t}(k',\tau) = 0$.*

Proposition 3 says that in the second task the agents work at the maximum effort until a time threshold and the principal offers a contract analogous to the one offered in the single task project. Proposition 3 implies that the optimal contract for each agent in the second stage can be summarized by one variable: the experimentation threshold, $T_i(k',\tau)$. This observation allows me to write the principal's two task problem as a standard optimal control problem, setting as a control the second task experimentation threshold. The proof of Proposition 3 is in section B.1 of the appendix.

The principal promises utility to the agent as a function of the history of the first stage. In the proof I show that, for any given promised utility, the optimal contract that satisfies limited liability involves a non-negative bonus at the beginning of the second task and a bonus contract for second task successes that takes the form of the one-task project optimal contract.

4.2 The first task

We saw that in a one task project the agents have to be given information rents to prevent them from delaying effort. Given that the project consists of two stages, the principal now may have to give rents to the agents to prevent them from free riding in the first task. The reason is that each agent expects a positive payoff once another agent completes the first step. This effect dampens the incentives of the agents in the first stage because they can free ride on their co-worker's efforts. This free-riding effect is present even though all agents' individual successes are observed by all players involved. We will see that in the optimal contract the agents receive an expected payoff that is non-increasing in the timing of the first period outcome.

The agent's problem

Let $v_{i,t}^{j}$ denote the expected payoff that agent i obtains in the second stage if j obtains a breakthrough at time t. Note that agent i's choice of effort in the first stage must depend on the continuation payoff in the following stage. In order to choose effort in the first stage agent i solves the following problem

$$\max_{a_{i,\cdot}^{1}\in[0,\bar{a}]}\int_{0}^{\infty}\left(\sum_{j}\left(w_{i,t}^{1,j}+v_{i,t}^{j}\right)a_{j,t}^{1}p_{t}-\kappa_{i}^{1}a_{i,t}^{1}\right)e^{-\int_{0}^{t}p_{s}^{1}a_{s}^{1}\,ds-rt}\,dt$$

Denote $y_{t}^{1}=\int_{0}^{t}a_{s}^{1}\,ds$ and $x_{0}^{1}=\log\left(\frac{1-\bar{p}^{1}}{\bar{p}^{1}}\right)$. Let denote $b_{i,t}=w_{i,1,t}^{i}+v_{i,t}^{i}$. $b_{i,t}$ is the total expected payoff–the bonus plus the payoff in the next task–that agent i receives when he attains a breakthrough at time t.

Proposition 4 (First task contract)**.** *There exists an absolutely continuous function $\tilde{\gamma}_{i,t}$ such that agent i's expected payoff from achieving a breakthrough at time t, $b_{i,t}$, satisfies the following differential equation*

$$\dot{b_{i,t}}=\left(b_{i,t}-\kappa_{i}^{1}\right)\left(a_{-i,t}+r\right)-\sum_{j\neq i}v_{i,t}^{j}a_{j,t}^{1}-\kappa_{i}^{1}re^{y^{1}+x_{0}^{1}}-r\tilde{\gamma}_{i,t}e^{y^{1}}+\dot{\tilde{\gamma}}_{i,t}e^{y^{1}}. \tag{6}$$

with boundary condition

$$\tilde{\gamma}_{i,T} = \left(b_{i,T} - \kappa_i^1\right) e^{-y^1} - \kappa_i^1 e^{x_0^1} - \int_{T_i}^{\infty} \sum_{j \neq i} v_{i,t}^j a_{j,t}^1 e^{-y_T - \int_T^t a_s \, ds - r(t-T)} \, dt.$$

where $T = \sup\{t | a_{i,t} > 0\}$ *and where* $\tilde{\gamma}_{i,t} > 0 \implies a_{i,t}^1 = \bar{a}_i$ *and* $\tilde{\gamma}_{i,t} < 0 \implies a_{i,t}^1 = 0$*. Also,* $w_{i,t}^{1,j} = 0$ *if* $j \neq i$*.*

Proposition 4 gives a necessary condition that relates an agent's expected payoff following a success in the first task to his choice of effort. In order for the agent's effort to be incentive compatible, equation (6) needs to hold. The Proposition is obtained by solving each agent's effort decision given an expected payoff, using optimal control. The function $\tilde{\gamma}_{i,t}$ is the multiplier in the agent's problem. The result is analogous to Proposition 2 in the single task case. In the single task case the multiplier $\tilde{\gamma}_{i,t}$ was always zero because the principal's cost always increases in $\tilde{\gamma}_{i,t}$. In the two task project, however, it is not always optimal to set $\tilde{\gamma}_{i,t}$ to zero. In the appendix I show that $\tilde{\gamma}_{i,t}$ is associated with the agent's incentive to exert effort at time t. When $\tilde{\gamma}_{i,t}$ is strictly positive the agent strictly prefers to exert effort. When $\tilde{\gamma}_{i,t}$ is zero, the agent is indifferent between all levels of effort and when it is negative the agent exerts zero effort. Equation (6) will serve as a constraint in the principal's problem while $\tilde{\gamma}_{i,t}$ will be a choice variable.

The principal's problem Proposition 3 characterizes second-task bonus contract. We only need to determine the second-task experimentation thresholds as a function of the history in the first task. Let $\mathbf{T}(k,\tau) = (T_1(k,\tau), T_2(k,\tau), \ldots, T_n(k,\tau))$ denote a vector of stopping times in the second stage if the first breakthrough was obtained by player k at time τ. Given a vector of timings $\mathbf{T}$ the expected transfer from breakthroughs, as agents exert maximum effort task from Proposition 3, is given by

$$\pi(\mathbf{T}(k,\tau)) = \sum_i \int_0^{T_i(k,\tau)} p_t \pi \bar{a}_i e^{-\int_0^t (p_s a_s + r)\, ds} \, dt + \pi^1.$$

The cost incurred by the agents in the second stage if the vector of stopping times is $\mathbf{T}(k,\tau)$ is given by

$$c(\mathbf{T}(k,\tau)) = \sum_i \int_0^{T_i(k,\tau)} \kappa_i \bar{a}_i e^{-\int_0^t (p_s a_s + r)\, ds} \, dt.$$

The principal chooses $a_{i,t}^1$ and $\mathbf{T}(i,t)$ for all agents i and times t to maximize

$$\sum_i \int_0^{T_i^1} \left(\pi(\mathbf{T}(i,t)) - c(\mathbf{T}(i,t)) - b_{i,t} - \sum_{j \neq i} v_{j,t}^i \right) a_{i,t}^1 e^{-y_t - rt} \, dt$$

subject to $\dot{y}_t = \sum_i a_{i,t}$ and $\dot{b}_{i,t} = (b_{i,t} - \kappa_i)(a_{-i,t} + r) - \sum_{j \neq i} v^j_{i,t} a^1_{j,t} - \kappa_i r e^{y^1 + x^1_0} - r\tilde{\gamma}_{i,t} e^{y^1} + \dot{\tilde{\gamma}}_{i,t} e^{y^1}$. (from equation (6)).

Each agent's expected utility in the second stage will depend on the time threshold T_i at which he stops working and will be given by $vi_{i,t}(T_i)$

$$v_{i,t}(T_i) = \frac{e^{-rT_i}\kappa_i \left(r - e^{T_i \bar{a}} r + \left(-1 + e^{rT_i}\right)\bar{a}\right)}{r(r - \bar{a})}(1 - \bar{p}).$$

$v_{i,t}(T_i)$ is the payoff agent i gets in the second task if he exerts effort until time threshold T_i and opposing agents all exert maximum effort until their deadlines, provided that the contract maximizes the principal's payoff. Note that $v_{i,t}(T_i)$ does not depend on other agents' experimentation thresholds.

4.3 Two symmetric agents

In what follows I describe the optimal contract for the project with two tasks when there are two symmetric agents. That is $\bar{a}_i = \bar{a}$, $\kappa^1_i = \kappa^1$ and $\kappa_i = \kappa$. The characteristics of this contract will depend on the parameter values and can be separated into three cases. In the first case, providing incentives for the first task is costly with respect to the expected payoff the agent receives from the second task. Thus, the principal has to reward agents who succeed in the first task with a bonus. I first discuss this case, then move on to the intermediate and low cost cases.

4.3.1 Costly first task incentives

We are in the costly first task incentives case if the agent receives a strictly positive bonus when he achieves a breakthrough. That is when $w^i_{i,1,t} > 0$ for every t.

We will see in Theorem 2 below that, in the costly first task incentives case the total expected payoff that agent i receives when he achieves a breakthrough at time t, $b_{i,t}$, is given by the following formula

$$b_{i,t} = \underbrace{w^1_{i,t}(T^1_i, T^1_{-i})}_{\text{single task contract}} + \underbrace{\int_t^\infty e^{-\int_t^\tau (r + a^1_{-i,s})\,ds} a^1_{-i,\tau} v_{i,\tau}(T^2_i(\tau))\,d\tau}_{\text{exp. payoff when slacking in first task}} \tag{7}$$

where $w_{i,t}(T_i, T_{-i})$ denotes the bonus wage of the one task project in which agent i stops working at time T_i and $-i$ stops at time T_{-i}.[19] $T^2_i(t)$ is the time at which agent i stops working at time t when

[19] In the notation of the two-task case $w^1_{i,t}(T^1_i, T^1_{-i}) = \kappa^1 + e^{\int_0^t a^1_{-i,s}\,ds + rt} \int_t^T e^{-rl} e^{\int_0^l a^1_{i,s}\,ds + x_0} r\kappa^1\,dl + \kappa^1 e^{-\int_t^T (r - a^1_{i,s})\,ds + x^1_0}$ where $x^1_0 = \log\left(\frac{(1-\bar{p}^1)}{\bar{p}^1}\right)$, $a^1_{k,t} = \bar{a}$ when $t \leq T^1_k$ for $k \in \{1, 2\}$. This bonus contract is analogous to the one defined by equation (5).

agent $-i$ succeeds at that time. We will see that while the first term is associated with procrastination rents, the second term compensates i to prevent him from free-riding on $-i$'s breakthroughs in the first task.

Agent i's stopping time when $-i$ succeeds at time t, $T_i^2(t)$, solves

$$\left(-\kappa + e^{\int_0^t a_{i,s}^1 ds}\left(-1+e^{T_i^2(t)\bar{a}}\right)\kappa(-1+\bar{p})+\bar{p}(\pi-\kappa)e^{-2T_i^2(t)\bar{a}}+\kappa\bar{p}\right) \\ -\bar{p}\int_{T_i^2(t)}^{V_a/\bar{a}-T_i^2(t)}(\pi-\kappa)e^{-\bar{a}T_i^2(t)-\bar{a}s-rs}ds \quad = \quad 0 \tag{8}$$

where $V_a = \left(-x_0 + \log\left(\frac{\pi-\kappa}{\kappa}\right)\right)$ is the total amount of experimentation at the efficient stopping belief. Define $T_i^{2*}(t) = V_a/\bar{a} - T_{-i}^2(t)$ and $\mathbf{T}(i,t) = \left(T_i^{2*}(t), T_{-i}^2(t)\right)$. Let T_1^1 and T_2^1 maximize

$$\sum_i \int_0^{T_i^1}\left(\pi(\mathbf{T}(i,t)) - c(\mathbf{T}(i,t)) - b_{i,t} - v_{i,t}\left(T_{-i}^2(t)\right)\right)a_{i,t}^1 e^{-y_t-rt}\,dt. \tag{9}$$

The following theorem describes the shape of the optimal contract.

Theorem 2 (Costly first task incentives)**.** *Suppose $b_{i,t} > v_{i,t}(T_i^{2*}(t))$ for each t.At the optimal contract in the project with two tasks:*

1. *Each agent i exerts maximum effort until time T_i^1 in the first task. If agent i achieves the first breakthrough at time t, he receives an expected payoff–including a bonus and the expected payoff in the next task–equal to $b_{i,t}$ with a bonus equal to $b_{i,t} - v_{i,t}(T_i^{2*}(t))$.*

2. *When agent i obtains the first breakthrough at time t, the second task bonus contract is defined by Proposition 3 with $T_i(i,t) = T_i^{2*}(t)$ and $T_{-i}(i,t) = T_{-i}^2(t)$. $T_{-i}^2(t)$ solves equation (8) and is decreasing in $\int_0^t a_{-i,s}^1 ds$.*

The assumption $b_{i,t} > v_{i,t}(T_i^{2*}(t))$ ensures that the expected payoff the agents receive in the second task does not surpass the expected payoff that the principal gives to the agent in the first task. Figure 5 (left) shows the expected payoff of agent i as a function of the first breakthrough for some parameter values. The contract illustrated in Figure 5 is such that both agents exert effort until the same time threshold in the first task.

$b_{i,t}$ and $v_{i,t}(T_i^{2*}(t))$ can be computed from primitives in closed form using equations 7, 8, 9 and the definition of $v_{i,t}$. In order to verify that one is in the costly first task incentives case, one can compute $b_{i,t}$ and $v_{i,t}$ using these equations and verify that the inequality holds.

Note that from equation (7) we have $b_{i,t} \geq w_{i,t}^1(T_i^1, T_{-i}^1)$. That is, the expected payoff received by the agent who achieves the first task is weakly greater than the bonus payment that the agent

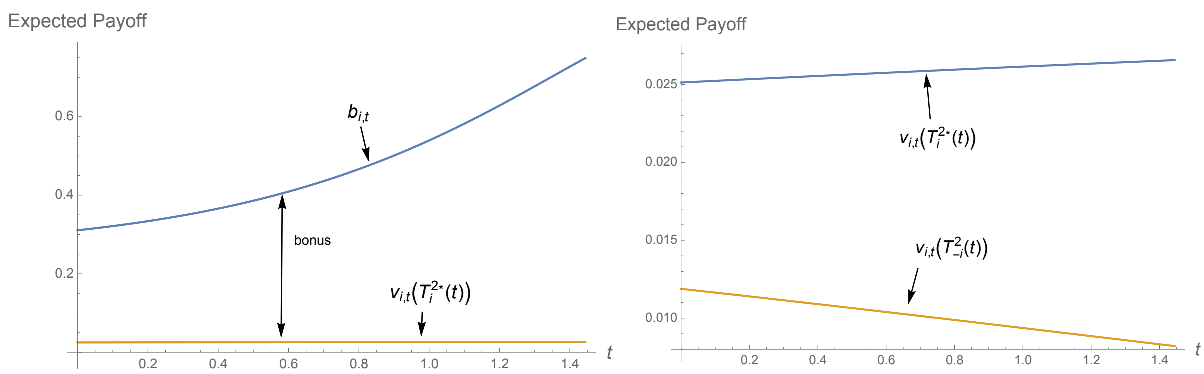

Figure 5: Left: Expected payoff ($b_{i,t}$), bonus and continuation payoff after the first discovery ($v_{i,t}(T_i^{2*}(t))$) as a function of time. Right: Expected payoff in the second task for agents i ($v_{i,t}(T_i^{2*}(t))$) and $-i$ ($v_{i,t}(T_{-i}^{2}(t))$) when agent i succeeds at time t ($v_{i,t}(T_i^{2*}(t))$) . Parameter values: $(\kappa^1, \kappa, \bar{a}, \bar{p}, \pi, n, r) = (1/4, 1/4, 1, 9/10, 5, 2, 1.5)$.

would receive when the first task is a one task project with experimentation thresholds (T_i^1, T_{-i}^1). The difference between the two is the expected payoff agent i would receive if he decided to shirk during the first task and hope for the other agent to bring them both to the second task. Whenever agent $-i$ exerts effort in the first stage and agent i receives a positive payoff after $-i$'s success, i has to be given an additional rent to prevent them from free-riding on $-i$'s efforts.

To gain intuition for why these rents occur, we refer to the dynamic programming heuristic. Consider the decision of the agent to shift effort ε from time interval $[t, t+dt]$ to time interval $[t+dt, t+2dt]$. The expected payoff of agent i at time t satisfies

$$V_{i,t} = \left(b_{i,t}(1 - e^{-p_t^1 a_{i,t}^1 dt}) - \kappa_i a_{i,t}^1 dt \right) + e^{-(r + p_t^1(a_i^1 + a_{-i,t}^1))dt} V_{i,t+dt} + v_{i,t}(T_i^2(t))(1 - e^{-p_t^1 a_{-i,t}^1 dt}).$$

Approximating the exponential with a second order Taylor expansion we obtain

$$\frac{\partial}{\partial (dt)^2} \left(\frac{\partial V_{i,t}}{\partial \varepsilon} \right) = \dot{b}_{i,t} - (a_{-i,t}^1 + r)(b_{i,t} - \kappa) + r\kappa e^{x_t^1} + p_t^1 v_{i,t}(T_i^2(t)) a_{-i,t}^1. \tag{10}$$

The last term in 10 is positive as long as $-i$ exerts effort and i exerts effort in the second task. Thus, if the principal offers expected payoff $b_{i,t} = w_{i,t}(T_i^1)$, the first three terms sum to zero, and agent i has incentives to shift effort to the future. In the two task case, agents get a positive surplus in the second task because they are given procrastination rents. If the principal were to only give them an expected payoff equal to the bonus wage in the one-task case the agents would prefer to not work for an instant and let the other agents achieve the first discovery.

Corollary 4. *In the costly first task incentives case the agents' contract has the following features:*

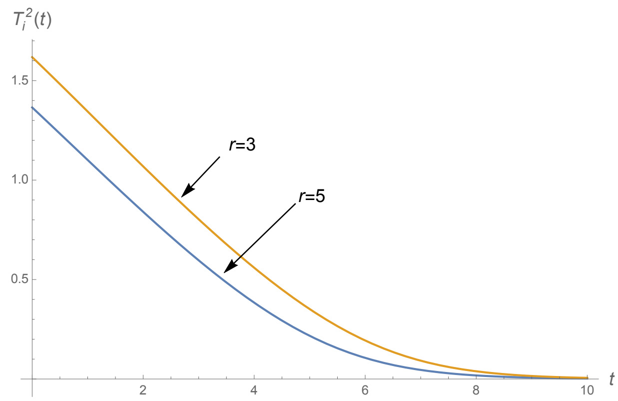

Figure 6: Experimentation stopping time in the second task of non-successful agent conditional on the timing of the first discovery. Parameter values: $(\kappa^1, \kappa, \bar{a}, \bar{p}^1, \bar{p}, \pi, \pi^1, n) = (1/4, 1/4, 1, 9/10, 9/10, 5, 0, 2)$.

- *The agent who succeeds in the first task is rewarded with more leeway to experiment (he experiments until the efficient belief), with reduced competition and with larger bonuses conditional on success in the second task.*

- *The agent who does not succeed in the first task, while his co-worker succeeds at time t, works until time threshold $T_i^2(t)$ which is decreasing in the total amount of effort i exerted in the first task, $\int_0^t a_{i,s}^1 ds$ and converges to zero as $\int_0^t a_{i,s}^1 ds$ converges to ∞.*

The principal faces a tradeoff between letting the losing agent work up to her desired amount of experimentation–the optimal stopping time in the one-task project–and decreasing the agents' rents from free-riding. The principal opts to distort the amount of experimentation down from her desired amount in the second task in order to reduce the rents from free-riding. To understand the intuition of this result, note that reducing experimentation from the optimal amount in the second task generates a second order loss–due to optimality–while reducing the bonus produces a first order gain. Figure 6 shows how the timing at which the losing player stops working in the second task changes with the timing of the first breakthrough. This distortion increases in the time at which the first breakthrough arrives because the principal discounts the experimentation in the second period and because agents who slack expect the first breakthrough to arrive relatively later.

The agent who succeeds in the first period is rewarded with a bonus but also with more leeway to work in the next task. In fact, the agent who succeeds in the first task works until the first best belief threshold in the second task. The losing agents experimentation threshold is decreasing in the time of the first breakthrough. Thus, the winning agent's expected payoff in the second task is increasing in the time of the first breakthrough, since he is assigned a lengthier experimentation

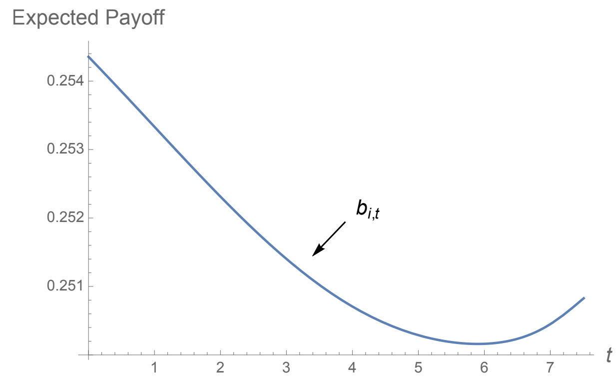

Figure 7: Expected payoff (including bonus) of agent who succeeds at time t. Parameter values: $(\kappa^1, \kappa, \bar{a}, \bar{p}^1, \bar{p}, \pi, \pi^1, n, r) = (1/4, 1/4, 1, 1-10^{-9}, 9/10, 5, 0, 2, 1.5)$.

period in the second task. Figure 5 (right) shows expected payoffs in the second task for winning and losing agents for some parameter values. The successful agent's overall payoff, however, considering the bonus, may increase or decrease in the timing of the first breakthrough. The function $w_{i,t}(T_i^1, T_{-i}^1)$ is always increasing in t but the term associated to free-riding rents is decreasing. When the first arm is relatively safe with respect to the second arm the free-riding term may dominate and the expected payoff after a breakthrough may be decreasing in the timing of the first breakthrough for some times. Figure 7 shows an example in which $b_{i,t}$ is non monotonic.

The previous discussion leads us to an important consequence of this model. An agent is rewarded not just with bonuses but with experimentation that is closer to the first best in the second task. The more responsibility that agents are assigned, the more information rents they have to be given to not choose the wrong actions. Thus, assigning more work to an agent is a form of reward. The principal faces the choice of rewarding an agent with just a bonus or with an assignment that involves more responsibility. She chooses the latter because an assignment that gives the agent the same payoff as a bonus also generates additional surplus arising from the successful agent's work. This observation provides a possible explanation for why firms use job assignments or promotions to reward workers instead of only monetary bonuses (see Baker, Jensen, and Murphy (1988) and Gibbons and Waldman (1999) for a discussion of this puzzle).

The experimentation time thresholds T_1^1 and T_2^1 that maximize the principal's payoff in equation 9 are not necessarily equal. It may be optimal for the principal to have projects start small, with fewer agents in the first task than the second one. This situation arises when one agent experiments in the first arm for less time conditional on no breakthrough than the other one. In some cases, one agent may not even participate in the first stage exploration.

Figure 8 illustrates a case in which contracts can be asymmetric. In the example as π^1 decreases, the asymmetry in the contracts offered to each agent increases. The dashed line represents

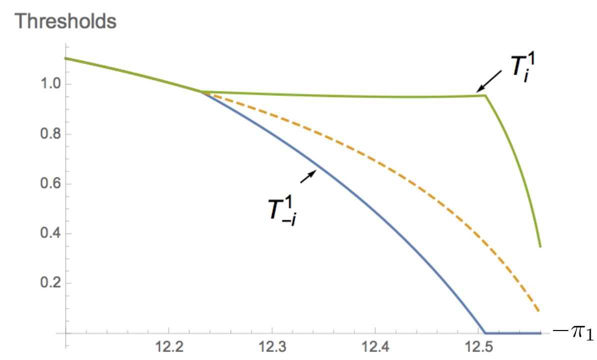

Figure 8: Asymmetric experimentation thresholds in the first task. Agent i's threshold is greater than agent $-i$'s for small values of π^1. Dashed line: Optimal symmetric experimentation threshold. Parameter values: $(\kappa^1, \kappa, \bar{a}, \bar{p}^1, \bar{p}, \pi, \pi^1, n, r) = (1/4, 1/9, 1, 0.99, 0.9, 5, 0, 2, 1.5)$.

the symmetric work threshold–which is not optimal for small values of π^1. For larger π^1 the two asymmetric thresholds collapse into the symmetric one. Intuitively, when the transfer after the first breakthrough is low, the value of the first breakthrough is not big enough to justify the high information rents agent $-i$ receives if i works until the first breakthrough. Thus, $-i$ works longer in the first task. In section (B.8) of the appendix I give a sufficient condition for the asymmetry of the contract. The condition is more likely to be satisfied when the first task is relatively safer–that is its prior of being good is higher. Intuitively, having more agents reduces the incentives to procrastinate–because of competition–but increases the incentive to free-ride. When the first arm has a high probability of being of good quality each agent is less able to affect his private belief about the task by choice of effort and, therefore, procrastination is less of a concern relative to free-riding.

We have seen that the principal distorts the agents' second task contracts. It is therefore natural to ask whether the principal would be better off hiring new agents for the second task. The answer is that if the principal had access to identical agents, but a fixed number of positions for agents, she would fire and replace all the agents that don't achieve a breakthrough in the first task and keep the agent who succeeds. It is never optimal to fire the agent who succeeds in the first task. This result is stated in the following Corollary.

Corollary 5 (Non-irreplaceable agents). *If the principal could costlessly replace some agents with identical ones for the second task, she would keep the agent who succeeds in the first task and replace the agent who does not. In the second stage, the agent who was present in the first task works until a longer time threshold.*

On the other hand, if the principal has access to an additional pool of agents in the final stage, and is given the option to either replace or add more agents, she would choose to not replace any agents and add as many agents as possible.

Suppose the first task is relatively safe and, thus, agents work until a late time threshold in the first task. The experimentation threshold of the losing agent in the second task goes to zero in the timing of the first breakthrough. Thus, the value of the losing agent's work in the second task is decreasing in the time of the first discovery. Suppose the principal can allocate agents to another task that gives less payoff than the second task of the original project when both agents work until T^* but gives the agents little information rents (for example a task that is very likely to be feasible). The principal may be better off allocating the losing agent to this alternative task instead when a first breakthrough arrives sufficiently late, because the losing agent experiments so little in the second task. The previous discussion is formalized in the following Corollary. Let task $\tilde{2}$ be identical to task 2 except that it gives transfer $\tilde{\pi}$ when completed and has prior probability of being good given by $\tilde{p}$.

Corollary 6. *For every $\tilde{\pi} < \pi$, there is a time $\bar{t}$ and $\tilde{p} > \bar{p}$ such that if agent i works at time $t > \bar{t}$ and agent $-i$ completes the first task at time t, then the principal assigns agent i to task $\tilde{2}$.*

Note that for small enough $\tilde{\pi}$ a single-task project consisting of task $\tilde{2}$ gives less payoff than a single-task project consisting of task 2.[20] In such case task $\tilde{2}$ is a task that would not be pursued by the principal in the absence of the first task. Note that the time at which the first milestone is achieved is bounded by the first task experimentation thresholds. When $\bar{p}^1$ is closer to one, the first task experimentation thresholds are larger. It is therefore possible that the first success will arrive later and, as a result, that non-successful agents will be assigned to less efficient tasks.

4.3.2 Cheap first task incentives

Now I turn to the the case in which the expected payoffs from the second stage are high enough to provide incentives in the first stage. The principal's preferred experimentation amount in the second task corresponds to the one we characterized in the one task case, given by equation (11) below. Incentives in the first task are cheap if the expected payoff an agent receives in the second task, when experimenting until the principal's preferred threshold, is above the expected payoff he needs to receive to exert effort until the efficient deadline in the first task. Each agent receives the same high expected payoff when the other agent succeeds as when they succeed. Because the expected payoff is so high, however, the agents are willing to exert maximum effort in the first

[20]An upper bound on the payoff of task $\tilde{2}$ in a single task project is given by the payoff when $\tilde{p} = 1$ and converges to zero as $\tilde{\pi}$ converges to zero.

stage in order to hasten the start of the second task. Thus, they do not need to be given rents for the first period effort. The principal does not need to pay a bonus after the first breakthrough, nor distort experimentation in the second stage from what she would choose if there was no first stage. Exploration in the first stage is chosen efficiently and no agents are kept from participating in the first task.

Let

$$T^{2*} = \frac{\ln\left(\frac{\pi-\kappa}{\kappa}\right) - \ln\left(\frac{1-\bar{p}}{\bar{p}}\right)}{(1+n)\bar{a}} \tag{11}$$

denote the threshold at which agents stop working optimally when the project only consists on the second task. Define $\mathbf{T}^* = (T^{2*}, T^{2*})$

$$\bar{T}^1 = \frac{\ln\left(\frac{\pi(\mathbf{T}^*)-c(\mathbf{T}^*)-\kappa}{\kappa}\right) - \ln\left(\frac{1-\bar{p}^1}{\bar{p}^1}\right)}{n\bar{a}}.$$

Define

$$b^*_{i,t} = w_{i,t}(\bar{T}^1, \bar{T}^1) + e^{(r+(n-1)t\bar{a})}\int_t^{\infty} e^{-(r+(n-1)t\bar{a})\tau} a^1_{-i,\tau} v_{i,\tau}(T^{2*})\, d\tau. \tag{12}$$

T^{1*} is the efficient stopping time in task one if agents work until T^{2*} in task two.

Theorem 3 (Cheap first-task incentives)**.** *If $b^*_{i,t} < v_{i,t}(T^{2*})$ at the optimal contract with two stages:*

1. *In the first stage all agents work until a breakthrough occurs. Agents do not receive a bonus after the first breakthrough.*

2. *In the second stage all agents exert maximum effort until a time threshold T^{2*}.*

The optimal contract when the incentive costs of the first stage are low enough is exactly as if the two tasks were independent of each other or the principal had different sets of agents to perform each task. The expected payoff of the agents after the first breakthrough does not depend on the history. The principal does not gain from replacing agents who do not succeed in the first task.

4.3.3 Intermediate incentive costs in the first task

The following theorem describes the optimal contract when the first task has intermediate costs. In this case there are times in which the agents do not receive bonuses when they obtain a breakthrough in the first task. However, successful and losing agents see their second stage experimentation thresholds distorted from the principal's preferred thresholds. Intuitively, at the times in which agents do not receive bonuses for discoveries the principal can distort the second stage thresholds in such a way that the agents' indifference between exerting effort between two consecutive instants is preserved. At other times, agents either receive bonuses as in Theorem 2 or do not receive bonuses nor see their experimentation distorted as in Theorem 3.

Theorem 4 (Intermediate cost)**.** *If $b_{i,t} > v_{i,t}(T_i^{2*}(t))$ and $b^*_{i,t} \geq v_{i,t}(T^{2*})$ at the optimal contract with two stages, then there are time threshold $t^1, t^2 \geq 0$ such that*

1. *For $t \in [t^1, t^2]$ the expected payoff is as in Theorem 2 but agents do not receive bonuses after the first breakthrough.*
2. *For $t \notin [t^1, t^2]$ the contract is either*
   (a) *As in Theorem 2 and agents receive bonuses for breakthroughs in the first task and the experimentation stopping times in the second task is given by $T_i^{2*}(t)$ for the winning player and $T_i^2(t)$ given by equation (8) for the losing player.*
   (b) *As in Theorem 3 and agents do not receive bonuses and their second-task experimentation is not distorted at time t.*

Theorem 4 says that in the intermediate cost case the optimal contract may have the features of the costly incentives case or the cheap incentives case in some time intervals. However, there must be a time interval in which the contract does not reward agents with bonuses but with assignments of experimentation in the second task. The limited liability constraint binds for these times. The principal would like to extract a payment from the agent who succeeds after the first round. The successful agent would obtain a positive payoff in expectation, but because only the agent who succeeds in the second task gets a bonus, extracting a payment from the winner of the first round does not satisfy limited liability. The contract in the intermediate cost case cannot be derived in closed form. The experimentation thresholds of the successful and the unsuccessful agent have to satisfy a joint optimality condition and at each time t the payoff function $b_{i,t}$ given by equation (7) must be equal to the successful agent's payoff in the second task, which, in turn, also depends on his experimentation threshold (see section B.4 in the appendix).

5 Extensions: One task project

5.1 Optimal disclosure of discoveries

Suppose now that when one agent achieves a breakthrough, it is observed by the agent and the principal but not commonly observed by the other agents.[21] We now ask whether the principal would disclose the breakthrough to the other agents. If the principal discloses immediately she avoids duplicated effort. But if she delays disclosure or does not disclose, and rewards agents who succeed after the first breakthrough, the agents' beliefs that they receive a reward may fall more slowly. As a result the principal can offer a lower bonus to the agents. I find that the principal will always choose to disclose the breakthrough immediately to all agents and will only pay a bonus to the agent that attains the first breakthrough.

Proposition 5 (Optimal disclosure)**.** *The optimal bonus contract and disclosure policy is such that the principal pays bonus $w_t^*(T^*)$ for the first breakthrough if it occurs at time $t \leq T^*$ and discloses it immediately.*

The proof is in the appendix in section C.1. Changing the disclosure policy affects the agent's optimal wage but also his belief update. I show that these effects precisely cancel so that the agent's expected payoff does not depend on the disclosure, conditional on a level of experimentation. If an agent continues to experiment after the task has already been completed, the principal has to at least compensate this agent for his incurred cost. As a result, it is better to avoid unnecessary effort by disclosing immediately.

Note that I restrict attention to disclosure policies in which the principal fully reveals that a breakthrough has occurred but may delay this disclosure. More generally, the principal could partially reveal that a breakthrough has occurred, as in Kamenica and Gentzkow (2011). In my setting it would be difficult for the principal to commit to such a policy, or to verifiably partially disclose a breakthrough.

5.2 Unobservable but verifiable discoveries

In the optimal contract agents receive a bonus that increases on the time at which a milestone is reached. Therefore, an agent may choose to delay the disclosure of a privately observed discovery in order to receive a higher bonus. I show that delaying disclosures is not optimal. Delaying disclosure has its costs. The agent discounts future bonuses and another agent may obtain a breakthrough

[21] In the next subsection I show that when the breakthrough is private to the agent the agent will always disclose it immediately to the principal.

in the meantime preventing the agent from receiving a prize. At the optimal contract these costs overcome the benefits from an increased bonus. To understand this result, note that the expected payoff of delaying disclosure until time t is given by $w_{i,t}e^{-n\bar{a}t-rt}$. This expected payoff decreases over time since

$$\frac{\partial w_{i,t}e^{-n\bar{a}t-rt}}{\partial t} = \kappa\left(-e^{-t((n-1)\bar{a}+r)}\right)\left(r\left(e^{nt\bar{a}+x_0}+1\right)+(n-1)\bar{a}\right) < 0.$$

Proposition 6 (Unobservable discoveries)**.** *Under the optimal contract of the one task project agents do not delay the disclosure of privately observed discoveries.*

5.3 Agents with heterogenous talents

We have seen that in the symmetric case the presence of other agents in the team has consequences for the rents they receive and the payoff of the principal. It is then natural to ask how asymmetry in players' capacities would affect the optimal contract and the payoffs of the players. Intuitively a player with a stronger opponent faces less temptation to procrastinate. A player with a weaker opponent faces a greater temptation.

In this section there are two players who have different maximum work capacities, $\bar{a}_i$, the flow cost of effort is κ for both agents. We have seen that the rents an agent gets depend on the number of other agents. In the symmetric case all agents stop working at the same time and receive the same bonus wage. We will see that the optimal contract is asymmetric and the agent who has the most capacity is the one who stops working earlier. It is costlier to prevent a faster agent from procrastinating since he faces less competition from the slow agent. The slow agent, in contrast, faces a large externality from the fast agent. As a result faster agents stop work earlier in the optimal contract.

In what follows we assume $n = 2$ and $\bar{a}_1 > \bar{a}_2$ and that the project has only one task.

Define

$$w_t^{1*} = \frac{\left(\bar{a}_1 + e^{(\bar{a}_2+r)t+\bar{a}_1 T-rT+x_0}\bar{a}_1 - \left(1+e^{(\bar{a}_2+\bar{a}_1)t+x_0}\right)r\right)\kappa}{\bar{a}_1 - r}$$

$$w_t^{2*} = \frac{\left(\left(1+e^{(\bar{a}_1+r)t+\bar{a}_2\hat{T}-r\hat{T}+x_0}\right)\bar{a}_2 - \left(1+e^{(\bar{a}_2+\bar{a}_1)t+x_0}\right)r\right)\kappa}{\bar{a}_2 - r}$$

$$w_t^{2**} = \frac{\left(\left(1+e^{rt+\bar{a}_1\tilde{T}+\bar{a}_2\hat{T}-r\hat{T}+x_0}\right)\bar{a}_2 - \left(1+e^{\bar{a}_2 t+\bar{a}_1\tilde{T}+x_0}\right)r\right)\kappa}{\bar{a}_2 - r}$$

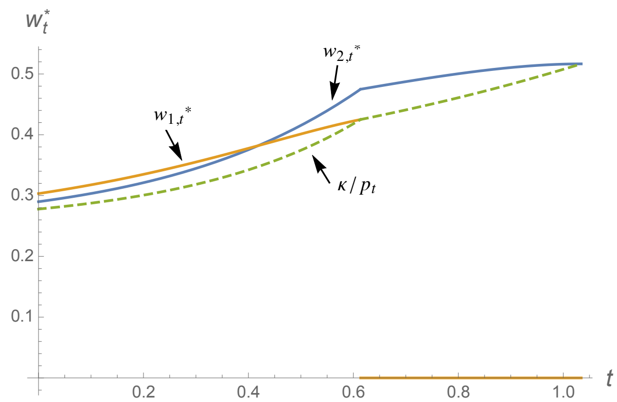

Figure 9: Bonus wages for asymmetric agents. Parameter values: $(\kappa, r, \bar{p}, \pi, \bar{a}_1, \bar{a}_2) = (1/4, 3, 9/10, 1, 2, 1)$

where $x_0 = \frac{1-\bar{p}}{\bar{p}}$, and

$$M(T) = T - \frac{\ln \frac{-r + \frac{e^{\bar{a}_2 T + 2\bar{a}_1 T + x_0}(\bar{a}_i + r)\kappa}{\pi - \kappa}}{\bar{a}_i}}{\bar{a}_2 + r} + \frac{\bar{a}_1 T + x_0 - \ln\left(-1 + \frac{\pi}{\kappa}\right)}{2\bar{a}_2}$$

Theorem 5 (Asymmetric agents)**.** *The optimal wage schemes, w_t^2 and w_t^1 are given by*

$$w_t^1 = w_t^{1*} \textit{ for } t \leq \tilde{T} \textit{ and } w_t^1 = 0 \textit{ for } t > \tilde{T}.$$

$$w_t^2 = w_t^{2*} \textit{ for } t \leq \tilde{T}\text{ , } w_t^2 = w_t^{2**} \textit{for } \tilde{T} \leq t \leq \hat{T} \textit{and } w_t^2 = 0 \textit{ for } t > T^*$$

with $a_{2,t} = \bar{a}_2$ for $t \leq \hat{T}$ and $a_{2,t} = 0$ thereafter and $a_{1,t} = \bar{a}_1$ for $t \leq \tilde{T}$ and $a_{1,t} = 0$ thereafter. $\tilde{T}$ solves $M(\tilde{T}) = 0$ and $\hat{T}$ is given by

$$\hat{T} = \frac{-\bar{a}_1 \tilde{T} - x_0 + \ln\left(\frac{\pi - \kappa}{\kappa}\right)}{2\bar{a}_2}.$$

Figure 5.3 shows an optimal contract for asymmetric agents. Agent 1 stops at time $T_1 > T_2$.

5.4 Agents with different costs

Assume now that there are two agents 1 and 2 with $\kappa_1 > \kappa_2$. The optimal contract takes the form of the symmetric case and solves the differential equation (3).

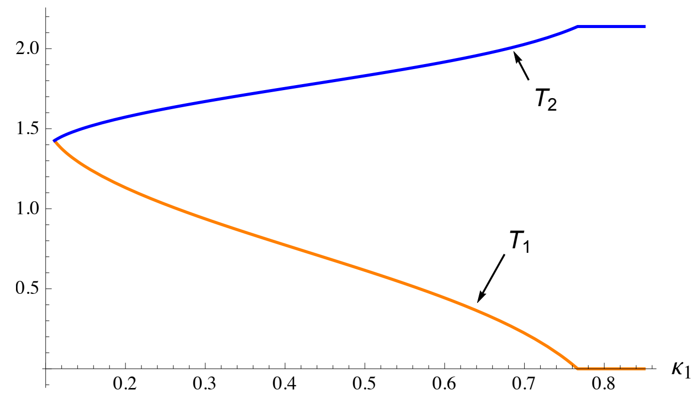

Figure 10: Experimentation threshold as a function of κ_1 where $(\kappa_2, \bar{a}, \bar{p}, \pi) = (1/9, 1, 9/10, 1, 2)$.

Define

$$w_{1,t}(T_1) = \kappa_1 + \frac{1-\bar{p}}{\bar{p}} \frac{\kappa_1 \left(-e^{nt\bar{a}}r + e^{r(t-T_1)+((-1+n)t+T_1)\bar{a}}\bar{a}\right)}{-r+\bar{a}}.$$

and

$$w_{1,t}(T_2) = \kappa_2 \ + e^{\int_0^t a_{1,s}ds + rt} \left(\frac{\kappa_2 r e^{x_0} \left(1 - e^{-rT_2 + \bar{a}T_2 + x_0}\right)}{r - \bar{a}} \right) + \kappa_2 e^{-(r-\bar{a})(T-t)+x_0}$$

where $a_{1,s} = \bar{a}$ if $s \leq T_1$ and $a_{1,s} = 0$ is $s > T_1$.

Proposition 7. *At the optimal contract with two agents with different costs the principal pays a bonus contracts given by $w_{i,t}(T_i)$ and each agent i works at maximum effort until experimentation threshold T_i with $T_1 \leq T_2$. The experimentation thresholds maximize the principal's payoff given that the bonus contracts are given by $w_{i,t}(T_i)$ for $i \in \{1,2\}$.*

Figure 10 shows the experimentation thresholds of agents 1 and 2 as κ_1 varies while κ_2 remains constant. As κ_1 increases T_1 decreases and T_2 decreases. For high enough values of κ_1 agent 1 is excluded from the project altogether. This is the case even though in the absence of agent 2 or if both agents had the same high cost agent 1 would experiment a strictly positive amount under the optimal contract. As the cost of agent 1 becomes large the bonus agent 1 has to receive to exert effort becomes large. As a result, at some point the principal prefers to rely only on agent 2's work even if it means that the breakthrough arrives relatively later.

The previous result indicates that if agents have heterogenous effort costs, the principal would not want to increase the number of agents unboundedly. The presence of low cost agents makes the higher cost agents redundant.

5.5 Agents with positive outside option

I now consider the case in which the agent's have a strictly positive outside option. In this case, the outside option does not coincide with the lower bound on wages imposed by the limited liability constraint. The agents have a strictly positive outside option, for instance, if they have access to alternative employment in which they also receive rents. Arguing as in section 3.3 I show that the principal offers a contract that has the form of the one-task optimal contract $w_t^*(T)$–as defined in equation (4)–plus a bonus W_0 at time zero. Let $T(\bar{V})$ be the experimentation threshold that gives agents expected payoff $\bar{V}$ under $w_t^*(T(\bar{V}))$. Formally, $T(\bar{V})$ is defined as the solution to

$$v(T(\bar{V})) = \frac{\kappa(1-\bar{p})\bar{a}e^{-rT(\bar{V})}\left(r\left(-e^{T(\bar{V})\bar{a}}\right)+\bar{a}\left(e^{rT(\bar{V})}-1\right)+r\right)}{r(r-\bar{a})} = \bar{V} \tag{13}$$

Recall $\bar{T} = \frac{\ln\left(\frac{\pi-\kappa}{\kappa}\right)-\ln\left(\frac{1-\bar{p}}{\bar{p}}\right)}{n\bar{a}}$ is the efficient experimentation threshold and $T^* = \frac{\ln\left(\frac{\pi-\kappa}{\kappa}\right)-\ln\left(\frac{1-\bar{p}}{\bar{p}}\right)}{(1+n)\bar{a}}$ is the principal preferred experimentation threshold in the one task project.

Proposition 8. *Suppose the principal contracts with n symmetric agents that have outside option* $\bar{V} > 0$. *The optimal bonus contract to each agent is given by* $w_t^*(T)$ *where*

$$T = \max\{\min\{\bar{T}, T(\bar{V})\}, T^*\}$$

and the transfer at time zero is

$$W_0 = \max\{0, \bar{V} - v(\bar{T})\}.$$

Proposition 8 states that when $\bar{V}$ is below the utility the agents receive in the contract characterized in the zero outside option case, the constraint does not bind and the contract is exactly as the one characterized in Theorem 1. If $\bar{V} > v(T^*)$ and $\bar{V} < v(\bar{T})$ the principal gives the agent more leeway to experiment and no bonus. When $\bar{V} > v(\bar{T})$ the agent works up to the efficient experimentation threshold and receives a bonus as well. Thus, when the agent's outside option is above the utility he receives in the principal's preferred contract, the agent is assigned a longer experimentation threshold. In this case, the outside option constraint binds in the principal's problem. The principal prefers to give the agent a higher expected payoff by giving him more leeway to experiment rather than a monetary bonus, whenever effort has positive marginal return. The reason is that an assignment that gives the agent the same payoff as a bonus also generates additional surplus arising from the agent's work. Therefore, the principal is better off assigning more responsibility to the agent rather than just rewarding him with a bonus.

$T(\bar{V})$ defined in equation (13) does not depend on the number of agents. However, the efficient experimentation threshold $\bar{T}$ does depend on n. Therefore, for every $\bar{V}$ there is a sufficiently large number of agents such the principal gives the agents bonuses at time zero. As $\bar{T}$ increases the bonuses increase, while the amount of work that each agent performs decreases. As a result, at some point the marginal return from an additional agent does not make up for the additional bonus at time zero and there is an optimal number of agents to include in the project.

Corollary 7. *The principal's payoff is single-peaked on the number of agents she hires.*

Keeping the outside option $\bar{V}$ fixed. The optimal number of agents for a project is increasing in the value of the project π, and decreasing in the discount rate r.

6 Extension: more general learning

I now analyze a more general setting in which agents learn about the project they are involved in as they work. The agents work affects the rate at which a verifiable signal–say for instance, a breakthrough or a breakdown–arrives.

Suppose there are K signals $\{s^i_1,\dots,s^i_K\}$ that can be produced by a player i. Signal k is produced by i at time t at instantaneous rate $\lambda^i_{k,t}a_{i,t}$. The rate $\lambda^i_{k,t}$ is governed by a differential equation that depends on the joint effort of all agents:

$$\dot{\lambda}^i_{k,t} = f^i_k(\sum_j a_{j,t}, \lambda^i_{k,t})$$

for $k \in \{1,\dots,K\}$ with f^i_k continuously differentiable and with $\lambda^i_{k,t}$ at time zero given by $\lambda^i_{k,0}$. Let $\lambda^i_{k,t}((\tilde{a}_{i,t})_t,(a_{-i,t})_t)$ denote $\lambda^i_{k,t}$ at time t when agent i chooses effort function $(\tilde{a}_{i,t})_t$ and agents other than i choose joint effort function $(a_{-i,t})_t$.

We will see that as long as there is a signal k such that $\frac{\partial f^i_k}{\partial a_{i,t}} \geq 0$ the principal can provide incentives for $a_{i,t} = \bar{a}_i$ while extracting full-surplus. The reason is that the principal can pay a bonus that exactly compensates each for the cost of effort if the first signal to arrive is signal k. Since the arrival rate of the signal in non-decreasing in effort the agent does not have incentives to procrastinate. By delaying effort the agent makes the arrival of a reward in the future less likely and sees his expected payoff diminished.

Let $w^{s^j_k}_{i,t}$ denote the bonus payment to agent i when agent j produces signal s^j_k. The payoff of agent i is given by

$$\int_0^t (\sum_{k,j} \lambda^j_{k,t} w^{s^j_k}_{i,t} a_{j,t} - \kappa_i a_{i,t}) e^{-\sum_{k\in\{1,\dots K\},j}\int_0^t \lambda^j_{k,s} a_{j,s}\, ds - rt}\, dt$$

where $a_{i,t}\lambda^i_{k,t}$ is the probability that agent i receives signal s_k at time t and $e^{-\sum_{k\in\{1,\ldots K\},j}\int_0^t \lambda^j_{k,s}a_{j,s}ds}$ is the probability that no signal has arrived until time t.

Proposition 9 (General learning)**.** *Suppose f^i_k is increasing in $a_{i,t}$. The principal can give incentives for $a_{i,t} = \bar{a}_i$ for every t by giving wage*

$$w^{s^i_k}_{i,t} = \frac{\kappa_i}{\lambda^i_{k,t}\left((\bar{a}_i)_t,(a_{-i,t})_t\right)}$$

and $w^{s^j_l}_{i,t} = 0$ when $l \neq k$ or $j \neq i$.

Proof. If an agent deviates from $a_{i,t} = \bar{a}_i$ and exerts lower effort in a positive measure set then $\lambda^i_{k,t} < \lambda^i_{k,t}\left((\bar{a}_i)_t,(a_{-i,t})_t\right)$, thereafter and the rewards from positive would be negative. Thus, once an agent deviates to effort below $\bar{a}_i$ he exerts zero effort forever after. Thus, the agent gets zero payoff from the deviation which is the same as the payoff from exerting maximal effort at all times. □

The previous Proposition applies to models in which there is no uncertainty about the quality of the arm because in this case the rate of arrival of breakthroughs is constant in the agents' work.

A converse of the previous result also holds, suppose there is a time interval $[t',t'']$ such that if each agent i has exerted maximum effort $\bar{a}_i$, $f^i_k(\sum_j a_{j,t},\lambda^i_{k,t})$ is decreasing in $a_{i,t}$ for $t \in [t',t'']$. Then, the optimal contract cannot extract full-surplus. The agents have to be given an information rent. This can be seen using the dynamic programming heuristic. It is without loss to assume that the principal only rewards agents for their own discoveries. Let $V_{i,t}$ denote the expected payoff of agent i at time t. $V_{i,t}$ must satisfy

$$V_{i,t} = \left(\sum_k \lambda^i_{k,t} w^{s^i_k}_{i,t} - \kappa_i\right) a_{i,t}dt + \left(1 - \left(r + \sum_{k,j}\lambda^j_{k,t}a_{j,t}\right)dt\right)V_{i,t+dt} + o(dt).$$

If the principal is extracting full surplus, $\left(\sum_k \lambda^i_{k,t} w^{s^i_k}_{i,t} - \kappa_i\right) = 0$ and $V_{i,t} = 0$ at every t. By exerting zero effort at time t, the agent can obtain a strictly positive surplus, as $V_{i,t} > 0$ for some interval (see section 3.2).

Proposition 10 (General learning)**.** *Suppose there is a time interval $[t',t'']$ such that if each agent j has exerted maximum effort $\bar{a}_j$ up to time t', $f^i_k(\sum_j a_{j,t},\lambda^i_{k,t})$ is decreasing in $a_{i,t}$ for $t \in [t',t'']$ and for each agent k. Then, agent i receives an expected payoff that is strictly above his cost of effort.*

Proposition 10 implies that many conclusions of my model apply to general setups in which agents may learn about the project they are involved in, as they work. An agent receives rents as long as the learning process is such that the rate at which verifiable signals arrive has slope decreasing in his effort at some time interval. Thus, any learning process in which, at any point during the project, the agent becomes more pessimistic about obtaining any verifiable signal (such as a success or a failure) as he exerts effort will imply that the principal cannot extract full-surplus. In such cases, competition is helpful to discipline the agents. The principal can use assignments of responsibility to reward agents and the agents may have to be given rents to prevent them from free-riding on other agents verifiable signals in early stages.

6.1 Application: good news, bad news model

As an application of the previous result, assume now that the players can learn that an arm is "bad"–and, therefore can never produce a breakthrough–by observing a "breakdown signal". As before, the probability that the task is good is $\bar{p}$ which is commonly known by all participants. The agents exert a privately observed costly effort over time $t \in \mathbb{R}^+$. Agent i exerts effort $a_{i,t} \in [0, \bar{a}_i]$ at time t at cost $\kappa_i a_{i,t}$. Agent i learns that the arm is bad if she receives a "breakdown signal" that arrives at time t at rate $\beta a_{i,t}$. If the arm is good agent i produces a breakthrough at time t at rate $a_{i,t}$. The probability at time t that an arm is good is denoted p_t. Both breakthroughs and breakdowns are publicly observed and verifiable.

By investing effort the agents may learn verifiably that the project is not feasible. For example, in a research project agents might learn that a problem has no solution or find a counterexample for a result that they were trying to obtain. Engineers developing a new product might learn that a significant element of it has already been patented or that the product cannot be produced at low enough cost. Scientists working on developing a drug might learn as they run trials that the drug has a serious adverse effect that will prevent it from getting approval from the FDA.

From the results stated in Proposition 9 the principal can extract full surplus. For example, suppose $\beta < 1$. As agents work, the belief that the arm is good p_t is decreasing over time whereas $(1 - p_t)$ is increasing. By making the payments contingent on the verifiable breakdowns the principal can condition on an event that becomes increasingly frequent as agents exert effort. Alternatively, if $\beta \geq 1$ the principal can make payments conditional on verifiable breakthroughs.

In a setting in which a bad arm gives breakdowns and a good arm does not give any signals, as in **?**, the principal can also extract full surplus for a team of agents to allocate their effort in the risky arm. In this case, the belief that the arm is bad decreases over time and, thus, the principal rewards agents for breakdowns. If the agents could surreptitiously tamper with the project so that it breaks down, the incentive scheme would not be optimal. In this model, however, the principal would have to give positive rents to incentivize agents to work in the safe arm.

7 Conclusions

This paper analyzes how to optimally design a contract that gives incentives to innovate to a group of agents. I show that incentives can be provided by simple history contingent bonus contracts. Agents receive information rents to prevent them from delaying effort over time. These rents are increasing with the amount of leeway to experiment that the agents are given. As a result and in order to reduce information rents, the principal has the agents stop experimentation early compared to the first best.

As projects require multiple successful experiments, the contracts have two novel characteristics. First, the agents receive rents to prevent them from free-riding on other agents' discoveries in early periods. Second, rewards and punishments are implemented by experimentation assignments. As a result, the optimal contracts for symmetric agents exhibit asymmetries that grow over time. To reduce the free-riding rents, the principal may exclude some agents from working, even in the absence of another profitable project. Agents contracts are sensitive to early successes. Agents who succeed see their experimentation assignments increased, receiving bigger bonuses when they succeed and having more opportunities to do so.

My paper has several empirical implications. First, within a firm workers who obtain successes or promotions are likely to be credited with future successes in the same project. Second, consider a project that requires substantial investments. If the first stage of a project is relatively safe, that is, its prior probability of being attainable is high, we expect to see fewer workers participating in the early stages. In this case, the rents to free-riding are high relative to the benefit of competition in early stages. In my model the principal is able to observe each agent's results. If the principal is less able to observe individual performance or there are other competing projects to which agents can be allocated, we should expect even fewer agents participating in the initial stages. Third, in expectation bonuses are higher in environments with risk than in environments in which the task is perfectly safe. Agents receive zero rents in safe projects. As the prior belief decreases from 1, agents' rents increase, and therefore their bonuses conditional on success must increase.[22] Finally, successful workers should be rewarded with promotions earlier in the project and bonuses later on. Workers who do not succeed are assigned less responsibility in the same project–compared to the successful ones–or are assigned to tasks that give them less information rents. Tasks that give less

[22] Holding the value fixed, eventually rents decline, however, as the initial probability becomes sufficiently low.

information rents can be tasks that carry less risk, are easier to perform (less costly), or have a slow arrival rate.

The current model assumes that agents' intermediate milestones are observable. It would be valuable to consider the case in which agents can withhold information about their successes in order to gain an advantage over their coworkers. In particular, it would be interesting to analyze the case in which there is imperfect observability of the agents' discoveries. In addition, it is informative to consider the case in which the principal may withhold information from agents about other agents' success.

In section 5 I analyzed the case in which either the principal or the other agents do not learn about an agent's achievement in the single task project. The optimal contract corresponds to the one that is optimal when discoveries are observed. However, it is unlikely that these results will go through when there is more than one stage. When an agent can choose to not disclose a breakthrough and can start working in the next task immediately, he faces less competition from the opponents. Not revealing to an agent that another agent has made a discovery may be cost-saving in the multiple milestone when free-riding rents are high.

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A Appendix: Model and benchmark

A.1 Proof of proposition 1 and lemma 2

Let $\tilde{w}_i : \mathscr{H}^t \to \mathbb{R}$ denote a payment scheme as a function of history for $i \in \{1, \dots, n\}$. The contract $\tilde{w}_i$ may consist of flows and transfers at different times depending on history.

We show that we can construct a contract

$$w = (w_{i,t}(h^t), W_{i,0})_{i,h^t \in \mathscr{H}^t}$$

where $w_{i,t}$ denotes the amount that agent i gets paid at time t and $W_{i,0}$ denotes the transfer at time zero that gives the same payoff to principal and agent after each history.

Let $h \in \mathscr{H}$ be a history until the end of the game in which breakthroughs arrive at times $\tau_1, \dots, \tau_J$ by agents $k_1, \dots, k_J$ for $0 \leq J \leq N$. If $J = 0$, h is a the history in which no breakthroughs are attained. Let $h^{\tau j}$ denote the history contained in h up to time τ_j. Define $\tau_0 = 0$ and let $w_i^j(\emptyset, h^{\tau_{j-1}})$ denote the discounted payoff that contract $\tilde{w}_i$ gives to agent i at the history in which the game ends with no verifiable signals at task j after history $h^{\tau_{j-1}}$.

Define:

$$W_{i,0} = \tilde{w}_i^1(\emptyset, h^0)$$

and

$$w_{i,\tau_j}(h^{\tau_j}) = \left(\tilde{w}_i^{j+1}(\emptyset, h^{\tau_j}) - \tilde{w}_i^j(\emptyset, h^{\tau_{j-1}})\right) e^{r\tau_j}.$$

By definition contract w is a bonus contract that gives the same expected payoff after each history to all players as contract $\tilde{w}_i$.

Lemma 2 follows immediately from the previous construction. Condition (2) only rules out wage schedules that given an agent a positive payoff when no breakthrough is attained. It is clear that such a contract cannot be optimal for the principal. By lowering the transfer after the history in which there is no success the agents' incentives are unaffected and the principal lowers her expenses.

A.2 Detailed computations for section (3.3)

The agent's payoff $V_{i,t}$ can be approximated as

$$\begin{aligned}V_{i,t} \;=\;& dt(pa_{i,t}w_{i,t}-\kappa a_{i,t})-\frac{1}{2}dt^2 w_{i,t}(pa_{i,t})^2+\\ & \left((1-(r+p(a_{i,t}+a_{-i,t}))dt+(r+p(a_{i,t}+a_{-i,t}))^2dt^2/2)\right)\times\\ & \Bigg(dt(pa_{i,t+dt}w_{i,t+dt}-\kappa a_{i,t+dt})-dt^2pa_{i,t+dt}w_{i,t}\left((1-p)(a_{i,t}+a_{-i,t})+\frac{pa_{i,t+dt}}{2}\right)+\\ & \Bigg(1-dt(p(a_{i,t+dt}+a_{-i,t+dt})+r)+\Bigg(\frac{1}{2}(p(a_{i,t+dt}+a_{-i,t+dt})+r)^2\;+\\ & +p(1-p)(a_{i,t}+a_{-i,t})(a_{i,t+dt}+a_{-i,t+dt})\Bigg)dt^2\Bigg)V_{i,t+2dt}\Bigg)+o(dt^3).\end{aligned} \tag{14}$$

From the previous expression

$$-\frac{\partial V_{i,t}}{\partial a_{i,t}}+\frac{\partial V_{i,t}}{\partial u_{i,t+dt}}=dt^2\left(-p_t\left(a_{-i,t}+r\right)\left(w_{i,t}-\kappa\right)+\kappa(-p_t)r+\kappa r\right)+dt\left(w_{i,dt+t}-w_{i,t}\right)+o(dt^3). \tag{15}$$

Dividing by dt^2 and taking the limit $dt\to 0$ we obtain

$$\dot{w}_i=(a_{-i}+r)(w_i-\kappa)-r\kappa e^x.$$

$\frac{\partial \Pi_{i,t}/\partial\varepsilon}{\partial (dt)^2}$ obtains from replacing $w_{i,t}=\pi$ in equation 15.

A.3 Proof of proposition 2

The principal chooses $w_{i,t}:\mathscr{H}_t\to\mathbb{R}_+$ and $a_i:\mathscr{H}_t\to[0,1]$ measurable with respect to history to maximize her profits.

The principal's problem is then

$$\max_{a_{i,t},w_{i,t}}\sum_i r\int_0^\infty p_t a_{i,t}(\pi-w_{i,t})e^{-\int_0^t(p_s a_s+r)\,ds}dt,$$

where, from IC, $a_i:\mathbb{R}_+\to[0,1]$ maximizes

$$r\int_0^\infty (p_t w_{i,t}-\kappa)a_{i,t}e^{-\int_0^t(p_s a_s+r)\,ds}dt.$$

The belief evolves as

$$\dot{p}_t=-p_t(1-p_t)(a_{i,t}+a_{-i,t})$$

where $a_{-i,t} = \sum_{j \neq i} a_{j,t}$, $a_s = \sum_i a_{i,s}$.

In what follows I consider the set of bonus schedules that satisfy necessary conditions for a given effort schedule $a_{i,s}$ for each agent i. I then find the bonus payments that minimize the principal's cost among the class of bonus schedules that satisfy the necessary conditions. Finally, I show that this bonus schedule satisfies sufficient conditions for optimality and is thus the optimal bonus schedule for a given effort $a_{i,s}$.

The agent's problem

Let $T_i = \inf_t \{a_{i,\tau} = 0, \tau \geq t\}$. $T_{i,}$ is the latest time at which effort is exerted by agent i. I make the technical assumption that $T_i < \mathscr{T}$ where $\mathscr{T}$ is an arbitrarily large finite time. Suppose the principal wants to implement effort $a_{i,s}$ for each agent i. Agent i's problem can be written as

$$\max_{a_{i,\cdot}} \int_0^{T_i} (w_{i,t} - \kappa) a_{i,t} \left(\bar{p} e^{-\int_0^t a_s \, ds} + (1 - \bar{p}) \right) e^{-rt} dt$$

where

$$p_t = \frac{\bar{p} e^{-\int_0^t a_s \, ds}}{\bar{p} e^{-\int_0^t a_s \, ds} + (1 - \bar{p})}.$$

Defining $y_t = \int_0^t a_s \, ds$ and replacing p_t into agent i's objective we obtain the following optimal control program for agent i

$$\max_{a_{i,\cdot}} \int_0^{T_i} \left(\bar{p} w_{i,t} e^{-y} - \kappa \bar{p} e^{-y} - \kappa (1 - \bar{p}) \right) a_{i,t} e^{-rt} dt$$

subject to

$$\dot{y} = a_i + a_{-i}.$$

The Hamiltonian for this problem is

$$H(a_{i,t}, x_t, \gamma_{i,t}) = \left(\bar{p} w_{i,t} e^{-y} - \kappa \bar{p} e^{-y} - \kappa (1 - \bar{p}) \right) a_{i,t} e^{-rt} + \eta_{i,t} (a_{i,t} + a_{-i,t}).$$

From Theorem 22.26 in page 465 of Clarke (2013), for any measurable $w_{i,t}$ there is an absolutely continuous function $\eta_{i,t}$ such that

$$\dot{\eta}_{i,t} = \bar{p} \left(w_{i,t} - \kappa \right) e^{-y} a_{i,t} e^{-rt}. \quad (16)$$

Also, $a_{i,t}$ maximizes

$$\left(\left(\bar{p}w_{i,t}e^{-y}-\kappa\bar{p}e^{-y}-\kappa(1-\bar{p})\right)e^{-rt}+\eta_{i,t}\right)a_{i,t}. \tag{17}$$

Denote $\gamma_{i,t}=((\bar{p}w_{i,t}e^{-y}-\kappa\bar{p}e^{-y}-\kappa(1-\bar{p}))+\eta_{i,t}e^{rt})$. From the previous expression, $\gamma_{i,t}>0 \implies a_{i,t}=\bar{a}$ and $\gamma_{i,t}<0 \implies a_{i,t}=0$. The boundary condition is

$$\gamma_{i,T_i}=\left(-\kappa-e^{-x_{T_i}}\kappa+e^{-x_{T_i}}w_{i,T_i}\right). \tag{18}$$

where $x_t=\int_0^t y_s\,ds+\log\left(\frac{1-\bar{p}}{\bar{p}}\right)$. Conditions (16) and (18) are necessary for the agent's choice of effort.

Given $\eta_{i,t}$, if $\gamma_{i,t}>0$ in a positive measure set the principal is better off by lowering $w_{i,t}$ so as to lower the expected payments to i, without affecting the effort $a_{i,t}$ that maximizes (17). Thus, of all wage schedules that satisfy agent i's necessary conditions for effort function $a_{i,s}$, the principal's preferred one is such that $\gamma_{i,t}=0$ or, equivalently,

$$\eta_{i,t}=-\left(\bar{p}w_{i,t}e^{-y}-\kappa\bar{p}e^{-y}-\kappa(1-\bar{p})\right)e^{-rt}. \tag{19}$$

We will see that, for a given effort function $a_{i,t}$, at the contract such that $\gamma_{i,t}=0$ the necessary conditions above are also sufficient. Thus, the agent's choice of effort under that contract is indeed $a_{i,t}$.

Replacing the expression for $\eta_{i,t}$ in equation (19) into equation (16), we obtain

$$0=r\kappa+e^{-x_t}(r(\kappa-w_{i,t})+(\kappa-w_{i,t})a_{-i,t}+\dot{w}_{i,t}). \tag{20}$$

Integrating equation (20) we obtain

$$\begin{aligned} w_{i,t} \;=\; & \kappa\left(1-e^{-\int_t^{\bar{t}}(r+a_{-i,s})\,ds}\right)+e^{\int_0^t a_{-i,s}\,ds+rt}\int_t^{\bar{t}} e^{-rl}e^{\int_0^l a_{i,s}\,ds+x_0}r\kappa\,dl \\ & +e^{-\int_t^{\bar{t}}(r+a_{-i,s})\,ds}\bar{w}. \end{aligned} \tag{21}$$

Thus, if a solution to the agent's problem exists conditions (16), (17) and (18) are necessary conditions for the agent's problem. We will see that these conditions are also sufficient and that a solution to the agent's problem exists.

Existence of solution to the agent's problem

Now I show that agent i's problem has a solution for each $w_{i,t}$. Agent i's problem can be written as

$$\max_{a_{i,\cdot}} \int_0^{T_i} (w_{i,t}-\kappa)a_{i,t}\left(\bar{p}e^{-\int_0^t a_s\,ds}+(1-\bar{p})\right)e^{-rt}dt$$

where

$$p_t = \frac{\bar{p}e^{-\int_0^t a_s\,ds}}{\bar{p}e^{-\int_0^t a_s\,ds}+(1-\bar{p})}.$$

Defining $y_t = \int_0^t a_s\,ds$ and replacing p_t into agent i's objective we obtain the following optimal control program for agent i

$$\max_{a_{i,\cdot}} \int_0^{T_i} \left(\bar{p}w_{i,t}e^{-y}-\kappa\bar{p}e^{-y}-\kappa(1-\bar{p})\right)a_{i,t}e^{-rt}dt$$

subject to

$$\dot{y} = a_i + a_{-i}$$

The complication in this problem is that $w_{i,t}$ might not be continuous. All we require is that it be Lebesgue measurable in t.

The running cost

$$\Lambda(t,x,a) = \left(\bar{p}w_{i,t}e^{-y_t}-\kappa\bar{p}e^{-y_t}-\kappa(1-\bar{p})\right)a_{i,t}e^{-rt}$$

is Lebesgue measurable, convex in a_i and lower semicontinuous in (y,a_i). The set of controls is bounded and the process $\dot{y}=a_{-i}$ and $a_i=0$ is admissible and makes the agent's objective finite. Thus, by Theorem 23.11 in page 481 of Clarke (2013) agent i's problem has a solution.

Sufficiency of Pontryagin's conditions Let's see that an effort schedule $a_{i,s}$ is optimal for the agent under the contract that is preferred by the principal among the contract that satisfy the necessary conditions. This is the contract such that $\gamma_{i,t}=0$ in equation (16) given the effort. In fact, let a_i^* denote the optimal effort that the principal would like to induce for agent i. Given $a_{i,t}=a_{i,t}^*$, only $\gamma_{i,t}=0$ solves (20).

Let's see that $\gamma_{i,t}=0$ and $a_{i,t}^*$ is the only solution to i's necessary condition (20) given the principal's optimal $w_{i,t}$. Suppose $\gamma_{i,0}>0$ then $a_{i,0}=\bar{a}_i$. From (20) $\gamma_{i,t}>0$ for all t since $a_{i,t}=\bar{a}_i$ for $t\le T_i$ and $r\kappa+e^{-x_t}(r(\kappa-w_{i,t})+(\kappa-w_{i,t})a_{-i,t}+\dot{w}_{i,t})=0$. However, $\gamma_{i,T_i}>0$ contradicts $\gamma_{i,T_i}=\left(-\kappa-e^{-x_{T_i}}\kappa+e^{-x_{T_i}}w_{i,T_i}\right)=0$ since $x_{T_i}=x_0+\sum_i\int_0^{T_i}a_{i,s}^*\,ds$.

Now, suppose $\gamma_{i,0}\le 0$ and let $t'=\text{argmin}_t\{a_{i,\tau}<\bar{a}_i|\tau\in(t,t+\varepsilon),\text{ for some }\varepsilon>0\}$. Since

$r\kappa + e^{-x_t}(r(\kappa - w_{i,t}) + (\kappa - w_{i,t})a_{-i,t} + \dot{w}_{i,t}) = 0$ for $t \in [0,t']$ we have $\gamma_{i,t'} \leq 0$. Also, there exists $\varepsilon > 0$ such that $r\kappa + e^{-x_t}(r(\kappa - w_{i,t}) + (\kappa - w_{i,t})a_{-i,t} + \dot{w}_{i,t}) < 0$ for $t \in (t', t' + \varepsilon)$. This implies $\gamma_{i,t} < 0$ for $t \geq t'$. However, $\gamma_{i,T_i} = \left(-\kappa - e^{-x_{T_i}}\kappa + e^{-x_{T_i}} w_{i,T_i}\right) > 0$ whenever $x_{T_i} < x_0 + \sum_i \int_0^{T_i} a^*_{i,s}\, ds$ which is a contradiction.

A.4 Proof of Theorem 1

I now show that the principal prefers to have the agents exert maximum effort until a deadline and that the contracts are symmetric. Note that the probability that no breakthrough has occurred up to time t, $e^{-\int_0^t (p_s a_s)\, ds}$, can be re-written as $(1-\bar{p})/(1-p_t)$ because $p_t \sum a_{i,t} = d\ln(1-p_t)/dt$. Consider the following optimal control program for the principal with state variables w_i, and x and control variables are a_i for each i.

$$\max_{a_i} \sum_i \int_0^{T_i} \frac{p_t a_{i,t}\,(\pi - w_{i,t})}{1-p_t} e^{-rt}(1-\bar{p})dt.$$

subject to

$$\dot{x}_t = a_{i,t} + a_{-i,t}$$

$$\dot{w}_i = (a_{-i} + r)(w_i - \kappa) - r\kappa e^x$$

$$a_i \in [0, \bar{a}]$$

We will solve this program by using Pontryagin's principle. Since

$$\frac{\partial \ln}{\partial t}\left(\frac{p_t}{1-p_t}\right) + \frac{\partial}{\partial t}\frac{p_t}{1-p_t} = \frac{\dot{p}_t}{(1-p_t)^2 p_t}$$

and

$$\frac{\partial}{\partial t}\left(\frac{p_t}{1-p_t}\right) = \frac{\dot{p}_t}{(1-p_t)^2},$$

the objective function can be re-written as

$$\sum_i \int_0^{T_i} \left(-\frac{\dot{p}_t}{(1-p_t)^2} - \frac{p_t a_{-i}}{(1-p_t)}\right)(\pi - w_{i,t})\, e^{-rt}(1-\bar{p})dt.$$

Integrating by parts and ignoring terms that don't include the maximization variables of the principal the objective function becomes

$$
\begin{aligned}
-\sum_i \int_0^{T_i} & e^{-rt} e^{-x}((r+a_{-i})\pi-(r+a_{-i})w_{i,t}+\dot{w}_{i,t}) \\
& -\sum_i e^{-x_{T_i}-rT_i}(\pi-w_{i,T_i})+\sum_i e^{-x_0}(\pi-w_{i,0}) \quad =
\end{aligned}
$$

$$
\begin{aligned}
-\int_0^{\infty} \sum_i & e^{-rt} e^{-x}((r+a_{-i,t})\pi-((r+a_{-i,t})\,\kappa+r\kappa e^{x})) \\
& +\sum_i e^{-x_0}(\pi-w_{i,0})-\sum_i e^{-x_{T_i}-rT_i}(\pi-w_{i,T_i})
\end{aligned}
$$

Let μ_i denote the co-state variable associated with the differential equation for w_i. Let γ be the co-state variable associated to x. Let $\mathbf{c}=(a_1,\ldots,a_n)$ be the vector controls, $\mathbf{p}=(\mu_1,\gamma)$ the vector of co-state variables and $\mathbf{x}=(x,w)$ the vector of state variables.

The Hamiltonian is given by

$$
\begin{aligned}
H_t(\mathbf{x},\mathbf{c},\mathbf{p}) \quad = \textstyle\sum_{i,T_i\geq t} \quad & e^{-rt}e^{-x_t}\left(-(r+a_{-i,t})\pi+(r+a_{-i,t})\,\kappa+r\kappa e^{x_t}\right) \\
& +\sum_{i,T_i\geq t}\mu_i\left((a_{-i,t}+r)(w_{i,t}-\kappa)-r\kappa e^{x_t}\right)+\gamma a_t \\
& +\sum_{i,T_i\geq t}\xi_i(\bar{a}-a_{i,t})+\sum_{i,T_i\geq t}\tilde{\xi}_i a_{i,t}.
\end{aligned} \tag{22}
$$

The evolution of the co-state variable of w_i is given by,

$$\dot{\mu_i}=-\mu_i(r+a_{-i,t}),$$

which implies

$$\mu_{i,t}=\mu_0 e^{-\int_0^t(a_{-i,s}+r)ds}.$$

The transversality condition at time zero for co-state variable μ_i is

$$\mu_{i,0}=e^{-x_0}.$$

At time T_i the wages of agent i jump down to zero and, therefore, the co-state variables may jump as well at those points. Define

$$g_i(x_\tau,\tau)=-(1+e^{x_\tau})\kappa \tag{23}$$

as the difference between the wage after the jump (to zero) and the wage before the jump and define also

$$h(x_\tau, w_\tau, \tau) = -\sum_{i,T_i=\tau} e^{-x_\tau - r\tau}(\pi - w_{i,\tau}). \tag{24}$$

From equation (74) in page 196 of Seierstad and Sydsæ ter (1987), the co-state at time T_i is given by

$$\mu_{i,T_i}^{-} = \frac{\partial h(x_{T_i}, w_{T_i}, T_i)}{\partial w_{i,T_i}} + \mu_{i,T_i}^{+} = e^{-x_{T_i} - rT_i} + \mu_{i,T_i}^{+} = e^{-x_0 - \int_0^{T_i}(a_{-i,s} + a_{i,s} + r)ds} + \mu_{i,T_i}^{+}.$$

and therefore $\mu_0 = e^{-x_0}$ and

$$\mu_{i,T_i}^{+} = e^{-x_0 - \int_0^{T_i}(a_{-i,s} + r)ds} - e^{-x_0 - \int_0^{T_i}(a_{-i,s} + a_{i,s} + r)ds}. \tag{25}$$

The evolution of the co-state variable of x is given by,

$$\dot{\gamma} = \sum_{i,T_i \geq t} \left((-(r + a_{-i,t})\pi + \kappa(r + a_{-i,t}))e^{-rt}e^{-x_t} + \kappa \mu_i r e^{x_t} \right)$$

From equation (74) in page 196 of Seierstad and Sydsæ ter (1987) at time T_i, γ_{T_i} jumps at time T_i and satisfies equation

$$\gamma_{T_i}^{-} = \sum_{j,T_j=T_i} \left(e^{-x_{T_i} - rT_i}(\pi - w_{j,T_i}) - \mu_{j,T_i}^{+} e^{x_{T_i}} \kappa \right) + \gamma_{T_i}^{+}. \tag{26}$$

Let n_t denote the number of agents that are still working at time t. If the effort $a_{i,t}$ is interior we have

$$(n_t - 1)(\kappa - \pi)e^{-x_t}e^{-rt} + \sum_{j \neq i, T_j \geq t} \mu_{j,t} \left(w_{j,t} - \kappa \right) + \gamma = 0. \tag{27}$$

Differentiating (27) with respect to t and replacing expressions for $\dot{\gamma}$, $\dot{x}$, we obtain that in an interval in which $a \in (0, \bar{a})$ we have

$$0 = e^{-rt}e^{-x}r(\kappa - \pi) + \kappa e^{x} r \mu_i. \tag{28}$$

Multiplying by $e^{rt}e^{x}$ and differentiating with respect to time we obtain

$$\dot{\mu}_i + r\mu_i + 2\dot{x}\mu_i = 0.$$

Replacing the expression for $\dot{\mu}_i$ and $\dot{x}$ we obtain

$$\mu_i(2a_i + a_{-i}) = 0.$$

Thus, unless $a_i = a_{-i} = 0$ we have $\mu_i = 0$ which contradicts (28).

Define $M = \{i | T_i \geq T_j, \forall j\}$ the set of agents who stop working at the latest time. Let $i \in M$ then $a_{i,t} = \bar{a}_i$ and

$$(|M|-1)(\kappa-\pi)e^{-x_{T_i}}e^{-rT_i} + \sum_{j\neq i, j\in M} \mu_{j,T_i}\left(w_{j,T_i}-\kappa\right) + \gamma_{T_i}^{-} \geq 0. \tag{29}$$

Note that $\gamma_{T_i}^{+} = 0$ because x is unrestricted after T_i and therefore replacing (26) and (25) the left hand side of (29) can be rewritten as

$$\begin{aligned}
&(|M|-1)(\kappa-\pi)e^{-x_{T_i}}e^{-rT_i} + \sum_{j\neq i, j\in M} e^{-x_0-\int_0^t (a_{-i,s}+r)ds}e^{x_{T_i}}\kappa + \\
&+\sum_{j,T_j=T_i}\left(e^{-x_{T_i}-rT_i}(\pi-w_{j,T_i}) - \left(e^{-x_0-\int_0^{T_i}(a_{-j,s}+r)ds} - e^{-x_0-\int_0^{T_i}(a_{-j,s}+a_{j,s}+r)ds}\right)e^{x_{T_i}}\kappa\right) = \\
&(|M|-1)(\kappa-\pi)e^{-x_{T_i}}e^{-rT_i} + \sum_{j\neq i, j\in M} e^{-x_0-\int_0^t (a_{-i,s}+r)ds}e^{x_{T_i}}\kappa + \\
&\sum_{j,T_j=T_i}\left(e^{-x_{T_i}-rT_i}(\pi-\kappa) - e^{-x_0-\int_0^{T_i}(a_{-i,s}+r)ds}e^{x_{T_i}}\kappa\right) = \\
&e^{-x_{T_i}-rT_i}(\pi-\kappa) - e^{-\int_0^{T_i}(-a_{i,s}+r)ds}\kappa \geq 0.
\end{aligned} \tag{30}$$

Let's see that $a_{i,t} > 0$ for $t \leq T_i$. In fact, the factor of $a_{i,t}$: $(|M|-1)(\kappa-\pi)e^{-x_t}e^{-rt} + \sum_{j\neq i, j\in M}\mu_{j,t}\left(w_{j,t}-\kappa\right) + \gamma_t$ has derivative

$$e^{-rt}e^{-x}r(\kappa-\pi) + \kappa e^{x} r\mu_i = e^{-rt}e^{-x}r(\kappa-\pi) + \kappa r e^{-\int_0^t(-a_{i,s}+r)ds} < e^{-rt}r\left(e^{-x_{T_i}}(\kappa-\pi) + e^{\int_0^{T_i} a_{i,s}ds}\kappa\right) \leq 0.$$

where the last inequality is justified by (30).Thus, the factor of $a_{i,t}$ in the Hamiltonian is strictly positive for $t < T_i$ since it is positive at T_i.

Now, consider the agents who stop working second to last. If $\hat{\imath}$ stops at that time the factor that multiplies $a_{\hat{\imath},t}$ in the Hamiltonian has derivative

$$e^{-rt}e^{-x_t}r(\kappa-\pi) + \kappa r e^{-\int_0^t(-a_{\hat{\imath},s}+r)ds} \leq e^{-rt}r\left(e^{-x_{T_i}}(\kappa-\pi) + e^{\int_0^{T_i} a_{i,s}ds}\kappa\right) < 0 \tag{31}$$

where the first inequality is justified by $a_{i,s} \geq a_{\hat{\imath},s}$ since $a_{i,s} = \bar{a}$ for $s \leq T_i$.

By replacing the condition $w_i(T_i)p_{T_i} = \kappa$ into (21) (which comes from $\gamma_{i,T_i} = 0$)

we obtain the following differential equation for the agents' bonus wages of agent i:

$$w_{i,t} = e^{\int_0^t (r+a_{-i,s})\,ds}\left(e^{-rt-\int_0^t a_{-i,s}\,ds}\kappa + e^{-rT_i+x_0+\int_0^{T_i} a_{i,s}\,ds}\kappa + \int_t^{T_i} e^{x_0-r\tau+\int_0^{\tau} a_{i,s}\,ds} r\kappa d\tau\right)$$

Replacing $a_{i,s} = \bar{a}$ since by the previous discussion agents exert the maximum effort we obtain the first order condition for the principal's choice of T_i. That is the first order condition for the agent who stops exerting effort last:

$$\frac{\partial \int_0^{T_i} p_t a_{i,t}(\pi - w_{i,t}) e^{-\int_0^t (p_s a_s + r)\,ds} dt}{\partial T_i} = e^{-x_{T_i}-rT_i}\left(\pi - e^{x_{T_i}+T_i\bar{a}}\kappa - \kappa\right)\bar{a}_i = 0 \tag{32}$$

The derivative is decreasing in T_i and therefore it has a unique solution. Now, suppose agent j has the second highest stopping time T_j and that $\tilde{n}-1$ agents in set $\tilde{I}$ work until time T_i. The first order condition with respect to T_j has to take into account the effect of increasing T_j on the wages of the agents who work until T_i and is given by

$$e^{-rT_j+\bar{a}T_j}\bar{a}\left((\pi-\kappa)\underbrace{\frac{e^{-\bar{a}\tilde{n}(T_j-T_i)}\left(r+\bar{a}\left(-2+e^{(r+\bar{a}(-1+\tilde{n}))\left(-T_i+T_j\right)}+\tilde{n}\right)\right)}{r+\bar{a}(-1+\tilde{n})}}_{*} e^{-x_{T_i}-T_i\bar{a}} - \kappa\right)$$

However, this expression cannot be zero for $T_j < T_i$. In fact, the term $*$ is strictly greater than one which together with (32) implies that the previous expression must be strictly positive. To see that $*$ is greater than one note that at $T_i = T_j$ it is equal to one. The derivative of the numerator in $*$ is given by

$$e^{\bar{a}\tilde{n}\left(T_i-T_j\right)}\bar{a}\left(e^{(r+\bar{a}(-1+\tilde{n}))\left(-T_i+T_j\right)}(-r+\bar{a}) + (r+\bar{a}(-2+\tilde{n}))\tilde{n}\right)$$

which is positive whenever $T_i > T_j$. Thus, the optimal contract is symmetric when the agents are symmetric.

We can now compute the optimal contract by solving its differential equation. From the law of motion of x_t we have $x_t = x_0 + n\bar{a}t$ with $x_0 = \frac{1-\bar{p}}{\bar{p}}$ and $\mu_i = \mu_i^0 e^{-(r+\bar{a}(n-1))t}$ from which obtain the following differential equation for the wage of agent i

$$\dot{w}_i - (\bar{a}(n-1)+r)w_i = -\kappa\left(re^{x_0+n\bar{a}t} + r + \bar{a}(n-1)\right).$$

The solution to this differential equation is

$$w_i(t) = \kappa + \frac{e^{x_0+nt\bar{a}} r\kappa}{r-\bar{a}} + e^{t(r+(-1+n)\bar{a})}C_i.$$

For a constant C_i to be determined. Agent i stops experimenting at a time T_i such that $w_i(T_i)p_{T_i} = \kappa$ where $p_t = \frac{1}{1+e^{x_0+nt\bar{a}}}$. The value of C_i is

$$C_i = \frac{e^{-rT_i+x_0+T_i\bar{a}}\kappa\bar{a}}{-r+\bar{a}}.$$

and replacing we obtain

$$w_i(t) = \kappa + \frac{e^{x_0}\kappa\left(-e^{nt\bar{a}}r + e^{r(t-T_i)+((-1+n)t+T_i)\bar{a}}\bar{a}\right)}{-r+\bar{a}}$$

By maximizing the principal's payoff over the threshold T_i we find that $T_i = T$ given by

$$T = \frac{-x_0 + \ln\left(\frac{\pi-\kappa}{\kappa}\right)}{(1+n)\bar{a}}.$$

Existence of solution to the principal's problem In what follows I show that the principal's program has a solution. Since there is a unique candidate solution that satisfies Pontryagin's condition this candidate solution must be the solution to the principal's relaxed problem. I then show that the effort that the principal would like to implement in the solution to her program is optimal for the agent given the wages. That is, at the optimal wage the differential equation for the wage and the agent's effort is not just necessary but also sufficient for the agent's optimality.

The principal's program has a solution by Theorem 18 in page 400 of Seierstad and Sydsæ ter (1987). In fact, let $U = [0,\bar{a}_1] \times \cdots \times [0,\bar{a}_n]$. The set

$$\begin{aligned} N(x,U,t) \quad = \quad & \Big\{\Big(\sum_i e^{-rt}e^{-x}((r+a_{-i,t})\pi - ((r+a_{-i,t})\,\kappa + r\kappa e^x) + \nu, \sum_i a_{i,t}, \\ & (a_{-1}+r)(w_1-\kappa) - r\kappa e^x, \ldots, (a_{-n}+r)(w_n-\kappa) - r\kappa e^x : \nu \le 0, a \in U\Big\} \end{aligned}$$

is convex for all (x,w,t) since it is linear in the controls and ν. Also, the set U is bounded and we can assume that $w_{i,t} \le \pi$ and $x_t \le x^{FB}$ with $x^{FB} = x_0 + \sum_i \int_0^{\bar{T}_i} \bar{a}_i\, ds$ where $\bar{T}_i$ is the timing at which agent i stops working in the first best.

A.4.1 Proof of Corollary 2

The derivative of w_t with respect to t is

$$\frac{e^{x_0}\kappa\left(-e^{nt\bar{a}}nr\bar{a} + e^{r(t-T)+((-1+n)t+T)\bar{a}}\bar{a}(r+(n-1)\bar{a})\right)}{\bar{a}-r}.$$

The numerator is negative iff

$$\frac{e^{(t-T)(r-\bar{a})}(r+(n-1)\bar{a})}{nr} \leq 1 \iff r \geq \bar{a}.$$

The derivative of w_t with respect to r is given by

$$\frac{e^{-rT+x_0+(-1+n)t\bar{a}}\kappa\left(-e^{rT+t\bar{a}}+e^{rt+T\bar{a}}(1-(t-T)(r-\bar{a}))\right)\bar{a}}{(r-\bar{a})^2}$$

which is negative iff $1-e^{-(t-T)(r-\bar{a})}-(t-T)(r-\bar{a}) \leq 0$ because $1+x \leq e^x$ for every x.

To see that $w_t^*(T^*)$ increases in $\bar{a}$ note that the derivative of $w_t^*(T^*)$ with respect to $\bar{a}$–taking into account that T^* depends on $\bar{a}$–is given by

$$\underbrace{\left(\frac{\kappa r\left(-nt\bar{a}+nrt+1\right)e^{-(t-T^*)(r-\bar{a})}-\kappa\left(\bar{a}\left(r-\bar{a}\right)\left((n-1)t+T^*\right)+r\right)}{(r-\bar{a})^2}-\frac{\kappa T^*}{\bar{a}}\right)}_{*}e^{\bar{a}((n-1)t+T^*)+r(t-T^*)+x_0}$$

When $T^*=t$ the previous expression is equal to $\frac{\kappa T^*(n\bar{a}-1)e^{nT^*\bar{a}+x_0}}{\bar{a}}$. Furthermore, the term $*$ increases in T^* since its derivative with respect to T^* is given by

$$\kappa r\left(\left(-nt\bar{a}+nrt+1\right)e^{(T^*-t)(r-\bar{a})}-1\right)\frac{1}{\bar{a}(r-\bar{a})} > 0.$$

To see that each agent's payoff increases in $\bar{p}$ note that x_0 decreases in $\bar{p}$ and that the derivative of an agent's bonus at time t with respect to x_0 is given by

$$\frac{\kappa e^{x_0}e^{nt\bar{a}}\left((n+1)r-(n\bar{a}+r)e^{(t-T)(r-\bar{a})}\right)}{(n+1)(r-\bar{a})} > 0.$$

A.4.2 Proof of Corollary 3

Let's see that the expected bonus conditional on it being paid is increasing in $\bar{p}$. The expected bonus is given by

$$\int_0^{T^*}\bar{a}e^{-\bar{a}nt-rt-x_0}\left(\kappa+\frac{\kappa e^{x_0}\left(\bar{a}e^{\bar{a}((n-1)t+T^*)+r(t-T^*)}-re^{\bar{a}nt}\right)}{\bar{a}-r}\right)dt \quad (1-\bar{p}) =$$

$$\kappa\bar{a}\left(-\frac{e^{-T^*(n\bar{a}+r)-x_0}}{n\bar{a}+r}+\frac{e^{-x_0}}{n\bar{a}+r}-\frac{e^{T^*(\bar{a}-r)}}{r-\bar{a}}+\frac{1}{r-\bar{a}}\right)(1-\bar{p}).$$

The probability that there is a breakthrough is given by

$$\int_0^{T^*} \bar{a}e^{-nt\bar{a}-x_0}\,dt \cdot (1-\bar{p}) = (1-\bar{p}) \cdot \frac{e^{-x_0} - e^{-nT^*\bar{a}-x_0}}{n}.$$

Thus, the expected bonus conditional on a success simplifies to

$$\frac{\kappa n\bar{a}e^{-rT^*}\left((n\bar{a}+r)\left(e^{rT^*}-e^{T^*\bar{a}}\right)e^{nT^*\bar{a}+x_0}+(r-\bar{a})\left(e^{T^*(n\bar{a}+r)}-1\right)\right)}{(r-\bar{a})\,(n\bar{a}+r)\,(e^{nT^*\bar{a}}-1)}. \tag{33}$$

Note that x_0 is increasing in $\bar{p}$ and T^* is increasing in $\bar{p}$. Let's see that the previous expression is decreasing in T^*. The first term is given by

$$\frac{\kappa n\bar{a}\left(e^{rT^*}-e^{T^*\bar{a}}\right)e^{nT^*\bar{a}-rT^*+x_0}}{(r-\bar{a})\,(e^{nT^*\bar{a}}-1)}.$$

Replacing T^* we obtain $nT^*\bar{a}+x_0 = \frac{n\log\left(\frac{\pi}{\kappa}-1\right)+x_0}{n+1}$. Taking the derivative of the previous expression and ignoring factors that do not depend on T^*, we obtain

$$\frac{\kappa n\bar{a}e^{-rT^*-((n+1)T^*\bar{a})}\left(r\left(-e^{-nT^*\bar{a}}\right)+\bar{a}\left(-ne^{T^*(r-\bar{a})}+e^{-nT^*\bar{a}}+e(n-1)\right)+r\right)}{(r-\bar{a})\,(e^{nT^*\bar{a}}-1)^2}.$$

The previous expression is zero at $T^*=0$ and it becomes negative for $T^*>0$. To see this note that the derivative of the term in parenthesis in the numerator is given by

$$n\bar{a}\,(\bar{a}-r)\left(-e^{-(n+1)T^*\bar{a}}\right)\left(e^{T^*\bar{a}}-e^{T^*(n\bar{a}+r)}\right),$$

which is strictly negative for $T^*>0$ iff and only if $r\geq\bar{a}$.

The derivative of the second term in equation (33), ignoring factors that do not depend on T^*, is given by

$$\frac{e^{-rT^*}\left(r\left(e^{nT^*\bar{a}}-1\right)-n\bar{a}\left(e^{rT^*}-1\right)e^{nT^*\bar{a}}\right)}{\left(e^{nT^*\bar{a}}-1\right)^2}.$$

The previous expression is zero at $T^*=0$ and negative for $T^*>0$. To see this note that the derivative of the term in parenthesis in the numerator is given by

$$n\bar{a}\left(e^{rT}-1\right)(n\bar{a}+r)\left(-e^{nT\bar{a}}\right)<0.$$

B Appendix: Project with two tasks

B.1 Second task: Proof of Proposition 3

Suppose that under the optimal contract each agent i gets expected utility $V_i(h^1)/(1-\bar{p})$ after history h^1 in the first period. We will see that the it is optimal for the principal to offer a contract of the form given by equation (5) and that agents work at maximum speed until a time threshold. In fact, if the principal were to offer a contract that does not satisfy equation (5) there is a contact that does satisfy equation (5) that gives the same expected payoff to all agents after the first breakthrough and weakly higher payoff to the principal. Consider the problem the principal as in the one stage setup of Theorem 1 with an additional integral constraint

$$V_i(h^1) \geq \int_0^{T_i} \left((w_{i,t} - \kappa) e^{-\int_0^t u_s ds - x_0} - \kappa \right) a_{i,t} e^{-rt} dt. \tag{34}$$

where T_i is the supremum time at which i stops working. (34).

We can define a new state variable $V_{i,t}$ with

$$\dot{V}_{i,t} = \left((w_{i,t} - \kappa) e^{-\int_0^t a_s ds - x_0} - \kappa \right) a_{i,t} e^{-rt}$$

setting $V_{i,0} = 0$ and

$$V_{i,T_i} \leq V_i(h^1). \tag{35}$$

Since the agents receive the same payoff if $V_{i,T_i} < V_i(h^1)$ agent i is given a bonus equal to $V(h^1) - V_{i,T_i}$ (which will be the salvage value in the optimal control problem).

The Hamiltonian of the modified problem is given by

$$\begin{aligned} \tilde{H}(\mathbf{x}, \mathbf{c}, \mathbf{p}) \quad = \textstyle\sum_{i,T_i \geq t} \quad & e^{-rt} e^{-x} \left(-(r + a_{-i,t}) \pi + (r + a_{-i,t}) \kappa + r \kappa e^x \right) \\ & + \sum_{i,T_i \geq t} \mu_i \left((a_{-i,t} + r)(w_i - \kappa) - r \kappa e^x \right) + \gamma u \\ & + \sum_i \tilde{\eta}_i \left((w_{i,t} - \kappa) e^{-x_t} - \kappa \right) a_{i,t} e^{-rt}. \end{aligned} \tag{36}$$

where $\tilde{\eta}_i$ is the multipliers associated with state variable $V_{i,t}$ and all other variables are the same as in the proof of Theorem 1. The law of motion of the multiplier $\tilde{\eta}_i$ is

$$\dot{\tilde{\eta}}_i = 0 \tag{37}$$

and, therefore, $\tilde{\eta}_i$ is constant.

The other co-state variables evolve according to

$$\dot{\mu}_{i,t} = -\mu_{i,t}(r + a_{-i,t}) - \tilde{\eta}_{i,t} e^{-x_t - rt} a_{i,t}$$

and

$$\dot{\gamma} = \sum_{i, T_i \geq t} \left((-(r + a_{-i,t})\pi + \kappa(r + a_{-i,t})) e^{-rt} e^{-x} + \kappa \mu_i r e^x + \tilde{\eta}_{i,t}(w_{i,t} - \kappa) e^{-x_t} a_{i,t} e^{-rt} \right).$$

The term that multiplies $a_{i,t}$ is given by

$$(n_t - 1)(\kappa - \pi) e^{-x} e^{-rt} + \sum_{j \neq i, T_j \geq t} \mu_j \left(w_j - \kappa \right) + \gamma + \tilde{\eta}_i \left((w_{i,t} - \kappa) e^{-x_t} - \kappa \right) e^{-rt} = 0. \tag{38}$$

where n_t denote the number of agents that are still working at time t. The derivative of this expression with respect to t is

$$e^{-rt - x_t} r(-\pi + \kappa) + e^{x_t} r \kappa \mu_{i,t}$$

The boundary condition for $\mu_{i,t}$ is $\mu_{i,0} = e^{-x_0}$ as in the one stage case and, therefore, we have

$$\mu_{i,t} = e^{\int_0^t (-r - a_{-i,s}) \, ds} \left(e^{-x_0} + \tilde{\eta}_i e^{-x_0} \left(e^{-\int_0^t a_{i,s} \, ds} - 1 \right) \right)$$

The boundary condition for $\tilde{\eta}_i$ is

$$\tilde{\eta}_{i,T_i} = \tilde{\eta}_i = 1 - \tilde{\mu}$$

where $\tilde{\mu} \geq 0$ is the multiplier associated to the constraint (35). Thus, we have

$$\mu_{i,t} = e^{\int_0^t (-r - u_{-i,s}) \, ds} \left(e^{-x_0} e^{-\int_0^t u_{i,s} \, ds} + \tilde{\mu} e^{-x_0} \left(1 - e^{-\int_0^t u_{i,s} \, ds} \right) \right). \tag{39}$$

By the same steps as in the proof of Theorem (1), and given equation (37), we can conclude that the effort must be either at the maximum or at zero. To see that the effort is exerted at the maximum up to a time threshold we can refer to equation (66) which is valid replacing the Hamiltonian H with the modified Hamiltonian $\tilde{H}$ given by equation (36). Equation (66) simplifies to

$$\left(\sum_{k \neq j} \left(e^{-rT_i} e^{-x_{T_i}} (\kappa - \pi) + \mu_k (w_{k,T_i} - \kappa) \right) + \gamma_{T_i}^- + \tilde{\eta}_i \left((w_{i,t} - \kappa) e^{-x_{T_i}} - \kappa \right) a_{i,t} e^{-rT_i}. \right) = 0 \tag{40}$$

which replacing $T_i = t$ has derivative with respect to t equal to

$$e^{-rt-x_t} r(-\pi+\kappa) + e^{x_t} r\kappa\mu_{i,t} \tag{41}$$

What follows is analogous to the analysis in the proof of Theorem 5. The expression in equation must be non-positive at T_i. Furthermore, from equation (39), the expression $e^{rt}\times$(41) is increasing in t, and as it is negative at T_i it must be negative for $t \leq T_i$. This observation establishes that the effort is at the maximum before it becomes zero.

Existence of solution to the principal's problem The principal's program has a solution by Theorem 18 in page 400 of Seierstad and Sydsæ ter (1987). In fact, let $U = [0, \bar{a}_1] \times \cdots \times [0, \bar{a}_n]$. The set

$$\begin{aligned} N((x, V_{i,\cdot}, U, t) \;\; = \;\; & \Big\{ \Big(\sum_i e^{-rt} e^{-x} ((r + a_{-i,t})\pi - ((r + a_{-i,t})\,\kappa + r\kappa e^x) + \nu, \sum_i a_{i,t}, \\ & (a_{-1} + r)(w_1 - \kappa) - r\kappa e^x, \ldots, (a_{-n} + r)(w_n - \kappa) - r\kappa e^x : \nu \leq 0, a \in U \Big\} \end{aligned}$$

is convex for all (x, w, t) since it is linear in the controls and ν. Also, the set U is bounded and we can assume that $w_{i,t} \leq \pi$ and $x_t \leq x^{FB}$ with $x^{FB} = x_0 + \sum_i \int_0^{\bar{T}_i} \bar{a}_i \, ds$ where $\bar{T}_i$ is the timing at which agent i stops working in the first best.

B.2 First task. Proof of Proposition 4

We can solve the agent's problem using optimal control. The Hamiltonian is given by

$$H_t = \left(w^i_{i,1,t} + v^i_{i,t} - \kappa_1\right) e^{-y^1} a^1_{i,t} \bar{p}^1 - \kappa^1 a^1_{i,t} (1 - \bar{p}^1) + \sum_{j \neq i} \left(w^j_{i,1,t} + v^j_{i,t}\right) a^1_{j,t} e^{-y^1} \bar{p} + \gamma_i \sum_i a^1_{i,1}.$$

By Pontryagin's principle γ_i is absolutely continuous and satisfies the following differential equation

$$\dot{\gamma}_i = r\gamma_i + \left(w^i_{i,1,t} + v^i_{i,t} - \kappa_1\right) e^{-y^1} a^1_{i,t} \bar{p} + \sum_{j \neq i} \left(w^j_{i,1,t} + v^j_{i,t}\right) a^1_{j,t} e^{-y^1} \bar{p}. \tag{42}$$

Define $\tilde{\gamma}_{i,t}$ so that

$$\gamma_{i,t} = -\left(w^i_{i,1,t} + v^i_{i,t} - \kappa^1\right) e^{-y^1} \bar{p}^1 + \kappa^1 (1 - \bar{p}^1) + \tilde{\gamma}_{i,t} \bar{p}.$$

Thus, $\tilde{\gamma}_{i,t} > 0$ implies $a_{i,t} > 0$ and $\tilde{\gamma}_{i,t} < 0$ implies $a_{i,t} < 0$. Replacing $\gamma_{i,t}$ in equation (42) we obtain

$$\begin{aligned}\dot{\tilde{\gamma}}_{i,t} &= r\tilde{\gamma}_{i,t} - e^{-y_t^1}\left(w_{i,1,t}^i + v_{i,t}^i - \kappa_1\right)(a_{-i,t} + r) + \sum_{j \neq i}\left(w_{i,1,t}^j + v_{i,t}^j\right)u_{j,t}^1 e^{-y_t^1} + \left(\dot{w}_{i,1,t}^i + \dot{v}_{i,t}^i\right)e^{-y_t^1} \\ &\quad + \kappa^1 r e^{x_0^1}. \end{aligned} \tag{43}$$

Let T denote the time at which the principal has the agents stop working. We allow for the possibility that the principal may want agent i to stop working earlier than other agents. Suppose at time $t > T_i$, $w_{i,1,t}^i + v_{i,t}^i = 0$. The salvage value at time T_i is given by $G(y_{T_i}, T_i)\bar{p}^1$ with

$$G(y_{T_i}, T_i) = \int_{T_i}^{\infty} \sum_{j \neq i} v_{i,t}^j a_{j,t}^1 e^{-y_{T_i} - \int_{T_i}^t a_s\, ds - rt}\, dt.$$

The boundary condition is

$$\gamma_{i,T_i} e^{-rT_i} = \frac{\partial G(y_{T_i}, T_i)}{\partial y_{T_i}} \bar{p}^1 = -\bar{p}^1 \int_{T_i}^{\infty} \sum_{j \neq i} v_{i,t}^j a_{j,t}^1 e^{-y_{T_i} - \int_{T_i}^t a_s\, ds - rt}\, dt$$

and thus,

$$\tilde{\gamma}_{i,T} = \left(w_{i,1,T}^i + v_{i,T}^i - \kappa^1\right)e^{-y^1} - \kappa^1 e^{x_0^1} - G(y_{T_i}, T_i)e^{rT_i} \tag{44}$$

We denote $b_{i,t} = w_{i,1,t}^i + v_{i,t}^i$. Solving the differential equation for $b_{i,t}$ and replacing condition (44) we obtain

$$\begin{aligned} b_{i,t} &= e^{\int_{T_i}^t (r + a_{-i,s}^1)\, ds}\left(\left(\tilde{\gamma}_{i,T} + G(y_{T_i}, T_i)e^{rT_i} + \kappa_1 e^{x_0^1}\right)e^{y_{T_i}^1}\right) + \kappa_1 \\ &\quad - e^{\int_{T_i}^t (r + a_{-i,s}^1)\, ds} \int_{T_i}^t e^{-\int_{T_i}^s (r + a_{-i,s}^1)\, ds} \sum_{j \neq i}\left(w_{i,1,s}^j + v_{i,s}^j\right) a_{j,s}^1\, ds + \\ &\quad e^{\int_{T_i}^t (r + a_{-i,s}^1)\, ds} \int_{T_i}^t e^{-\int_{T_i}^s (r + a_{-i,s}^1)\, ds} e^y \left(\dot{\tilde{\gamma}}_{i,s} - r\gamma_{i,s} - \kappa_1 r e^{x_0^1}\right) ds \end{aligned}$$

Integrating by parts and simplifying we obtain

$$\begin{aligned} b_{i,t} &= e^{-\int_t^{T_i} (r + a_{-i,s}^1)\, ds} e^{y_{T_i}^1 + x_0^1} \kappa_1 + \kappa_1 + \sum_{j \neq i} \int_t^{T_i} e^{-\int_t^s (r + a_{-i,s}^1)\, ds}\left(w_{i,1,s}^j + v_{i,s}^j\right) u_{j,s}^1 + \\ &\quad + \int_t^{T_i} e^{-\int_t^s (r - a_{i,s}^1)\, ds}\left(\tilde{\gamma}_{i,s} a_{i,s}^1 + \kappa_1 r\right) ds + \tilde{\gamma}_{i,t} + G(y_{T_i}, T_i) e^{-\int_t^{T_i} (r + a_{-i,s}^1)\, ds + rT_i + y_{T_i}^1}. \end{aligned} \tag{45}$$

Analogously, the principal increases her payoff by setting $w_{i,1,t}^j = 0$ for $j \neq i$.

If the wage schedule satisfies equation (6) then $\tilde{\gamma}_{i,t}$ is the unique solution to (43) which implies that if the agent's problem has a solution for a given wage then Pontryagin's principle is also sufficient.

B.3 Detailed computations for section 4.2

Using a second degree Taylor expansion, agent is expected payoff at time t can be written as

$$\begin{aligned}
\tilde{V}_{i,t} &= V_{i,t} + -\frac{1}{2}dt^2 v_{i,t}^{-i}(pa_{-i,t})^2 + dt(pa_{-i,t}v_{i,t}^{-i}) + \\
&\left(\frac{1}{2}dt^2(p(a_{i,t}+a_{-i,t})+r)^2 + dt(-(p(a_{i,t}+a_{-i,t})+r)) + 1\right) \times \\
&\left(dt(pa_{-i,t+dt}v_{i,t+dt}^{-i}) - dt^2 pa_{-i,t+dt}v_{i,t+dt}^{-i}\left((1-p)(a_{i,t}+a_{-i,t}) + \frac{pa_{-i,t+dt}}{2}\right) + \right. \\
&(1 - dt(p(a_{i,t+dt}+a_{-i,t+dt}) + r) \\
&\left. dt^2\left(\frac{1}{2}(p(a_{i,t+dt}+a_{-i,t+dt})+r)^2 + (1-p)p(a_{i,t}+a_{-i,t})(a_{i,t+dt}+a_{-i,t+dt})\right)\right)\tilde{V}_{i,t}\Bigg),
\end{aligned}$$

where $V_{i,t}$ is given by equation (14), replacing $w_{i,t}$ by $b_{i,t}$. Thus, we obtain

$$-\frac{\partial \tilde{V}_{i,t}}{\partial a_{i,t}} + \frac{\partial \tilde{V}_{i,t}}{\partial a_{i,t+dt}} = -\frac{\partial V_{i,t}}{\partial a_{i,t}} + \frac{\partial V_{i,t}}{\partial a_{i,t+dt}} + \left(p_t a_{-i,t} v_{i,t}^{-i}\right) dt^2 + o(dt^3).$$

B.4 First task. The principal's problem

I now state the principal's problem, write it as optimal control problem given the continuation contract derived in Proposition 3 and prove the Theorems in section 4.

The principal's problem To write the problem of the principal subject to the differential equation of the agents' bonus wages. For each i we add the constraint

$$b_{i,t} - v_{i,t}^{i}(\mathbf{T}(i,t)) \geq 0 \tag{46}$$

which requires that the payoff the agent gets after succeeding at time t is at least the continuation payoff from time t as necessary under limited liability.

We can assume $\tilde{\gamma}_{i,t} \geq 0$ since for any contract with $\tilde{\gamma}_{i,t} < 0$ for t in some set Θ there is a payoff equivalent contract that gives the same incentives to the agents with $\tilde{\gamma}_{i,t} = 0$ for $t \in \Theta$. Also, as long as $b_{i,t} > v_{i,t}^{i}(\mathbf{T}(i,t))$ the principal sets $\tilde{\gamma}_{i,t} = 0$. If not, by decreasing $\tilde{\gamma}_{i,t}$ slightly the principal can give incentives for the same effort at lower cost.

Additionally we require for each i

$$\tilde{\gamma}_{i,t}(a^1_{i,t} - \bar{a}) \geq 0 \tag{47}$$

$$\tilde{\gamma}_{i,t}a^1_{i,t} \geq 0. \tag{48}$$

These two constraints are equivalent to $\tilde{\gamma}_{i,t} > 0 \implies a_{i,t} = \bar{a}$ and $\tilde{\gamma}_{i,t} < 0 \implies a_{i,t} = 0$, as is required from agent i's problem.

In order to solve the ensure that the principal's problem has a solution we will make the technical assumption that the derivative of the agent's multiplier $|\dot{\gamma}_{i,t}|$ is bounded by an arbitrarily large constant M.

I will use optimal control to solve the principal's problem in the first stage. The state variables are $y^1_t = \int_0^t a^1_s\, ds$, the first period bonus wage $b_{i,t}$, the multiplier from the agent's problem $\tilde{\gamma}_{i,t}$. The control variables at time τ are the second period time thresholds $\mathbf{T}(k,\tau) = (T_1(k,\tau), T_2(k,\tau), \ldots, T_n(k,\tau))$, each agent i's effort $a_{i,\tau}$ and $\dot{\tilde{\gamma}}_{i,t}$ which we denote $d_{i,t}$. The differential equations for the state variables are

$$\dot{y}^1_s = \sum_i a^1_{i,s} \tag{49}$$

$$\dot{b}_{i,t} = \left((b_{i,t} - \kappa)(a_{-i,t} + r) - \sum_{j \neq i} v^j_{i,t} u^1_{j,t} - \kappa r e^{y^1 + x^1_0} - r\tilde{\gamma}_{i,t} e^{y^1} + d_{i,t} e^{y^1} \right) \tag{50}$$

$$\dot{\tilde{\gamma}}_{i,t} = d_{i,t}. \tag{51}$$

The Hamiltonian is given by

$$\begin{aligned} H^{RR} \;=\; & \sum_i \left(\pi(\mathbf{T}(i,t)) - c(\mathbf{T}(i,t)) - b_{i,t} - \sum_{j \neq i} v^i_{j,t}(\mathbf{T}(i,t)) \right) a^1_{i,t} e^{-y^1_t - rt} \bar{p}^1 + \gamma_t \sum_i a^1_{i,t} \\ & + \sum_i \gamma_{i,t} \left((b_{i,t} - \kappa)(a_{-i,t} + r) - \sum_{j \neq i} v^j_{i,t}(\mathbf{T}(j,t)) a^1_{j,t} - \kappa r e^{y^1_t + x^1_0} - r\tilde{\gamma}_{i,t} e^{y^1_t} + d_{i,t} e^{y^1_t} \right) + \\ & + \sum_i \left(\eta_{i,t} d_{i,t} + \beta^1_{i,t} \tilde{\gamma}_{i,t}(a^1_{i,t} - \bar{a}) + \beta^2_{i,t} \tilde{\gamma}_{i,t} a^1_{i,t} + \tilde{\xi}_{i,t}\left(b_{i,t} - v^i_{i,t} \right) \right) \end{aligned}$$

where γ_t is the multiplier associated to y_t, $\gamma_{i,t}$ is the multiplier associated to $b_{i,t}$, $\eta_{i,t}$ is the multiplier associated to $\tilde{\gamma}_{i,t}$ and $\beta^m_{i,1}$ for $m \in \{1,2\}$ and $\tilde{\xi}_{i,t}$ are associated to the constraints.

Evolution of co-state variables By Pontryagin's principle

$$\dot{\gamma}_t = \sum_i \left(\left(\pi(\mathbf{T}(i,\tau)) - c(\mathbf{T}(i,\tau)) - b_{i,t} - \sum_{j\neq i} v^i_{j,t}(\mathbf{T}(i,\tau)) \right) a^1_{i,t} e^{-y^1_t - rt} \bar{p}^1 + \gamma_{i,t} \kappa r e^{y^1_t + x^1_0} + r\tilde{\gamma}_{i,t} e^{y^1_t} - d_{i,t} e^{y^1_t} \right). \tag{52}$$

and

$$\dot{\gamma}_{i,t} = a_{i,t} e^{-y^1_t - rt} \bar{p}^1 - \gamma_{i,t} \left(a_{-i,t} + r\right) - \tilde{\xi}_{i,t}. \tag{53}$$

$$\dot{\eta}_{i,t} = r\gamma_{i,t} e^{y^1_t} - \beta^1_{i,t}(a^1_{i,t} - \bar{a}) - \beta^2_{i,t} a^1_{i,t}. \tag{54}$$

We have the following condition at T_i:

$$b_{i,T_i} = \kappa(1 + e^{y^1_{T_i} + x^1_0}) + G(y^1_{T_i}, T_i) e^{y^1_{T_i} + rT_i} + \tilde{\gamma}_{i,T_i} e^{y^1_{T_i}} \tag{55}$$

Let $\tilde{\mu}$ be the multiplier associated with constraint (55). The boundary conditions are:

$$\gamma_{i,0} = 0$$

$$\gamma_{i,T_i} = -\tilde{\mu}$$

$$\eta_{i,0} = 0$$

$$\eta_{i,T_i} = \tilde{\mu} e^{y^1_{T_i}}$$

$$\gamma_T = \tilde{\mu} \left(\tilde{\gamma}_{i,T_i} e^{y^1_{T_i}} + e^{y^1_{T_i} + x^1_0} \kappa \right)$$

Maximization with respect to $d_{i,t}$ Whenever $|d_{i,t}| \neq M$, $d_{i,t}$ the terms that multiply $d_{i,t}$ must be zero. Thus,

$$\gamma_{i,t} e^{y^1_t} + \eta_{i,t} = 0. \tag{56}$$

Suppose $\tilde{\gamma}_{i,t} > 0$ and $a_{i,t} = \bar{a}$ for t in a time interval $[t_0, t_1]$ then $\beta^j_{i,t} = 0$ for $j \in \{1,2\}$ in that time interval. Differentiating equation (54) with respect to t and combining it with (56) we obtain

$$-\left(a_{i,t} + a_{-i,t}\right) \gamma_{i,t} - \dot{\gamma}_{i,t} = r\gamma_{i,t}$$

and thus, $\gamma_{i,t} = \gamma_{i,t_0} e^{-\int_{t_0}^t (r + a_s)\, ds}$ for $t \in [t_0, t_1]$. Replacing into equation (53) we obtain

$$\tilde{\xi}_{i,t} = a^1_{i,t} e^{-y^1_t - rt} \bar{p}^1 + a^1_{i,t} \gamma_{i,t_0} e^{-\int_{t_0}^t (r + u^1_s)\, ds} \tag{57}$$

Maximization with respect to $T_i(k,\tau)$ Let $k \neq i$, if $a^1_{k,\tau} > 0$. The derivative of the agent i's payoff with respect to $T_i(k,\tau)$ can be derived by replacing the wage equation (5) and is given by $e^{-T_i(k,\tau)r}\left(-1+e^{T_i(k,\tau)\bar{a}_i}\right)\kappa\bar{a}_i(1-\bar{p})$. Thus, the first order condition for maximization with respect to $T_i(k,\tau)$ is given by

$$
\begin{aligned}
&(\pi-\kappa)\,\bar{p}e^{-T_i(k,\tau)(r+n\bar{a}_i)} - \bar{p}\int_{T_i(k,\tau)}^{T_k(k,\tau)} (\pi-\kappa)\,e^{-y_s-rs}\,ds - (1-\bar{p})e^{-T_i(k,\tau)r}\kappa \\
&-e^{-T_i(k,\tau)r}\left(e^{T_i(k,\tau)\bar{a}_i}-1\right)(1-\bar{p})\kappa - \gamma_{i,t}e^{y^1_t+rt}e^{-T_i(k,\tau)r}\left(e^{T_i(k,\tau)\bar{a}_i}-1\right)(1-\bar{p})\kappa\frac{1}{\bar{p}^1} \quad = \quad 0
\end{aligned}
\tag{58}
$$

For the maximization with respect to $T_i(i,\tau)$ note that if $\tilde{\xi}_{i,\tau}=0$ then the $T_i(i,\tau)$ solves

$$
\max\int_0^{T_i(i,\tau)} \bar{a}_i\,(p_t\pi-\kappa)\,e^{-\int_0^t (p_s a_s + r)\,ds}\,dt
$$

which corresponds to the planers problem and therefore corresponds to the efficient belief threshold for i.

If $\tilde{\xi}_{i,\tau}>0$ then $T_i(i,\tau)$ solves

$$
\max\int_0^{T_i(i,\tau)} \bar{a}_i\,(p_t\pi-\kappa)\,e^{-\int_0^t (p_s a_s + r)\,ds}\,dt - \tilde{\xi}_{i,\tau}v^i_{i,\tau}.
$$

If $\tilde{\gamma}_{i,\tau}>0$ replacing $\tilde{\xi}_{i,\tau}$ from equation (57) and x_0 the first order condition is given by

$$
\left((\pi-\kappa)\,e^{-rT_i(i,\tau)-\sum_k T_k(i,\tau)\bar{a}-x_0} - \kappa e^{-rT_i(i,\tau)}\right)\bar{p}^1 - \left(\bar{p}^1+\gamma_{i,t_0}e^{-\int_0^{t_0}(r+a_s)\,ds}\right)e^{-rT_i(i,\tau)}\left(e^{\bar{a}T_i(i,\tau)}-1\right)\kappa = 0
$$

Thus, if $\gamma_{i,t_0}=0$ the threshold is $2T_i(i,\tau)=\frac{-x_0+\mathrm{Log}\left(\frac{\pi-\kappa}{\kappa}\right)}{\sum_k T_k(i,\tau)\bar{a}}$. That is, agent i works until the inefficient belief threshold that was optimal in the one stage game.

If $\gamma_{i,t_0}>0$ the condition becomes

$$
\left((\pi-\kappa)\,e^{-\sum_k T_k(i,\tau)\bar{a}-x_0} - \kappa e^{\bar{a}T_i(i,\tau)}\right)\bar{p}^1 - \gamma_{i,t_0}e^{-\int_0^{t_0}(r+a^1_s)\,ds}\left(e^{\bar{a}T_i(i,\tau)}-1\right)\kappa = 0
$$

which gives $T_i(i,\tau)<\frac{1}{2}\frac{-x_0+\mathrm{Log}\left(\frac{\pi-\kappa}{\kappa}\right)}{\sum_k T_k(i,\tau)\bar{a}}$. Since it is better for the principal to set $T_i(i,\tau)=\frac{1}{2}\frac{-x_0+\mathrm{Log}\left(\frac{\pi-\kappa}{\kappa}\right)}{\sum_k T_k(i,\tau)\bar{a}}$ when $\tilde{\gamma}_{i,t}>0$–and this choice does not affect the bonus contract in other times–then it must be the case that $\gamma_{i,t_0}=0$.

Maximization with respect to $a^1_{i,t}$ The term that multiplies $a^1_{i,t}$ in the Hamiltonian is given by

$$\left(\pi(\mathbf{T}(i,t)) - c(\mathbf{T}(i,t)) - b_{i,t} - \sum_{j\neq i} v^i_{j,t}(\mathbf{T}(i,t))\right) e^{-y^1_t - rt} + \gamma_t + \sum_{j\neq i} \gamma_{j,t}\left(b_{j,t} - \kappa^1 - v^i_{j,t}(\mathbf{T}(i,t))\right). \tag{59}$$

Existence of solution to the principal's problem In order to prove existence of the solution I will re-write the problem so that the principal's payoff is in terms of the expected payoff each agent gets in the optimal contract. Denote

$$W^2_i(v_i) = \int_0^{T_i(v_i)} (p_t\pi - \kappa)\, a_{i,t} e^{-\int_0^t p_s a_s - rt}\, dt$$

where $T_i(v_i)$ solves

$$\frac{e^{-rT_i(v_i)}\kappa\left(r - e^{T_i(v_i)\bar{a}} r + \left(-1 + e^{rT_i(v_i)}\right)\bar{a}\right)}{r(r-\bar{a})}(1-\bar{p}) = v_i. \tag{60}$$

With this notation the principal's objective for the first task becomes

$$\int_0^{\infty} \sum_i \left(\sum_j W^2_i(v^i_{j,t}) - b_{i,t} - \sum_{j\neq i} v^i_{j,t}(\mathbf{T}(i,t))\right) a^1_{i,t} e^{-y^1_t - rt}\bar{p}^1.$$

Denote the integrand in the previous expression as $f_0(\mathbf{x},\mathbf{a},t)$, where $\mathbf{x}$ is a vector that contains the state variables and $\mathbf{a}$ the control variables. Denote $f_i(\mathbf{x},\mathbf{a},t)$ for $i \in \{1,\ldots,3\}$ for the differential equations given by equations (49) through (51) and $f(\mathbf{x},\mathbf{a},t) = (f_i(\mathbf{x},\mathbf{a},t))^3_{i=1}$. I will now establish existence of a solution to the principal's problem by referencing Theorem 18 in page 400 of Seierstad and Sydsæ ter (1987). Define the set

$$N(\mathbf{x},U,t) = \{(f_0(\mathbf{x},\mathbf{a},t) + \nu, f(\mathbf{x},\mathbf{a},t)) : \nu \leq 0, \mathbf{a} \in U\}$$

where U denotes the set of controls.

Let's see that $N(\mathbf{x},U,t)$ is convex. First note that $W^2_i(v^i_{j,t})$ is concave in $v^i_{j,t}$. In fact, from equation (60) we have

$$T'_i(v_i) = \frac{e^{rT_i(v_i)}}{\kappa\bar{a}\left(e^{\bar{a}T_i(v_i)} - 1\right)(1-\bar{p})}$$

and

$$T_i''(v_i) = -\frac{e^{2rT_i(v_i)}\left((\bar{a}-r)\,e^{\bar{a}T_i(v_i)}+r\right)}{\kappa^2\bar{a}^2\left(e^{\bar{a}T_i(v_i)}-1\right)^3(1-\bar{p})^2}$$

The second derivative of $W_i^2(v_i)$ with respect to v_i is given by

$$\bar{p}(\kappa-\pi)\left((\bar{a}n+r)T_i'(v_i)^2 - T_i''(v_i)\right) - \kappa(\bar{p}-1)e^{\bar{a}nT_i(v_i)}\left(rT'(v_i)^2 - T_i''(v_i)\right).$$

Replacing the expressions for $T_i'(v_i)$ and $T_i''(v_i)$ into the previous expression we obtain

$$\frac{e^{2rT_i(v_i)}\left(\bar{p}(\pi-\kappa)\left(-\left((n+1)e^{\bar{a}T_i(v_i)}-n\right)\right)-\kappa(\bar{p}-1)e^{\bar{a}(n+1)T_i(v_i)}\right)}{\bar{a}\kappa^2(\bar{p}-1)^2\left(e^{\bar{a}T_i(v_i)}-1\right)^3} < 0.$$

To see that $N(\mathbf{x},U,t)$ is convex consider two controls $\mathbf{a} = (a_{i,t}, d_{i,t}, v^i_{j,t}, T_i(i,t))_{i,j}$ and $\tilde{\mathbf{a}} = (\tilde{a}_{i,t}, \tilde{d}_{i,t}, \tilde{v}^i_{j,t}, \tilde{T}_i(i,t))_i$, and reals $\nu, \nu' \leq 0$ and let's see that

$$\beta\left(f_0(\mathbf{x},\mathbf{a},t)+\nu, f(\mathbf{x},\mathbf{a},t)\right) + (1-\beta)\left(f_0(\mathbf{x},\tilde{\mathbf{a}},t)+\nu', f(\mathbf{x},\tilde{\mathbf{a}},t)\right) \tag{61}$$

is in $N(\mathbf{x},U,t)$ for every β. Since $W_i^2(v_i)$ is concave, f_0 and f are concave in $\mathbf{a}$, thus, (61) is in $N(\mathbf{x},U,t)$.

B.5 Costly incentives in the first task

We need to prove that the principal sets $\tilde{\gamma}_{i,t} = 0$ and $a_{i,t} = \bar{a}$.

Note first that $b_{i,t}$ in equation (45) increases in $\tilde{\gamma}_{i,t}$. Since $b_{i,t} > v_{i,t}(T_i^{2*}(t))$ at the best choice of experimentation threshold in the first task and the maximum amount of experimentation that is profitable for i to perform in the second task, the principal has to give a strictly positive bonus to the agent in case of success. Setting $\tilde{\gamma}_{i,t} = 0$ reduces the bonus.

Thus, solving for the multipliers we obtain

$$\gamma_{i,t} = \bar{p}^1 e^{-\int_0^t (r+a^1_{-i,s})\,ds}\left(1-e^{-\int_0^t a^1_{i,s}\,ds}\right) \tag{62}$$

and replacing in equation (58) we obtain the condition in equation (8). Let's see that equation (8) implies that $\frac{\partial T_i}{\partial t} < 0$. First, note that

$$(\pi-\kappa)\,\bar{p}e^{-T_i(k,\tau)(r+n\bar{a}_i)} - \bar{p}\int_{T_i(k,\tau)}^{T_k(k,\tau)} (\pi-\kappa)\,e^{-y_s - rs}\,ds$$

is decreasing in T_i. It's derivative with respect to T_i is given by

$$\left(re^{r(V_a/\bar{a}-T_i)}\left((\pi-\kappa)e^{x_0-2T_i\bar{a}}\left(\bar{a}\left(e^{(2T_i-V_a/\bar{a})(\bar{a}+r)}-2\right)-r\right)+\kappa\left(\bar{a}+r\right)\right)\right)\frac{\bar{a}e^{-rV_a/\bar{a}}}{r+\bar{a}}<0.$$

The inequality is justified because $T_k \geq T_i$ and because $\pi-\kappa/p_{T_i}=\left(-\kappa e^{2T_i\bar{a}}-\kappa+\pi\right)>0$.

Now, to see that the principal sets $a^1_{i,t}=\bar{a}$. Suppose $a^1_{i,t}<\bar{a}$ at some interval. Define

$$\pi(T_{-i})=\int_0^{T_{-i}}p_t(\pi-w_t^*(T_{-i}))\bar{a}e^{-\int_0^t(p_sa_s+r)\,ds}\,dt.$$

$\pi(T)$ is the expected payoff that the principal receives in task two from agent $-i$'s work in that task. Define

$$\tilde{\pi}(T_i)=\int_0^{T_i}(p_t\pi-\kappa)\bar{a}e^{-\int_0^t(p_sa_s+r)\,ds}\,dt.$$

The principal's payoff from agent i's work in the first stage can be approximated as

$$\Pi_{i,t}=\left(p_t^1\left(\pi(T_{-i})+\tilde{\pi}(T_i)\right)-\kappa-\left(p_t^1b_{i,t}-\kappa\right)\right)a^1_{i,t}(1-e^{-a^1_{i,t}dt})+e^{-(r+a_{i,t}+a_{-i,t})dt}\Pi_{i,t+dt}$$

Replacing $\Pi_{i,t+dt}$ recursively and replacing the exponentials by their second order Taylor expansion we obtain

$$\frac{\partial}{\partial(dt)^2}\left(\frac{\partial\Pi_{i,t}}{\partial\varepsilon}\right)=\frac{d\left(\pi(T_{-i})+\tilde{\pi}(T_i)\right)}{dt}p_t^1-(a^1_{-i,t}+r)\left(\left(\pi(T_{-i})+\tilde{\pi}(T_i)\right)-\kappa\right)p_t^1+ \qquad (63)$$
$$r\kappa e^{x_t^1}p_t^1+\underbrace{\frac{\partial}{\partial(dt)^2}\left(\frac{\partial V_{i,t}}{\partial\varepsilon}\right)}_{=0}<0$$

The inequality follows from $\frac{d(\pi(T_{-i})+\tilde{\pi}(T_i))}{dt}=\gamma_{i,t}\frac{\partial v^k_{i,t}}{\partial T_i}\frac{\partial T_i}{\partial t}$ from the maximization of the Hamiltonian with respect to T_i, $\frac{\partial T_i}{\partial t}<0$ and $\frac{\partial v^k_{i,t}}{\partial T_i}=e^{-T_i(k,\tau)r}\left(-1+e^{T_i(k,\tau)\bar{a}_i}\right)\kappa\bar{a}_i(1-\bar{p})>0$, and from $p_t^1\left(\pi(T_{-i})+\tilde{\pi}(T_i)\right)-\kappa>0$, since otherwise the principal does not have agent i exert effort at time t.

Thus, the principal does not want to delay effort and the agents exert maximum effort until a time threshold.

B.6 Cheap incentives in the first task

When $b^*_{i,t} < v_{i,t}(T^{2*})$ the principal does not have to give any bonuses after the first milestone and the second task bonuses and experimentation thresholds are the principal's preferred ones. To see that the principal prefers that the agents exert full-effort until the efficient threshold note that since the payoff of the agent and the principal are constant in t we obtain as before

$$\frac{\partial}{\partial(dt)^2}\left(\frac{\partial \Pi_{i,t}}{\partial \varepsilon}\right) = -(a^1_{-i,t}+r)\left((\pi(T_{-i})+\tilde{\pi}(T_i))-\kappa\right)p^1_t + r\kappa e^{x^1_t}p^1_t + \underbrace{\frac{\partial}{\partial(dt)^2}\left(\frac{\partial V_{i,t}}{\partial \varepsilon}\right)}_{=0} < 0. \quad (64)$$

Thus, the principal does not want to delay the agents' work.

B.7 Intermediate costs case

If $b_{i,t} = v^i_{i,t}(\mathbf{T}(i,t))$ and the principal's payoff is decreasing in $T_i(i,t)$ (fixing the other agents' stopping times) then we must also have $\tilde{\gamma}_{i,t} = 0$. If not, the principal can lower $\tilde{\gamma}_{i,t}$ and $T_{i,}(i,t)$ and incentivize the same effort at lower cost. Thus, $\tilde{\gamma}_{i,t}$ can only be non-zero when $T_{i,}(i,t)$ maximizes the principal's payoff in the second period. This payoff can only be maximized in an interval if $\tilde{\xi}_{i,t} = \bar{a}e^{-\int_0^t a_s ds - rt}$, and thus, $\gamma_{i,t}$ is constant in that interval, and therefore, $T_i(-i,t)$ and $T_i(i,t)$ are constant in that interval. These thresholds cannot remain constant when $\tilde{\gamma}_{i,t} = 0$ because $b_{i,t}$ is not constant when $\tilde{\gamma}_{i,t} = 0$. Thus, when $b_{i,t} = v^i_{i,t}(\mathbf{T}(i,t))$ it is either the case that $\tilde{\gamma}_{i,t} > 0$ and $T(i,t)$ and $T(-i,t)$ are constant (by the arguments when maximizing over $T_i(k,t)$) or $\tilde{\gamma}_{i,t} = 0$. If $\tilde{\gamma}_{i,t} > 0$ and the experimentation thresholds in the second task are not set at the principal's preferred ones (given by the solution of the one task model), then the principal can lower the agents' payment slightly without affecting their incentives by bringing the experimentation thresholds closer to her preferred ones.

To see that the principal sets the agents' efforts at the maximum until a threshold note that from the previous discussion for every time t either (63) or (64) holds.

B.8 Conditions for an asymmetric contract

Let T^S solve the following equation

$$v^{-i}_{i,T}\left(T^2_i(T^S)\right)e^{T^S\bar{a}} + \kappa^1 e^{3T^S\bar{a}+x^1_0} + \kappa^1 + c\left(T^{2*}_i(T^S), T^2_{-i}(T^S)\right) - \pi\left(T^{2*}_i(T^S), T^2_{-i}(T^S)\right) = 0$$

This equation corresponds to the first order condition with respect to the experimentation threshold in the first stage assuming both agents stop at the same time.

Proposition 11. *A sufficient condition for the first task contract to be asymmetric is*

$$-\kappa^1 e^{T\bar{a}+x_0^1}\left(-\bar{a}e^{T(\bar{a}+r)+2T^S\bar{a}}+(\bar{a}+r)\,e^{T^S(\bar{a}+r)+2T\bar{a}}+r\left(-e^{T^S(3\bar{a}+r)}\right)\right) \\ +v_{i,T}^{-i}\left(T_i^2(T)\right)\left(e^{T(2\bar{a}+r)}-e^{\bar{a}(T+T^S)+rT^S}\right)\bar{a} \;<\; 0 \tag{65}$$

for $T\in[T^S-\varepsilon,T^S]$ *for some* ε.

The expression in (65) corresponds to the first order condition with respect to the stopping time of the agent who stops first. If the expression is negative then $T=T^S$ is not the optimal first stopping time.

C Appendix: Extensions

C.1 Optimal disclosure of discoveries: Proposition 5

Let $\mathscr{D}$ denote the space of potential disclosure policies of discoveries by the principal. Discoveries are verifiable by all the agents. A disclosure policy $d(h^t)\in\mathscr{D}$ is a function of the history $h^t\in\mathscr{H}^t$ and is a process that is adapted to the σ-algebra of public histories. I assume that disclosures fully reveal that a breakthrough has occurred. The space of possible disclosure policies is very large. Examples of policies are: disclose discovery as soon as it arrives with probability one, disclose a discovery two seconds after it arrives with probability q and then disclose at some Poisson rate after that, not disclose a breakthrough if it arrives before some time $\tilde{t}$ and disclose it right away thereafter.[23] Proposition 2 allows me to simplify the problem considerably. A disclosure policy translates in a non-decreasing measurable process $\int_0^t a_{-i,s}\,ds$, as a function of t, from the viewpoint of agent i. Thus, Proposition 2 characterizes the wage i must receive. Let T denote the supremum of the times at which agent i exerts positive effort. Solving the differential equation given by equation (3) in Proposition 2 I obtain

$$w_{i,t}=\kappa\left(\exp\left(-\int_t^T(r-a_{i,s})\,ds+\int_0^t a_s\,ds+x_0\right)+1\right)+e^{rt+\int_0^t a_{-i,s}\,ds}\int_t^T \kappa r e^{-r\tau+\int_0^\tau a_{i,s}\,ds+x_0}\,d\tau.$$

[23] Because I assume that disclosures are perfect, I do not consider policies in which the principal partially discloses a breakthrough. A partial disclosure policy is for instance one in which the principal flips a coin at some time and sends a signal in the event that either the flip is heads or there was a breakthrough. In this case, conditional on a signal the belief that the opponent has had a success would rise but not to one.

The agent's payoff is given by

$$\int_0^T (p_t w_{i,t} - \kappa) a_{i,t} e^{-\int_0^t (p_s a_s + r)\,ds} dt$$

Replacing the expression for $w_{i,t}$, i's expected payoff becomes

$$\int_0^T \bar{a} \left(\int_t^T \kappa r e^{\int_t^\tau a_{i,s}\,ds - r\tau} d\tau + \kappa e^{-rt} \left(e^{-\int_t^T (r - a_{i,s})\,ds} - 1 \right) \right) dt.$$

Thus, i's payoff does not depend on the process $\int_0^t a_{-i,s}\,ds$. That is, the agent's payoff does not depend on the choice of disclosure policy under the optimal contract.

Let's see that the principal can never gain from not disclosing right away. Suppose the principal chooses disclosure policy $\int_0^t \tilde{a}_{-i,s}\,ds$ and that effort is given by $\int_0^t a_{-i,s}\,ds$. Let p_t denote the principal's belief and $\tilde{p}_t$, agent i's belief. The principal's payoff from agent i's work can be written as

$$\int_0^T (p_t \pi e^{-\int_0^t p_s a_s\,ds} - \kappa \left(\bar{p} e^{-\int_0^t (a_{i,s} + \tilde{a}_{-i,s})\,ds} + (1 - \bar{p}) \right)) a_{i,t} e^{-rt} dt$$
$$- \int_0^T \left((p_t w_{i,t} - \kappa) \bar{p} e^{-\int_0^t (a_{i,s} + \tilde{a}_{-i,s})\,ds} - \kappa (1 - \bar{p}) \right) a_{i,t}\,dt.$$

The last integral in the previous expression does not depend on the disclosure policy. However, the first integral does. When an agent works after another agent has found a discovery the principal has to compensate the agent for the cost of effort but does not gain anything, in reduced costs, from the duplicated effort.

C.2 Asymmetric agents. Proof of Theorem 5

If both agents are working together the Hamiltonian is given by equation (22) with an appropriate modification of the upper bounds on efforts. Thus, the agents are exerting their maximum efforts until they stop when their expected payment equals κ. The time at which the players stop is found by maximizing over the stopping times of each agent.

The earlier proof that shows that the multiplier $\gamma_{i,t}$ is positive goes through in the asymmetric case. To see that there are no intervals in which zero effort is exerted note that from Theorem 7 in page 196 of Seierstad and Sydsæ ter (1987) (equation (77)) a necessary condition for optimality of the wage schedule is that

$$H(\mathbf{x}(T_i^+),\mathbf{c}(T_i^+),\mathbf{p}(T_i^+)) - H(\mathbf{x}(T_i^-),\mathbf{c}(T_i^-),\mathbf{p}(T_i^-)) = \frac{\partial h(x_{T_i}, w_{T_i}, T_i)}{\partial T_i}, \tag{66}$$

where h is defined in equation (24).

Let M_i denote the set of agents that stop work at time T_i. The previous expression translates into

$$\sum_{j\notin M_i}\left(\gamma_{T_i}^- - \gamma_{T_i}^+\right)a_j + \sum_{j\in M_i}\left(\sum_{k\neq j}\left(e^{-rT_i}e^{-x_{T_i}}(\kappa-\pi) + \mu_k(w_{k,T_i}-\kappa)\right) + \gamma_{T_i}^-\right)\bar{a}_j + \\ + \sum_{j\in M_i} e^{-rT_i - x_{T_i}}\left(-(r+\sum_{j\notin M_i} a_j)(\pi-\kappa) + r\kappa e^{x_{T_i}} + \mu_j e^{x_{T_i}}\sum_{j\notin M_i} a_j\kappa\right) = \\ -\sum_{i\in M_i} e^{-x_{T_i}-rT_i}(\pi - \kappa(1+e^x))r$$

where we replaced $w_{i,T_i} = \kappa(1+e^x)$. Replacing using equations (26), and (25) in the previous expression and simplifying we obtain

$$\sum_{j\in M_i}\left(\sum_{k\neq j}\left(e^{-rT_i}e^{-x_{T_i}}(\kappa-\pi) + \mu_k(w_{k,T_i}-\kappa)\right) + \gamma_{T_i}^-\right)\bar{a}_j = 0.$$

Note that each one of the terms inside the first positive must be greater or equal than zero. If the term is strictly less than zero for some j then $a_{j,T_i} = 0$ which contradicts the definition of T_j. This means that we must have

$$\left(\sum_{k\neq j}\left(e^{-rT_i}e^{-x_{T_i}}(\kappa-\pi) + \mu_k(w_{k,T_i}-\kappa)\right) + \gamma_{T_i}^-\right) = 0 \tag{67}$$

for every $j \in M_i$. Setting $t = T_i$ in the left hand side of the previous expression and taking the derivative with respect to t, as in equation (28) many terms cancel and we obtain

$$e^{-rt}e^{-x}r(\kappa-\pi) + \kappa e^x r e^{-rt} e^{-\int_0^t a_{-j,s}\,ds - x_0}. \tag{68}$$

If this derivative is positive then at $t \in [t', T_i]$ for t' close enough to T_i, the effort of agent j at time t, $a_{j,t}$, must be zero since the term that multiplies $a_{j,t}$ in the Hamiltonian would be negative. This observation contradicts the optimality of the contract because the principal would be better off setting $T_i = t'$. Thus, the derivative must be negative. However, (68)$\cdot e^{rt}$ is increasing in t which implies the derivative in (68) is negative for every $t < T_i$. Thus, whenever the left hand side of equation (67) is zero at time T_i, $a_{j,t} > 0$ for $t \leq T_i$ and there cannot be intervals with zero effort.

Suppose $\bar{a}_i > \bar{a}_j$ and $T_i \geq T_j$ To see that the agent with the highest arrival rate needs to be the first to stop working note that the first order condition of the principal's payoff with respect to T_i is given by

$$\bar{a}_i e^{-T_i(\bar{a}_i+r)-\bar{a}_j T_j - x_0}\left(\pi - \kappa\left(e^{2T_i\bar{a}_i+\bar{a}_jT_j+x_0}+1\right)\right) = 0 \tag{69}$$

The first order condition with respect to T_j is given by

$$(\pi-\kappa) - e^{2T_i\bar{a}_i+\bar{a}_jT_j+x_0}\kappa \underbrace{\frac{(\bar{a}_i+r)e^{\bar{a}_i(T_j-T_i)-T_i\bar{a}_i+\bar{a}_jT_j}}{\left(\bar{a}_i e^{(\bar{a}_i+r)(T_j-T_i)}+r\right)}}_{*}. \tag{70}$$

However, the term $*$ is strictly less than one whenever $T_i > T_j$ which contradicts equation (32). In fact, when $T_i = T_j = T$ $*$ is equal to $e^{T(\bar{a}_j-\bar{a}_i)} < 1$. The derivative of $*$ is given by

$$-\frac{\bar{a}_i(\bar{a}_i+r)e^{T_j(\bar{a}_i+\bar{a}_j)-2\bar{a}_iT_i}\left((\bar{a}_i-r)e^{(\bar{a}_i+r)(T_j-T_i)}+2r\right)}{\left(\bar{a}_i e^{(\bar{a}_i+r)(T_j-T_i)}+r\right)^2} < 0.$$

which implies $*$ is less than one and equations (69) and (70) cannot both be satisfied simultaneously.

C.3 Positive Outside Option: Proof of Proposition 5.5

It follows from the proof of Proposition 3 that the bonus contract to agent i is given by $w_t^*(T_i)$ for some experimentation time threshold T_i.

The Lagrangian of the principal's problem is

$$\max_{(T_i,W_{i,0})_i}\sum_{i=1}^{n}\left(\left(-W_{i,0}+\int_0^{T_i} p_t\left(\pi - w_t^*(T_i)\right)a_{i,t}e^{-\int_0^t p_s a_s\,ds - rt}\,dt\right)\right.$$
$$\left.+\lambda_i\left(W_{i,0}+\int_0^{T_i}\left(p_t w_t^*(T_i)-\kappa\right)a_{i,t}e^{-\int_0^t p_s a_s\,ds-rt}\,dt-\bar{V}\right)+\mu_i W_{i,0}\right)$$

where λ_i is the multiplier associated to the constraint on each agent's expected payoff and μ_i is associated to the constraint $W_{i,0} \geq 0$.

The first order conditions with respect to $W_{i,0}$ is

$$-1+\lambda_i+\mu_i = 0$$

If $W_{i,0} > 0$ then $\mu_i = 0$ and $\lambda_i = 1$. The first order condition with respect to T_i is

$$p_t \pi - \kappa = 0.$$

If $\lambda_i \neq 0$ and $\mu_i \neq 0$, we have $T_i = T(\bar{V})$.
If $\lambda_i = 0$ then $T_i = T^*$.