

Indeterminacy in Sovereign Debt Markets: A Quantitative Analysis*

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August 2015

Abstract

In this paper we use a benchmark model of sovereign debt to quantify the importance of non-fundamental risk in driving interest rate spreads during the euro-area sovereign debt crisis. Our model features debt maturity choices, risk averse lenders and rollover crises à la Cole and Kehoe (2000). In this environment, lenders' expectations of a default can be self-fulfilling, and their beliefs contribute to variation in interest rate spreads along with economic fundamentals. We show that maturity choices of the government are informative about the prospect of a future self-fulfilling crisis. The government can reduce the occurrence of these inefficient runs by lengthening the maturity of its debt. Hence, when rollover problems are pressing, we should observe an increase in debt maturity. After fitting the model to observed maturity choices, we find that rollover risk accounts on average for 23% of Italian interest rate spreads during the episode, 14% in 2012:Q2. Our results have implications for the effects of liquidity provisions like the OMT program announced by the European Central Bank during the summer of 2012.

Keywords: Sovereign Debt Crises, Rollover Risk, Indeterminacy, Risk Premia.

*First draft: 02/12/2015. Preliminary, comments welcomed. We thank Pooyan Ahmadi, Manuel Amador, Cristina Arellano, Javier Bianchi, Russell Cooper, Cosmin Ilut, Gaston Navarro, Daniel Neuhann, Monika Piazzesi, Felipe Saffie, Vania Stavrakeva, Cédric Tille and seminar participants at Chicago Booth International Macro conference, SCIEA 2015, University of Rochester conference on the European Sovereign Debt Crisis, Konstanz Seminar for Monetary Theory and Policy, Rome Junior conference on Macroeconomics, University of Zurich conference on the Economics of Sovereign Debt, SED 2015 and NBER Summer Institute 2015. Parisa Kamali provided excellent research assistance. All errors are our own.

1 Introduction

The summer of 2012 marked one of the major developments of the Eurozone sovereign debt crisis. After a period of sharp increases, in August 2012, interest rate spreads of peripheral countries declined to almost their pre-crisis level. These declines have been attributed to the establishment of the Outright Monetary Transaction (OMT) program, a framework through which the European Central Bank (ECB) could purchase government bonds of members of the euro-area. One reading of these events is that the establishment of the OMT program was successful in dealing with coordination failures among bondholders. By promising to act as a lender of last resort, the argument goes, the ECB reduced the scope for self-fulfilling debt crises, bringing back bond prices to the value justified by economic fundamentals.

This is not, however, the only interpretation. Indeed, the high interest rate spreads observed in Europe could have purely been the results of poor economic conditions. A credible announcement by the ECB to sustain prices in secondary markets above their actuarially fair value would still produce a decline in interest rate spreads. Unlike the coordination failure view, this second interpretation may induce governments to over borrow and delay structural reforms, as well as posing balance sheet risk for the ECB. Therefore, any assessment of these interventions needs first to address a basic question: were interest rate spreads in the euro-area periphery the result of self-fulfilling beliefs, or were they due to bad economic fundamentals? This paper takes a first step toward answering this question by bringing a benchmark model of sovereign borrowing with self-fulfilling rollover crisis to the data and applying it to the debt crisis in the euro area. We show that debt maturity choices of the government are informative about the prospect of future self-fulfilling crises. After fitting the model to Italian data, we find that rollover risk accounts on average for 23% of the fluctuations in interest rate spreads during the episode, 14% in the quarter prior to the OMT announcements.

We consider the canonical model of sovereign borrowing in the tradition of [Eaton and Gersovitz \(1981\)](#), [Aguiar and Gopinath \(2006\)](#) and [Arellano \(2008\)](#). In our environment, a government issues debt of different maturities in order to smooth out endowment risk. We follow [Cole and Kehoe \(2000\)](#) and assume that the government cannot commit to repay its debt within the period. This opens the door to self-fulfilling debt crises: if lenders expect the government to default and do not buy new bonds, the government may find it too costly to service the stock of debt coming due, thus validating the expectations of lenders. This can happen despite the fact that a default would not be triggered if lenders held more optimistic expectations about the government's willingness to repay. These rollover

crises can arise in the model when the stock of debt coming due is sufficiently large and economic fundamentals are sufficiently weak.

As commonly done in the literature, we assume that this indeterminacy is resolved by the realization of a coordination device. In our set up, default risk varies over time because of “fundamental” and “non-fundamental” uncertainty. Specifically, default risk may be high because lenders expect the government to default in the near future irrespective of their behavior. Or, it may be high because of the expectation of a future rollover crisis. The goal of our analysis is to distinguish these different sources of default risk.

The first contribution of this paper is to point out that the maturity choices of the government are informative for this purpose. Our argument builds on two properties of canonical sovereign debt model. First, when default risk mostly reflects the prospect of a future rollover crisis, the government has incentives to *lengthen* its debt maturity: by doing so, it reduces the amount of debt that needs to be serviced in the near future, thus decreasing the possibility of a self-fulfilling crisis. Second, when default risk is purely the result of bad economic fundamentals, the government has incentives to *shorten* the maturity of its debt. Absent rollover risk, in fact, the maturity composition of debt balances two forces. On the one hand, as emphasized by [Arellano and Ramanarayanan \(2012\)](#) and [Aguiar and Amador \(2014\)](#), short term debt is desirable because it allows the government to raise more resources from the lenders.¹ On the other hand, long term debt is desirable because it offers better insurance properties. When fundamental default risk is high, the benefits of issuing short term debt increase because the government wants to raise as much resources as possible. Moreover, [Dovis \(2014\)](#) shows that the need to hold long term debt for insurance reasons falls. Hence the government finds it optimal to shorten its debt maturity when facing high fundamental default risk. These two properties imply that fundamental and non-fundamental sources of default risk predict different behavior for debt maturity: it should increase when rollover risk is an important driver of interest rate spreads, and it should decline otherwise. Therefore, maturity choices are informative about the underlying sources of default risk.

The second contribution of our paper is to make this insight operational. A key problem in using this identifying restriction is that the relationship between interest rate spreads and debt duration is not only a product of government’s incentives, but it depends on lenders’ attitude toward risk. [Broner, Lorenzoni and Schmukler \(2013\)](#) document that risk premia over long term bonds typically increase during sovereign crises. Neglecting these

¹When the stock of debt is mostly short term, the government has less incentives to borrow because the associated increase in interest rates is applied on a large fraction of the stock of debt. Shortening the maturity of debt *today* is a device to discipline the *future* borrowing behavior of the government, and it allows the government to obtain better terms from the lenders.

shifts could undermine our identification strategy: rollover risk could be driving interest rate spreads and yet we could observe a shortening of debt maturity simply because lenders demand high compensation to hold long term risky bonds. To address this issue, we allow for time-varying term premia by introducing shocks to the lenders' stochastic discount factor. In doing so we follow a large literature on affine models of the term structure of interest rates (Piazzesi, 2010), specifically the exponentially Gaussian approach of Ang and Piazzesi (2003).²

We apply our framework to the recent sovereign debt crisis in Italy. We calibrate the lenders' stochastic discount factor by matching the behavior of risk premia on long term German's zero coupon bonds. Specifically, we ask the model to replicate the Cochrane and Piazzesi (2005) predictive regressions as well as the behavior of the risk-free rate over our sample. Implicit in our approach is the assumption that financial markets in the euro area are sufficiently integrated and that the lenders in our model are the marginal investors for other assets beside Italian government securities. The parameters of the government's decision problem are calibrated following previous research in the area.

We next measure the importance of non-fundamental risk during the recent sovereign debt crises. Specifically, we apply the particle filter to our model and we estimate the path of the state variable over the sample. Given this path, we decompose observed interest rate spreads into a component reflecting the expectation of a future rollover crisis and a component due to the fundamental shocks. We document that the combination of high risk premia and bad domestic fundamentals account for most of the run-up in interest rate spreads observed during the 2011-2012 period. Moreover, we show that neglecting the information content of maturity choices results in substantial uncertainty over the split between fundamental and non-fundamental sources of default risk, as the model lacks identifying restrictions to discipline the risk of a rollover crisis.

Finally, we show how our results can be used to interpret the OMT announcements. We model OMT as a price floor schedule implemented by the ECB. These interventions can be designed so that they eliminate the possibility of rollover crises without an ECB intervention of the equilibrium path, resulting in a Pareto improvement. Our question is whether the ECB followed this benchmark. To test for this hypothesis, we use the model to construct the counterfactual Italian spread that would prevail in the absence of rollover risk, and we compare it with the actual spread observed after the policy announcements.

²In a related paper, Borri and Verdelan (2013) study a sovereign debt model where lenders have time-varying risk aversion *à la* Campbell and Cochrane (1999). In a previous version of the paper we followed this route considering a more flexible specification of the external habit model that allows for time-variation in term premia (Wachter, 2006; Bakaert, Engstrom and Xing, 2009). Such formulation delivers similar results to the one that we currently use, but it is computationally more challenging.

The model interprets the sharp decline in interest rate spreads observed after these announcements as reflecting to a large extent a prospective subsidy offered by the ECB to peripheral countries. This is a reflection of the negligible role that rollover risk has in the second quarter of 2012.

This paper contributes to the literature on multiplicity of equilibria in sovereign debt models. Previous works in this area like [Alesina, Prati and Tabellini \(1989\)](#), [Cole and Kehoe \(2000\)](#), [Calvo \(1988\)](#), and [Lorenzoni and Werning \(2013\)](#) have been qualitative in nature. More recently, [Conesa and Kehoe \(2012\)](#), [Aguiar, Chatterjee, Cole and Stangebye \(2015\)](#) and [Navarro, Nicolini and Teles \(2015\)](#) considered more quantitative models featuring multiple equilibria. To best of our knowledge, this is the first paper that conducts a quantitative assessment of the importance of rollover risk in driving interest rate spreads in a particular application.³ The main innovation relative to the existing literature is our identification strategy based on the behavior of debt maturity around default crises.

More generally, the paper is related to quantitative analysis of sovereign debt models. Papers that are related to our work include [Arellano and Ramanarayanan \(2012\)](#), [Chatterjee and Eyigungor \(2013\)](#), [Hatchondo, Martinez and Sosa Padilla \(2015\)](#), [Bianchi, Hatchondo and Martinez \(2014\)](#) and [Borri and Verdhelan \(2013\)](#). Relative to the existing literature, our model features rollover risk, endogenous maturity choices and risk aversion on the side of the lenders. Our analysis shows that the behavior of debt duration is necessary for the identification of rollover risk, while shocks to the stochastic discount factor of the lenders are necessary to control for confounding demand factors that may undermine our identification strategy. Our modeling of the maturity choices differ from previous research and builds on recent work by [Sanchez, Saprizo and Yurdagul \(2015\)](#) and [Bai, Kim and Mihalache \(2014\)](#). Specifically, the government in our model issues portfolios of zero coupon bonds with an exponentially decaying duration. The maturity choice is discrete, and it consists on the choice of the decaying factor. This modeling feature simplifies the numerical analysis of the model relative to the canonical formulation of [Arellano and Ramanarayanan \(2012\)](#).

Our analysis on the effects of liquidity provisions is related to [Roch and Uhlig \(2014\)](#) and [Corsetti and Dedola \(2014\)](#). These papers show that these policies can eliminate self-fulfilling debt crisis when appropriately designed. We contribute to this literature by using our calibrated model to test whether the drop in interest rates spreads observed after the announcement of OMT is consistent with the implementation of such policy or whether it signals a prospective subsidy paid by the ECB.

³There is also a reduced form literature that addresses this issue, see [De Grauwe and Ji \(2013\)](#).

Finally, our paper is related to the literature on the quantitative analysis of indeterminacy in macroeconomic models, see the contributions of [Jovanovic \(1989\)](#), [Farmer and Guo \(1995\)](#) and [Lubik and Schorfheide \(2004\)](#). The closest in methodology is [Aruoba, Cuba-Borda and Schorfheide \(2014\)](#) who use a calibrated New Keynesian model solved numerically with global methods to measure the importance of beliefs driven fluctuations for the U.S. and Japanese economy.

Layout. The paper is organized as follows. Section 2 presents the model. Section 3 discusses our key identifying restriction, and Section 4 presents an historical example supporting our approach. Section 5 describes the calibration of the model and presents an analysis of its fit. Section 6 uses the calibrated model to measure the importance of rollover risk during the Italian sovereign debt crisis. Section 8 analyzes the OMT program. Section 9 concludes.

2 Model

2.1 Environment

Preferences and endowments: Time is discrete, $t \in \{0, 1, 2, \dots\}$. The exogenous state of the world is $s_t \in S$. We assume that s_t follows a Markov process with transition matrix $\mu(\cdot | s_{t-1})$. The exogenous state has two types of variables: *fundamental*, $s_{1,t}$, and *non-fundamental*, $s_{2,t}$. The fundamental states are stochastic shifters of endowments and preferences while the non-fundamental states are random variables on which agents can coordinate. These coordination devices are orthogonal to fundamentals.

The economy is populated by lenders and a domestic government. The lenders value flows according to the stochastic discount factor $M(s_t, s_{t+1})$. Hence the value of a stochastic stream of payments $\{d\}_{t=0}^{\infty}$ from time zero perspective is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} M_{0,t} d_t, \quad (1)$$

where $M_{0,t} = \prod_{j=0}^t M_{j-1,j}$.

The government receives an endowment (tax revenues) $Y_t = Y(s_t)$ every period and decides the path of spending G_t . The government values a stochastic stream of spending $\{G_t\}_{t=0}^{\infty}$ according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(G_t), \quad (2)$$

where the period utility function U is strictly increasing, concave, and it satisfies the usual assumptions.

Market structure: The government can issue a portfolio of non-contingent defaultable bonds to lenders in order to smooth fluctuations in G_t . For tractability, we restrict the portfolios that the government can issue to be portfolios of zero-coupon bonds (ZCBs) indexed by (B_t, λ_t) . A portfolio (B_t, λ_t) at the end of period t corresponds to a stock of $(1 - \lambda_t)^{j-1} B_t$ zero-coupon bond of maturity $j \geq 1$ outstanding. The variable $\lambda_t \in [0, 1]$ captures the duration of the government stock of debt, and it can be interpreted as its decay factor. Higher λ_t implies that debt payments are concentrated at shorter maturities. For instance, if $\lambda_t = 1$, then all the debt is due next period. The variable B_t controls the face value of debt. Specifically, the total face value of debt is B_t / λ_t .

If we let $q_{t,j}$ be the price of a zero-coupon bond of maturity j at time t , the value of a portfolio (B_t, λ_t) is

$$\sum_{n=1}^{\infty} q_{t,n} (1 - \lambda_t)^{n-1} B_t.$$

The timing of events within the period follows [Cole and Kehoe \(2000\)](#): the government issues a new amount of debt, lenders choose the price of newly issued debt, and finally the government decides to default or not, $\delta_t = 0$ or $\delta_t = 1$ respectively. Differently from the timing in [Eaton and Gersovitz \(1981\)](#), the government does not have the ability to commit not to default within the current period. As we will see, this opens the door to self-fulfilling debt crisis.

The budget constraint for the government when he does not default is

$$G_t + B_t \leq Y_t + \Delta_t, \tag{3}$$

where Δ_t is the net issuance of new debt given by

$$\Delta_t = \sum_{n=1}^{\infty} q_{t,n} \left[(1 - \lambda_{t+1})^{n-1} B_{t+1} - (1 - \lambda_t)^n B_t \right]. \tag{4}$$

If a government enters the period with a portfolio (B_t, λ_t) and wants to exit the period with a portfolio (B_{t+1}, λ_{t+1}) , the government must issue additional $(1 - \lambda_{t+1})^{n-1} B_{t+1} - (1 - \lambda_t)^n B_t$ zero coupon bonds of maturity n .⁴

⁴When $(1 - \lambda_{t+1})^{n-1} B_{t+1} - (1 - \lambda_t)^n B_t$ is negative the government is buying back the ZCB of maturity n . Buy backs of government securities under our formulation are necessary whenever the government wants to shorten the duration of the debt. This is an unrealistic feature of the model as buy backs are hardly observed in the data, but it allows for a greater numerical tractability.

We assume that if the government defaults, he is excluded from financial markets and he suffers losses in output. We denote by $\underline{V}(s_{1,t})$ the value for the government conditional on a default. Lenders that hold inherited debt and the new debt just issued do not receive any repayment.⁵

2.2 Recursive Equilibrium

2.2.1 Definition

We now consider a recursive formulation of the equilibrium. Let $\mathbf{S} = (B, \lambda, s)$ be the state today and $\mathbf{S}'$ the state tomorrow. The problem for a government that has not defaulted yet is

$$V(\mathbf{S}) = \max_{\delta \in \{0,1\}, B', \lambda', G} \delta \{U(G) + \beta \mathbb{E}[V(\mathbf{S}') | \mathbf{S}]\} + (1 - \delta) \underline{V}(s_1) \quad (5)$$

subject to

$$\begin{aligned} G + B &\leq Y(s_1) + \Delta(\mathbf{S}, B', \lambda'), \\ \Delta(\mathbf{S}, B', \lambda') &= \sum_{n=1}^{\infty} q_n(s, B', \lambda') \left[(1 - \lambda')^{n-1} B' - (1 - \lambda)^n B \right], \end{aligned}$$

where $q_n(s, B', \lambda')$ is the price of a defaultable ZCB of maturity n given that the realization s for the exogenous state and the government's choices for the new portfolio is (B', λ') , and $[(1 - \lambda')^{n-1} B' - (1 - \lambda)^n B]$ is the net issuance of ZCB of maturity n .

The lender's no-arbitrage condition requires that

$$\begin{aligned} q_1(s, B', \lambda') &= \delta(\mathbf{S}) \mathbb{E} \{ M(s_1, s'_1) \delta(\mathbf{S}') | \mathbf{S} \} \\ q_n(s, B', \lambda') &= \delta(\mathbf{S}) \mathbb{E} \{ M(s_1, s'_1) \delta(\mathbf{S}') q_{n-1}(s', B'', \lambda'') | \mathbf{S} \} \text{ for } n \geq 2 \end{aligned} \quad (6)$$

where $B'' = B'(s', B', \lambda')$ and $\lambda'' = \lambda'(s', B', \lambda')$. The presence of $\delta(\mathbf{S})$ in equation (6) implies that new lenders receive a payout of zero in the event of a default today.

A *recursive equilibrium* is value function for the borrower V , associated decision rules $\{\delta, B', \lambda', G\}$ and a pricing functions $q = \{q_n\}$ such that $\{V, \delta, B', \lambda', G\}$ are a solution of the government problem (23) and the pricing functions satisfies the no-arbitrage conditions (6).

⁵This is a small departure from [Cole and Kehoe \(2000\)](#), since they assume that the government can use the funds raised in the issuance stage. Our formulation simplifies the problem and it should not change its qualitative features. The same formulation has been adopted in other works, for instance [Aguar and Amador \(2014\)](#).

2.2.2 Multiplicity of equilibria and Markov selection

We now show that there are multiple recursive equilibria in this model. Following [Cole and Kehoe \(2000\)](#), when inherited debt is sufficiently high, a coordination problem can generate a “run” on debt, whereby it is optimal for an atomistic investor not to lend to the government if the other investors are also not buying the bonds. This can happen despite the fact that the atomistic investor would lend to the government if the other lenders would.

In order to better understand this form of strategic complementarity, suppose that lenders expect the government to default today. By equation (6), the price of any portfolio (B', λ') is zero. The expectation of the lenders is validated in equilibrium if default is optimal from the government’s viewpoint. This second condition is met if⁶

$$U(Y - B) + \beta \mathbb{E}[V((1 - \lambda)B, \lambda, s') | \mathbf{S}] < \underline{V}(s_1), \quad (7)$$

that is if the government finds optimal to default when he cannot issue new debt. From the above expression it is clear that for each (λ, s) there exists a level of inherited debt B sufficiently high such that the inequality in (7) is satisfied. Condition (7) depends on V , an equilibrium object. In the Appendix, we show that we can find sufficient conditions on the primitives of the model for (7) to be satisfied in some region of the state space, thus establishing the presence of multiple equilibria.

Debt crisis may thus be self-fulfilling: lenders may extend credit to the sovereign and there will be no default, or the lenders may not roll-over government debt, in which case the sovereign would find it optimal to default. Therefore, the outcomes are *indeterminate* in this region of the state space. We follow most of the literature and use a parametric mechanism that selects among these possible outcomes. In order to explain our selection mechanism, it is useful to partition the state space in three regions. Following the terminology in [Cole and Kehoe \(2000\)](#), we say that the borrower is in the *safe zone*, $\mathcal{S}^{\text{safe}}$, if the government does not find optimal to default even if lenders do not rollover his debt. That is,

$$\mathcal{S}^{\text{safe}} = \left\{ \mathbf{S} : U(Y - B) + \beta \mathbb{E}[V((1 - \lambda)B, \lambda, s') | \mathbf{S}] \geq \underline{V}(s_1) \right\}.$$

We say that the borrower is in the *crisis zone*, $\mathcal{S}^{\text{crisis}}$, if (B, λ, s) are such that it is not optimal for the government to repay debt during a rollover crisis but it is optimal to repay if the

⁶If condition (7) is not satisfied, instead, no coordination problem among lenders can arise. This is because if lenders decide to run, and so $q = 0$, it is still optimal for the government to repay his debt. Thus, lenders have no incentive to run: it is optimal for an individual lender to lend at a positive price even if other lenders do not and so $q = 0$ cannot be an equilibrium price.

lenders roll it over. That is,

$$\mathcal{S}^{\text{crisis}} = \left\{ \mathbf{S} : U(Y - B) + \beta \mathbb{E} [V((1 - \lambda)B, \lambda, s') | \mathbf{S}] < \underline{V}(s_1) \text{ and } \max_{B', \lambda'} U(Y - B + \Delta(\mathbf{S}, B', \lambda')) + \beta \mathbb{E} [V(B', \lambda', s') | \mathbf{S}] \geq \underline{V}(s_1) \right\}.$$

Finally, the residual region of the state space, the *default zone*, $\mathcal{S}^{\text{default}}$ is the region of the state space in which the government defaults on his debt regardless of lenders' behavior,

$$\mathcal{S}^{\text{default}} = \left\{ \mathbf{S} : \max_{B', \lambda'} U(Y - B + \Delta(\mathbf{S}, B', \lambda')) + \beta \mathbb{E} [V(B', \lambda', s') | \mathbf{S}] < \underline{V}(s_1) \right\}.$$

Indeterminacy in outcomes arises only when the economy is in the crisis zone.

The selection mechanism works as follows. Without loss of generality, let the non-fundamental state, s_2 , be $s_2 = (p, \xi)$. Whenever the economy is in the crisis zone, lenders roll-over the debt if $\xi \geq p$. In this case, there are no run on debt and $\delta(\mathbf{S}) = 1$ by our definition of crisis zone. If $\xi < p$, instead, the lenders do not roll-over the government debt. We will assume that ξ is an i.i.d. uniform on the unit interval while p follows a first order Markov process, $p' \sim \mu_p(\cdot | p)$. Given these restrictions, we can interpret p as the probability of having a rollover crises this period conditional on the economy being in the crisis zone. While p is relevant for selecting between outcomes in the crisis zone today, the relevant state variable that determines how perspective rollover risk affects interest rate spreads today is the expected realization of p in the next period, $\pi = \mathbb{E}(p' | p)$.

Conditional on this selection rule, the outcome of the debt auctions are unique in the crisis zone once we adopt this selection rule. However, we cannot assure that the equilibrium value function, decision rules and pricing functions are unique as the operator that implicitly defines a recursive equilibrium may have multiple fixed points. In order to overcome this issue, we restrict our attention to the limit of the finite horizon version of the model. Under our selection rule, the finite horizon model features a unique equilibrium and so does its limit.

The equilibrium outcome is a stochastic process

$$\mathbf{y} = \{\lambda(s^t, B_0, \lambda_0), B(s^t, B_0), \delta(s^t, B_0, \lambda_0), G(s^t, B_0, \lambda_0), q(s^t, B_0, \lambda_0)\}_{t=0}^{\infty}$$

naturally induced by the recursive equilibrium objects. The outcome path depends on properties of the selection, i.e. the process for $\{p_t\}$, and on the realization of the non-fundamental state s_2 . In our quantitative analysis we will use information from government's choices in order to infer properties of the inherently unobservable $\{p_t\}$ process,

and to assess whether rollover risk was an important driver of Italian spreads in the recent crisis. As we will argue in the next section, government's choices regarding the maturity of debt are going to be informative for our exercise.

3 Maturity Choices and Sources of Default Risk

In this section, we explain why maturity choices provide information that is useful to distinguish between fundamental and non-fundamental sources of default risk. The key insight is that if rollover risk is large then the government has an incentive to "exit" from the crisis zone. As first showed in [Cole and Kehoe \(2000\)](#), to achieve this objective the government can lengthen the maturity of his debt since long term debt is less susceptible to runs. Hence, we should expect the government to lengthen debt maturity if rollover risk is high. On the contrary, previous research - for instance [Arellano and Ramanarayanan \(2012\)](#), [Aguiar and Amador \(2014\)](#) and [Dovis \(2014\)](#) - has shown that a shortening of maturity is typically an optimal response of the sovereign when facing a default crises driven by fundamental shocks: a shortening of debt maturity around a debt crisis would then indicate a more limited role for rollover risk.

In what follows we illustrate these insights using numerical illustrations from a calibrated version of our model with risk neutral investors, $M_{t,t+1} = 1/(1+r)$.

3.1 Maturity choices in absence of rollover risk

We start from the case in which rollover risk is absent, $\{p_t\}$ is identically equal to zero. Previous works on incomplete market models without commitment have emphasized two channels as the main determinants of the maturity composition of debt in the face of default risk: *insurance* and *incentives*.

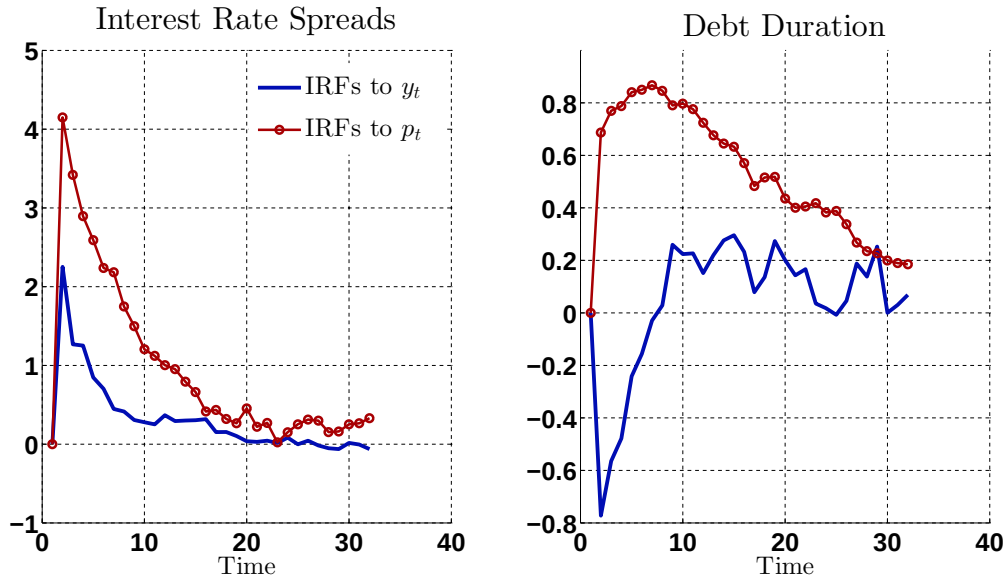
The *insurance channel* refers to the fact that long term debt is a better asset than short term debt to provide the government with insurance against shocks. Capital gains and losses imposed on holders of long term debt can approximate wealth transfers associated with state contingent securities, as the market value of debt falls when the marginal utility of the government is high. This channel leads the government to issue bonds of longer duration.

The *incentives channel* pushes the government to issue relatively more short term debt because it is a better instrument to raise resources from lenders. Intuitively, when inherited debt is long-term, the government has more incentives to issue new debt. This is because

higher interest rates are applicable only to the new issuances, not on the stock of existing debt. When debt is short-term, the ex-post incentive for the government to issue more debt - and therefore increase the probability of future default- are lower because the higher interest rates are levied on the whole stock of debt. In equilibrium, the price of long term debt is more sensitive to new issuances relative to the price of short term debt because lenders anticipate higher future default risk for the former.⁷

In absence of rollover risk, the relative strength of these two forces over time shapes the optimal portfolio decision for the government. Figure 1 plots the response of interest rate spreads and debt duration to a negative income shock in a calibrated version of our model. We can see that when the prospect of a default increase (interest rate spreads go up) the government shortens the maturity of its debt.

Figure 1: The dynamics of interest rate spreads and debt duration



Notes: The blue solid line reports impulse response functions (IRFs) of interest rate spreads and debt duration to a 3 standard deviations income shock in the model without rollover risk. The red circled line reports IRFs to a 3 standard deviation increase in p_t . IRFs are calculated by simulation, and they are expressed as deviation from the ergodic mean. Interest rate spreads are expressed in annualized percentages while debt duration in years.

This preference for shorter maturities in the face of “fundamental” default risk arises because of two reasons. First, incentives not to dilute outstanding debt are stronger the higher is the risk of default. Indeed, in states when output is low and/or inherited debt is high, the government would like to issue more debt in order to smooth out consumption. As argued earlier, short term debt is a better instrument for this purpose because its price

⁷The *debt-dilution* problem is not present if we consider the best SPE (which is history dependent) in which reputational costs prevent the government from deviating from its promised path of debt issuance.

is less sensitive to new issuances. See [Aguiar and Amador \(2014\)](#) for a similar argument. Second, the need to hold long term debt for insurance reasons falls when default risk increases. As discussed in [Dovis \(2014\)](#), this happens because pricing functions are more sensitive to shocks when the economy approaches the default region. Hence the larger ex-post variance of the price of long-term debt allows for more insurance because the market value of long term debt falls more in future bad states.

3.2 Maturity choices with rollover risk

We now turn to the analysis of the maturity choice in presence of rollover risk. The government has an additional reason to actively manage the maturity of its debt. When $\pi = \mathbb{E}[p'|p] > 0$, a rollover crisis can occur with positive probability if the economy happens to be in the crisis zone next period. Since these outcomes are inefficient from the government's perspective, the government has an incentive to reduce the likelihood of falling into the crisis zone next period. As emphasized in [Cole and Kehoe \(2000\)](#), this can be achieved by reducing debt issuance and/or by lengthening the maturity of issued debt.

The logic of why lengthening the maturity of debt issued today helps avoiding the crisis zone in the next period can be best understood by looking at the condition defining the crisis zone,

$$U(Y' - B') + \beta \mathbb{E}[V((1 - \lambda')B', \lambda', s'')|\mathbf{S}'] < \underline{V}(s'_1). \quad (8)$$

Suppose the government today lengthens the maturity of its debt while keeping the amount of resources it raises constant. This is achieved by increasing λ' and reducing B' by the appropriate amount. By doing so, the government reduces the payments coming due in the next period at the cost of increasing future payments and reducing the continuation value $\beta \mathbb{E}[V((1 - \lambda')B', \lambda', s'')|\mathbf{S}']$. It is easy to show that this variation increases the left hand side of (8). The borrower is "credit constrained" in that the marginal utility of consumption next period when there is no rollover crises is higher than the marginal reduction in expected utility from period two onward. Therefore, lengthening debt maturity reduces the likelihood of falling into the crisis zone next period.

The circled line in [Figure 1](#) plots the response of interest rate spreads and debt duration to an increase in p_t . As expected, an increase in the probability of future rollover crises leads to an increase in debt maturity. This stands in sharp contrast to what happens in the model conditional on an increase in fundamental default risk.

In sum, this discussion suggests that when the sources of default risk are fundamental,

interest rate spreads increase and the duration of debt declines. When default risk arises because of the prospect of a rollover crisis, instead, the government lengthens its debt maturity.

4 A case study: Italy in the early 1980s

Before turning to the quantitative analysis, it is useful to discuss in more details our main identifying restriction. Our approach builds on the hypothesis that governments would respond to heightened rollover risk by actively lengthening the maturity of their debt. However, previous cross-country studies have shown that the maturity of new issuances typically shortens around default crises ([Broner et al., 2013](#); [Arellano and Ramanarayanan, 2012](#)), and examples of governments extending the life of their debt in turbulent times are not well documented in the literature. In this section we discuss in details one of these examples. Using a narrative approach, we show how the Italian government in the early 1980s responded to heightened rollover risk (or refinancing risk in the Treasury parlor) by lengthening the duration of public debt, and we explain how this historical episode supports our identification strategy.

Two main factors at the beginning of the 1980s contributed to place the Italian government at risk of a roll-over crisis. First, the average residual maturity of government debt collapsed, going from a peak value of 9.2 years in 1972 to 1.1 years in 1980.⁸ At that time, the Italian government needed to refinance the entire stock of debt, roughly 60% of gross domestic product, within the span of a year. Second, and in an effort to increase the independence of the central bank, a major institutional reform freed the Bank of Italy from the obligation of buying unsold public debt in auctions. This effectively meant that the government couldn't rely anymore on the central bank to finance its maturing debt and spending needs, and it had to use primarily private markets.⁹

The short duration of government debt coupled with the loss of central bank financing exposed the Italian government to rollover risk. Auction markets at the time were not well developed, and private demand of treasuries was weak and volatile ([Campanaro and Vittas, 2004](#)). Table 1 reports two statistics: i) the average ratio between the demand of Italian treasury bills by private operators in auctions and the target set by the Treasury,

⁸These low values were the results of the chronic inflation of the 1970s which discouraged investors from holding long duration bonds that were unprotected from inflation risk, see [Pagano \(1988\)](#).

⁹Starting from 1975, the Bank of Italy was required to act as a residual buyer of all the public debt that was unsold in the auctions. This resulted in a massive increase in the share of public debt held by the Bank of Italy, reaching a maximum of 40% in 1976. See [Tabellini \(1988\)](#) for a discussion of the historical context underlying the "divorce" between the Bank of Italy and the Italian Treasury.

and ii) the average ratio between the quantity of bond sold in the auctions and the target set by the Treasury between 1981 and 1986.¹⁰ We can see how in 1981 and in 1982 private demand of government bonds was substantially lower than the amount offered, and this was exposing the Italian government to refinancing risk as it was not mandatory for Bank of Italy to buy unsold public debt anymore. The potential of a default crisis became evident in the last quarter of 1982, when the weak demand in the auctions of government debt led the Treasury to hit the limit of the overdraft account it had with the Bank of Italy.¹¹ The refusal of the newly independent Bank of Italy to buy unsold bonds in the auctions led to a budgetary crisis. While the Parliament later voted a law that allowed a temporal overshoot of the overdraft account (Scarpelli, 2001), the event revealed to policymakers the risks implicit in rolling over large amounts of debt in short periods of time.

Table 1: **Auctions of Italian Treasury bills in the 1980s**

	Private demand/Offered	Sold/Offered
1981	0.55	0.93
1982	0.71	0.93
1983	0.78	0.98
1984	0.90	0.98
1985	0.88	0.98
1986	0.84	0.98

Notes: Our calculations from Bank of Italy, Supplements to the Statistical Bulletin-Financial Markets.

In such a context, the early 1980s saw a rapid increase in interest rate differentials between Italian and German government securities: as we can see from the circled line in the left panel of Figure 2, between January 1980 and March 1983, interest rate spreads rose from 500 to 1300 basis points.¹² In light of the extremely short maturity of the stock of government debt, the institutional changes occurring at the time, and the low private demand for debt in auctions, it is plausible to believe that these tensions in the Italian bond markets were partly reflecting fears of rollover crises (or refinancing risk in the Treasury parlance). In this respect, the response of the government is consistent with the predictions

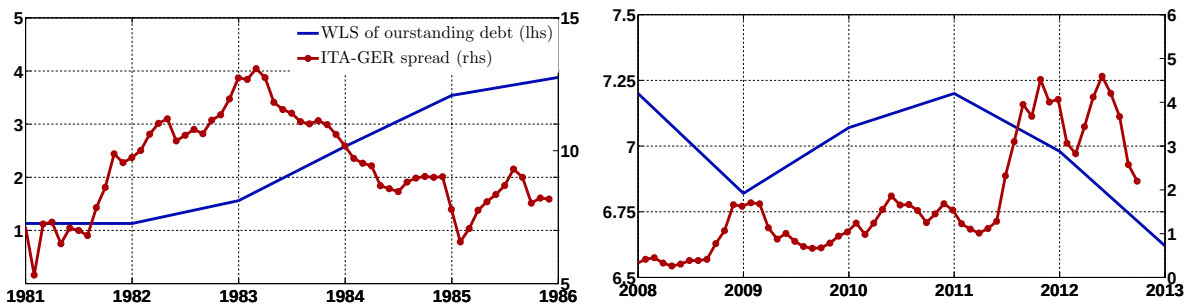
¹⁰The two differ because of the purchases in the auctions of Treasury bills by the Bank of Italy.

¹¹This account allowed the Italian Treasury to directly borrow from the Bank of Italy up to a limit of 14% of the expenditures budgeted for the current year.

¹²As Italy and Germany did not have a common currency at the time, these interest rate differentials reflect currency risk along with the risk of outright repudiation. To best of our knowledge, it is not possible to separate these two components of the spreads using existing methodologies (Du and Schreger, 2015) because of the unavailability of cross-currency swaps data for the early 1980s. It is worth noticing, however, that Italy and Germany were part of the European Monetary System at the time, an exchange rate regime which allowed for limited realignments between the currencies of their members.

of our model. As documented in [Alesina et al. \(1989\)](#) and in [Scarpelli \(2001\)](#), the Italian government actively pursued throughout the 1980s a policy to extend the life of its debt. Specifically, the Treasury introduced a new type of bonds whose interest payments were indexed to the prevailing nominal rate, thus offering to bondholders a protection from inflation risk. These *Certificati di Credito del Tesoro* (CCT) had longer maturity than the *Buoni Ordinari del Tesoro* (BOT), and they quickly replaced the latter as the main instrument used by the government to finance its spending needs.¹³ The solid line in the left panel of Figure 2 shows that the weighted average life of Italian government debt more than tripled within the span of four years, going from 1.13 years in 1981 to 3.88 years in 1986.

Figure 2: Debt duration and Interest rate spreads: 1980s vs 2010s



Notes: The solid line stands for the weighted-average life of the outstanding central government debt. Data are reported in years (right hand side), and they are obtained from the Italian Treasury. The circled line reports the yields differential between an Italian and a German zero coupon government bonds with a duration of twelve months. Data are reported in annualized percentages (left hand side), and they are obtained from Bank of Italy and Bundesbank.

Through the lens of the model, the actions of the Italian Treasury reduced its exposure to rollover risk without increasing the incentive to inflate away the debt. Consistent with this view, we can observe from the left panel of Figure 2 that as the maturity of the stock of debt increased, the interest rate spreads between Italian and German government securities started to decline in 1983. Overall, this episode provides support to our identification strategy: when rollover problems are pressing, governments have incentives to manage the maturity of their debt in order to minimize the risk of facing a run.

For comparison, Figure 2 report these variables during the latest years. The dynamics of interest rate spreads and debt duration appear different from the experience of the 1980s. The right panel of Figure 2 shows that the weighted average life of government debt decreased by roughly one year during the 2011-2014 period. Moreover, auctions of government debt during those years did not show signs of lack of demand, as the demand

¹³Indexed securities like CCT are not subject to refinancing and rollover problem but are essentially equal to short term debt for the incentive to generate ex-post inflation because any effort to generate ex-post inflation will not reduce the real value of debt. See [Missale and Blanchard \(1994\)](#).

of both short term debt, and that of longer term bonds above the minimal price was always well above the amount that the Treasury planned to issue. Given the discussion of this section, this cursory look at the data suggests that the recent experience does not square well with an interpretation that emphasizes roll-over risk as the major source of the current crisis. In what follows, we will make the analysis more formal and we will use the structural model to measure the contribution of rollover risk in the run-up of Italian spreads during this recent episode.

5 Quantitative Analysis

We now apply our framework to Italian data. This section proceeds in three steps. Section 5.1 describes the parametrization of the model and our empirical strategy. Section 5.2 describes the data. Section 5.3 reports the results of our calibration and some indicators of model fit.

5.1 Parametrization and Calibration Strategy

5.1.1 Lenders' stochastic discount factor

It is common practice in the sovereign debt literature to assume risk neutrality on the lenders' side. This specification, however, is not desirable given our objectives. First, several authors have argued that risk premia are quantitatively important to account for the volatility of sovereign spreads (Borri and Verdelan, 2013; Longstaff, Pan, Pedersen and Singleton, 2011). Assuming risk neutrality implies that other unobserved factors in the model, for instance p_t , would need to absorb the variations in this component of the spread. Second, sovereign debt crisis are typically accompanied by a significant increase in term premia (Broner et al., 2013). Neglecting these shifts could undermine our identification strategy: rollover risk could be driving interest rate spreads of peripheral countries in the euro-area and yet we could observe a shortening in debt maturity simply because high term premia made short term borrowing relatively cheaper.

Therefore, we introduce a stochastic discount factor that allows us to fit the behavior of risk premia over long term bonds observed in Europe over the period of analysis. Specifically, we follow Ang and Piazzesi (2003) and assume that $m_{t,t+1} = \log M_{t,t+1}$ is

given by the conditionally Gaussian process

$$\begin{aligned} m_{t,t+1} &= -(\delta_0 + \delta_1 \chi_t) - \frac{1}{2} \lambda_t^2 \sigma_\chi^2 - \lambda_t \varepsilon_{\chi,t}, \\ \chi_{t+1} &= \mu_\chi (1 - \rho_\chi) + \rho_\chi \chi_t + \varepsilon_{\chi,t} \quad \varepsilon_{\chi,t} \sim \mathcal{N}(0, \sigma_\chi^2), \\ \lambda_t &= \lambda_0 + \lambda_1 \chi_t. \end{aligned} \tag{9}$$

When enriched with a process for payouts, one can use $m_{t,t+1}$ along with the pricing formula in equation (1) to express asset prices as a function of model parameters and of the state variable χ_t .

In order to understand how this formulation generates shifts in risk premia over long term assets, it is useful to discuss the pricing of non-defaultable ZCBs. As shown in [Ang and Piazzesi \(2003\)](#), one can express the the log price of a non-defaultable ZCB maturing in n periods as a linear function of χ_t ,

$$q_t^{*,n} = a_n + b_n \chi_t, \tag{10}$$

where a_n e b_n are functions of the model's parameters (See Appendix B). Fluctuations in χ_t generate movements in the level and in the slope of the yield curve. These fluctuations in bond prices can give rise to risk premia on long term bonds. In fact, we can write the excess log returns on a bond maturing in n periods as¹⁴

$$\mathbb{E}_t[rx_{t+1}^n] + \frac{1}{2} \sigma_t^2 [rx_{t+1}^n] = -\text{cov}_t[m_{t,t+1}, q_{t+1}^{*,n-1}]. \tag{11}$$

From the above equation we can see that long term bonds earn a risk premium when lenders expect these assets to depreciate in "bad" times, that is when $\text{cov}_t[m_{t,t+1}, q_{t+1}^{*,n-1}] < 0$. Using equation (10) and the definition of stochastic discount factor in (9) we can write this risk premium as

$$-\text{cov}_t[m_{t,t+1}, q_{t+1}^{*,n-1}] = \lambda_t b_{n-1} \sigma_\chi^2. \tag{12}$$

Equation (12) makes clear that λ_t is the market price of risk in this model, converting a unit covariance between innovations to the pricing kernel and innovations to bond prices into excess returns. Different choices of model parameters imply different behavior for risk premia on long term bonds. For example, when $\lambda_0 = \lambda_1 = 0$, the lenders are risk neutral and risk premia on long term bonds are identically zero. When $\lambda_1 \neq 0$, risk

¹⁴In order to derive this equation, we make use of the lenders' no-arbitrage condition $\mathbb{E}_t[e^{m_{t,t+1} + rx_{t+1}^n}] = 0$, of the definition of excess log returns $rx_{t+1}^n = q_{t+1}^{*,n-1} - q_t^{*,n} + q_t^{*,1}$, and of the joint log-normality of the pricing kernel and excess returns.

premia will vary over time with the state variable χ_t . These movements in risk premia over non-defaultable ZCBs will interact with the government decision problem and will generate movements in default risk on sovereign bonds, on the premium lenders demand to hold these assets and ultimately on the government debt maturity choices. We will discuss these interactions in Section 6.3.

For future reference, we let $\theta_1 = [\delta_0, \delta_1, \lambda_0, \lambda_1, \mu_\chi, \rho_\chi, \sigma_\chi]$ denote the parameters governing the lenders' stochastic discount factor. It is important to stress that in our environment the stochastic discount factor is exogenous with respect to the risk of a government default. Section x discusses the implications of this exogeneity assumption for our exercise.

5.1.2 Government's decision problem

The government period utility function is CRRA

$$U^{\text{gov}}(G_t) = \frac{G_t^{1-\sigma} - 1}{1-\sigma},$$

with σ being the coefficient of relative risk aversion. The government discounts future flow utility at the rate β . If the government enters a default state, he is excluded from international capital markets and he suffers an output loss τ_t . These costs of default are a function of the country's income, and they are parametrized following Chatterjee and Eyigungor (2013),

$$\tau_t = \max\{0, d_0 e^{y_t} + d_1 e^{2y_t}\}.$$

If $d_1 > 0$, then the output losses are larger when income realizations are above average. We also assume that, while in autarky, the government has a probability ψ of reentering capital markets. If the government reenters capital markets, it pays the default costs and starts his decision problem with zero debt.

The country's endowment $Y_t = \exp\{y_t\}$ follows the stochastic process,

$$y_{t+1} = \rho_y y_t + \rho_{y\chi}(\chi_t - \mu_\chi) + \sigma_y \varepsilon_{y,t+1} + \sigma_{y\chi} \varepsilon_{\chi,t+1}, \quad \varepsilon_{y,t+1} \sim \mathcal{N}(0, 1). \quad (13)$$

In this formulation, output of the domestic economy depends on the factor χ_t and on its innovations, allowing us to match the observed correlation between risk premia and real economic activity over our sample.

The probability of lenders not rolling over the debt in the crisis zone follows the stochas-

tic process $p_t = \frac{\exp\{\tilde{p}_t\}}{1+\exp\{\tilde{p}_t\}}$, with $\tilde{p}_t$ given by

$$\tilde{p}_{t+1} = (1 - \rho_p)p^* + \rho_p\tilde{p}_t + \sigma_p\varepsilon_{p,t+1}, \quad \varepsilon_{p,t+1} \sim \mathcal{N}(0, 1). \quad (14)$$

We let $\theta_2 = [\sigma, \beta, d_0, d_1, \psi, \rho_y, \rho_{y\chi}, \sigma_y, \sigma_{y\chi}, p^*, \rho_p, \sigma_p]$ denote the parameters associated to the government decision problem.

5.1.3 Calibration strategy

Our calibration strategy consists in choosing $\theta = [\theta_1, \theta_2]$ in two steps. In the first step, we choose θ_1 in order to match the behavior of risk premia over non-defaultable long term bonds, measured using the term structure of German's ZCB. We focus on non-defaultable bonds rather than on the bonds issued by our government because of two main reasons. First, we can calibrate these parameters without solving the government decision problem, which is numerically challenging. Second, this approach does not require us to specify the unobserved default intensity process, that would otherwise confound the measurement of the price of risk. Implicit in our approach is the assumption that the lenders are "marginal" for pricing other financial assets in the euro area beside Italian government securities.

In the second step, and conditional on θ_1 , we calibrate θ_2 by matching some basic facts about the Italian economy. In view of our previous discussion, we place empirical discipline on the $\{p_t\}$ process by making sure that the calibrated model replicates the joint behavior of interest rate spreads and the duration of debt for the Italian economy.

5.2 Data

We use the Bundesbank online database to obtain information on the term structure of ZCB for Germany. Specifically, we collect monthly data on the parameters of the [Nelson and Siegel \(1987\)](#) and [Svensson \(1994\)](#) model, and we generate nominal bond yields for all maturities between $n = 1$ and $n = 20$ quarters. We convert these monthly series at a quarterly frequency using simple averages. We use the OECD *Main Economic Indicators* database to obtain a series for inflation, defined as the year-on-year percentage change in the German CPI index. These series, available for the period 1973:Q1-2013:Q4, are used in the first step of our procedure to calibrate θ_1 .

The endowment process y_t is mapped to linearly detrended log real Italian GDP. The quarterly GDP series is obtained from the OECD *Main Economic Indicators*. The interest rate spread series is the CDS spread on an Italian 6 months government bonds, obtained

from *Markit*. We map this series to the interest rate spread on a one period ZCB. Our indicator for debt duration is the weighted-average life of outstanding bonds issued by the Italian central government. This indicator, obtained from the Italian Treasury, is mapped in the model to $\frac{1}{\lambda}$.¹⁵ These series are used in the second step to calibrate θ_2 .

5.3 Results

The results are organized in two sections. First, we describe the parametrization of the pricing model. Then, we discuss the calibration of the parameters governing the government decision problem.

5.3.1 Calibration of the pricing model

We choose the parameters of the lenders' stochastic discount factor to fit the behavior of risk premia on long term German ZCB over our sample. Specifically, we calibrate θ_1 to match key features of the predictive regressions of [Cochrane and Piazzesi \(2005\)](#) (henceforth C-P). In order to explain the procedure, let $rx_{t+1}^n = q_{t+1}^{*,n-1} - q_t^{*,n} + q_t^{*,n}$ be the *realized* excess log returns on a ZCB maturing in n quarters, $f_t^n = q_t^{*,n-1} - q_t^{*,n}$ the time t log forward rate for loans between $t + n - 1$ and $t + n$, and $y_t^1 = -q_t^{*,1}$ the log yield on a ZCB maturing next quarter. We denote by $\mathbf{rx}_{t+1}$ and $\mathbf{f}_t$ vectors collecting, respectively, excess log returns and log forward rates for different n s. We proceed in two stages. In the first stage, we estimate by OLS a regression of the average excess returns across maturities on all the forward rates in $\mathbf{f}_t$,

$$\overline{rx}_{t+1} = \gamma_0 + \gamma' \mathbf{f}_t + \bar{\eta}_t. \quad (15)$$

In the second stage, we estimate the regressions

$$rx_{t+1}^n = a_n + b_n(\hat{\gamma}_0 + \hat{\gamma}' \mathbf{f}_t) + \eta_t^n, \quad (16)$$

where $\hat{\gamma}$ is the OLS estimator derived in the first stage. C-P document that the linear combination of log forward rates obtained in the first stage has predictive power for the second stage regressions when applied to U.S. bond data. [Dahlquist and Hasseltoft \(2013\)](#) confirm this pattern for other countries, including Germany. Risk premia on a ZCB maturing in n period can then be measured using the fitted value of this second stage

¹⁵The weighted-average life of a bond is the weighted average of the times of the principal repayments. In our model this is exactly $\frac{1}{\lambda}$.

regression: from equation (16) we can see that *expected* excess returns on a bond maturing in n period equal $\mathbb{E}_t[rx_{t+1}^n] = \alpha_n + \beta_n(\hat{\gamma}_0 + \hat{\gamma}'\mathbf{f}_t)$.

Our procedure consists in calibrating θ_1 so that the pricing model defined by the system in (9) satisfies three properties:

1. The factor χ_t equals $\hat{\gamma}_0 + \hat{\gamma}'\mathbf{f}_t^{\text{model}}$, where $\hat{\gamma}_0$ and $\hat{\gamma}$ are the OLS point estimates in equation (15) and $\mathbf{f}_t^{\text{model}}$ are the log forward rates generated by the model.
2. The model implied coefficients of equation (16) are equal to the OLS point estimates $(\hat{\alpha}_n, \hat{\beta}_n, \hat{\sigma}_{\eta^n})$, for a five year bond ($n = 20$).
3. The mean and standard deviation of y_t^1 in the model matches that in the data.

The first requirement allows us to interpret χ_t as a shock directly moving risk premia on long term bonds,¹⁶ and it gives us a direct mapping between the state χ_t and the data, $\hat{\gamma}_0 + \hat{\gamma}'\mathbf{f}_t$. The second requirement implies that our pricing model replicates the estimated time series of $\mathbb{E}_t[rx_{t+1}^{20}]$ once we feed it with $\chi_t = \hat{\gamma}_0 + \hat{\gamma}'\mathbf{f}_t$. The third requirement makes sure that the behavior of the risk free rate generated by the model is consistent with that observed the data.

Table 2: **Summary statistics: yields and holding period returns**

	Mean	Standard deviation	Sharpe Ratio
$y_t^1 - \text{infl}_t$	2.16	1.93	
$y_t^{20} - \text{infl}_t$	2.94	1.72	
rx_{t+1}^4	0.21	2.05	0.11
rx_{t+1}^8	0.94	4.22	0.22
rx_{t+1}^{12}	1.54	6.08	0.25
rx_{t+1}^{16}	2.02	7.70	0.26
rx_{t+1}^{20}	2.40	9.14	0.26

Notes: The sample period is 1973:Q1-2013:Q4. Variables are reported as annualized percentages (multiplied by a factor 400).

We use our data on the term structure of German's ZCBs to construct time series for the realized excess log returns and the log forward rates for $n = 4, 8, 12, 16, 20$, following the above definitions. Table 2 reports summary statistics on yields and realized excess returns at different horizons. We can verify that the yield curve slopes up on average: yields on 5 years bonds are, on average, 80 basis points higher than yields on bonds maturing next quarter. We can also see that long term bonds earn a positive excess return over our sample. For example, holding a 5 year bond and selling it off next quarter earns,

¹⁶As we show in Appendix B, equation (16) holds exactly in our pricing model for any n .

on average, an annualized premium of 2.40% relative to investing the same amount of money in a bond that matures next quarter. Excess returns on long term bonds increase monotonically with n , and so does their Sharpe ratio.

Table 3 reports the results of the C-P regressions. The top panel reports OLS estimates of equation (15), where $\bar{r}\bar{x}_{t+1}$ are realized excess log returns averaged across $n = 4, 8, 12, 16, 20$ and the vector $\mathbf{f}_t$ includes the risk free rate and the log forward rates for our five maturities. The bottom panel reports the individual bond regressions of equation (16). Differently from the analysis of [Cochrane and Piazzesi \(2005\)](#) on U.S. data, the estimated vector $\hat{\gamma}$ is not “tent” shaped. However, we confirm using German data the finding that a single linear combination of log forward rates has predictive power for excess log returns, and that the sensitivity of the latter to this factor (the estimated b_n ’s) increases with maturity.

Table 3: [Cochrane and Piazzesi \(2005\)](#) regressions

	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	R^2
Estimates of equation (15)	-0.002 (-0.27)	-1.65 (-2.89)	5.00 (2.92)	-21.70 (-2.10)	47.20 (1.58)	-45.18 (-1.19)	16.53 (0.95)	0.12
	a_n			b_n		R^2		
Estimates of equation (16)	4		-0.001 (-2.06)		0.46 (5.48)			0.20
	8		-0.000 (-0.37)		0.77 (4.92)			0.13
	12		0.000 (0.14)		1.02 (4.60)			0.11
	16		0.001 (0.30)		1.27 (4.55)			0.11
	20		0.001 (0.34)		1.48 (4.56)			0.11

Notes: The sample period is 1973:Q1-2013:Q4. Robust t -statistics in parenthesis.

Table 4 reports the numerical values of the parameters in θ_1 guaranteeing that our stochastic discount factor satisfies the three properties discussed above. See Appendix B for a discussion of the procedure we use to match these empirical targets.

5.3.2 The government's decision problem

We next turn to the calibration of $\theta_2 = [\sigma, \beta, d_0, d_1, \psi, \rho_y, \rho_{y\chi}, \sigma_y, \sigma_{y\chi}, p^*, \rho_p, \sigma_p]$. We fix σ to 2, a conventional value in the literature, and we set $\psi = 0.0492$, a value that implies an average exclusion from capital markets of 5.1 years following a sovereign default, in line with the evidence in [Cruces and Trebesch \(2013\)](#).

We use our output series and the linear combination of log forward rates $\chi_t = \hat{\gamma}_0 + \hat{\gamma}'\mathbf{f}_t$ to estimate the output process in equation (13). We estimate a VAR(1) for $[\chi_t, y_t]$, restricted so that χ_t is exogenous with respect to y_t . As $\rho_{y\chi}$ turns out to be not significantly different from zero in this specification, we also impose the restriction $\rho_{y\chi} = 0$. The point estimates of this restricted VAR are $\rho_y = 0.939$, $\sigma_{y\chi} = -0.002$ and $\sigma_y = 0.008$.

While in a future draft we plan of calibrating the remaining parameters to match basic facts about the price, quantity and duration of Italian public debt, in this draft we borrow their value from previous research. Our calibration for $\{p_t\}$ implies an annualized probability of rollover crisis of 2% (assuming that the economy is in the crisis zone) and it allows for large and persistent deviations from this value.

Table 4: **Model calibration**

	Numerical Value	Empirical Targets
δ_0	0.003	C-P predictive regressions and risk free rate
δ_1	0.242	C-P predictive regressions and risk free rate
λ_0	-0.228	C-P predictive regressions and risk free rate
λ_1	-2322.111	C-P predictive regressions and risk free rate
μ_χ	0.008	C-P predictive regressions and risk free rate
ρ_χ	0.906	C-P predictive regressions and risk free rate
σ_χ	0.008	C-P predictive regressions and risk free rate
ρ_y	0.939	Restricted VAR(1) for $[\chi_t, y_t]$
$\rho_{y\chi}$	0.000	Restricted VAR(1) for $[\chi_t, y_t]$
σ_y	0.008	Restricted VAR(1) for $[\chi_t, y_t]$
$\sigma_{y\chi}$	-0.002	Restricted VAR(1) for $[\chi_t, y_t]$
σ	2.000	Conventional value
ψ	0.049	Cruces and Trebesch (2013)
β	0.950	Chatterjee and Eyigungor (2013)
d_0	-0.180	Chatterjee and Eyigungor (2013)
d_1	0.240	Chatterjee and Eyigungor (2013)
$\frac{\exp\{p^*\}}{1+\exp\{p^*\}}$	0.005	Numerical Exploration
ρ_p	0.950	Numerical Exploration
σ_p	0.500	Numerical Exploration

6 Decomposing Italian Spreads

We now use the calibrated model to measure the importance of rollover risk in driving Italian spreads during the period of analysis. We proceed in two steps. In the first step, we use our calibrated model along with the data presented in Section 5 to estimate a time series for the exogenous shocks, $\{y_t, \chi_t, p_t\}$. In the second step, we use the estimated path for the state variables and the model equilibrium conditions to calculate the component of interest rate spreads that is due to rollover risk. This exercise is reported in Section 6.1. In order to highlight the information content of maturity choices, Section 6.2 repeats the experiment, this time excluding the debt duration series from the set of observables. Finally, Section 6.3 discusses the role of risk averse lenders for generating our results.

6.1 Measuring rollover risk

Our model defines the nonlinear state space system

$$\begin{aligned} \mathbf{Y}_t &= \mathbf{g}(\mathbf{S}_t; \theta) + \eta_t \\ \mathbf{S}_t &= \mathbf{f}(\mathbf{S}_{t-1}, \varepsilon_t; \theta), \end{aligned}$$

with $\mathbf{Y}_t$ being a vector of measurements, η_t classical measurement errors, the state vector is $\mathbf{S}_t = [B_t, \lambda_t, y_t, \chi_t, p_t]$ and ε_t are innovations to structural shocks. The first part of the system collects measurement equations, describing the behavior of observable variables while the second part collects transition equations, regulating the law of motion for the potentially unobserved states. We estimate the realization of the model state variables by applying the particle filter to the above system (Fernández-Villaverde and Rubio-Ramírez, 2007).¹⁷ The set of measurements $\mathbf{Y}_t$ includes the time series for interest rate spreads, linearly detrended real GDP, the previously estimated series for χ_t , and the weighted-average life of Italian debt. It is important to stress that the estimation of $[y_t, \chi_t]$ is disciplined by “actual” observations because the measurement equation incorporates empirical counterparts of these shocks. The truly unobservable process is the realization of p_t .

Equipped with the estimated path for the model state variables, we next use the structural model to measure the contribution of rollover risk to interest rate spreads. For this purpose, we use the lenders’ Euler equation and express interest rate spreads on a ZCB

¹⁷The measurement errors are Gaussian. The variance for the measurement error associated to interest rate spreads is set at 2.5% of the sample variance of the observables. The measurement errors for the other variables is set at 20% of their respective sample variance. The number of particles adopted is 50000.

maturing next period as follows

$$\begin{aligned} \frac{r_{1,t} - r_t^*}{r_{1,t}} &= \Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{default}}\} + \Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{crisis}}\}\mathbb{E}_t[p_{t+1}] \\ &\quad - \text{Cov}_t\left(\frac{M_{t,t+1}}{\mathbb{E}_t[M_{t,t+1}]}, \delta_{t+1}\right). \end{aligned} \tag{17}$$

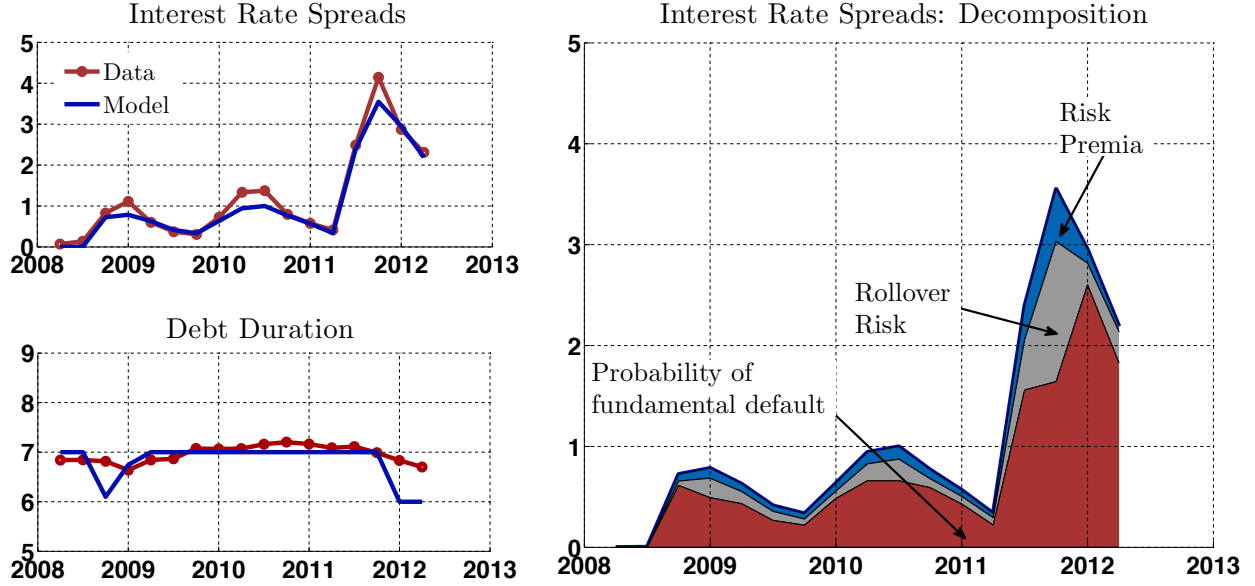
The first two components in equation (17) represent the different sources of default risk in the model. As discussed in Section 2, the government can default because of two events. First, if $\mathbf{S}_{t+1} \in \mathcal{S}^{\text{default}}$, the government finds it optimal to default irrespective of the behavior of lenders. Second, the government may be in the crisis zone next period, in which case the conditional probability of observing a default is $\mathbb{E}_t[p_{t+1}]$. Finally, $\text{Cov}_t\left(\frac{M_{t,t+1}}{\mathbb{E}_t[M_{t,t+1}]}, \delta_{t+1}\right)$ reflects the premium that lenders demand to hold risky government securities. Our objective is to construct a time series for these three components of the interest rate spreads.

The left panels of Figure 3 reports the behavior of spreads and debt duration in the data and in the model. The model closely matches the behavior of interest rate spreads. This is not surprising because the variance of the measurement errors associated to this series in the state space model is small. The model tracks closely the behavior of the debt duration series, which decreases in the latter part of the sample. Between 2011:Q1 and 2012:Q2, the weighted average life of outstanding Italian debt dropped by half a year.¹⁸ The model captures this pattern, but it cannot quantitatively fit this step because the grid for λ in the numerical solution is such that duration changes at least one year, see Appendix A.

The right panel of Figure 3 reports the model implied decomposition of equation (17). The red shaded area represents the conditional probability of falling into the default region next period, the gray shaded area reports the conditional probability of a rollover crisis, and the blue shaded area denotes risk premia. From the figure, we can see that the risk premium component explains, on average, roughly 10% of the variation in interest rate spreads over our sample. The bulk of the variation in interest rate spreads arise because of fluctuations in the conditional probability of a fundamental default. Finally, rollover risk accounts for up to 38% of the observed movements in spreads, although its role is negligible at the end of the sample.

¹⁸While this may seem a small variation, it is important to stress that we are measuring the duration of the outstanding *stock* of debt. Hence, the change in duration for the flows (net issuances) are substantially larger.

Figure 3: Contribution of rollover risk to interest rate spreads



Notes: The top left panel reports CDS spreads on 6 months Italian government bonds along with the point estimates for interest rate spreads on a one period ZCB implied by the model. The bottom left panel reports the same information for the weighted-average life of outstanding government debt. The right panel reports the decomposition of the filtered interest rate spreads given by equation (17). The red area represents $\{Pr_t\{S_{t+1} \in \mathcal{S}^{default}\}\}$, the gray area $\{Pr_t\{S_{t+1} \in \mathcal{S}^{crisis}\}\}$, and the blue area $\{Cov_t\left(\frac{M_{t,t+1}}{E_t[M_{t,t+1}]}\right)\}$.

6.2 The information content of maturity choices

We now repeat the filtering experiment, this time excluding the debt duration series from the set of observables. Table 5 reports several statistics for this specification. Specifically, the point estimates for the average of the three components of the interest rate spreads over the sample, along with the 5th and 95th percentile. We also report, as a comparison, the same statistics for the experiment of Section 6.1.

Absent data on debt duration, the model does not have clear identifying restrictions that can be used to discipline p_t , and it attributes to this term variation in interest rate spreads that can not be accounted by $[y_t, \chi_t]$. Even in this specification, though, the model assigns *on average* a fairly limited role to rollover risk. This depends on the fact that detrended real GDP during the 2008-2012 period was well below average, and the χ_t factor signals increases in risk premia over the episode, see Figure x. Hence, the model requires little variation in p_t to fit the Italian spreads.

However, Table 5 documents also substantial uncertainty in this decomposition. This can be seen by looking at the standard errors of the three components. For example, the rollover risk component can account, on average, for almost all the variation in interest

rate spreads (113 basis points vs an average spreads in the data of 120 basis points).

Table 5: **Interest rate spreads decomposition**

		Average	5 th Percentile	95 th Percentile
No duration	$\Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{default}}\}$	0.74	0.15	1.20
	$\Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{crisis}}\}$	0.32	0.00	1.13
	$\text{Cov}_t\left(\frac{M_{t,t+1}}{\mathbb{E}_t[M_{t,t+1}]}\right)$	0.10	0.03	0.25
Benchmark	$\Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{default}}\}$	0.75	0.55	1.13
	$\Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{crisis}}\}$	0.21	0.00	0.45
	$\text{Cov}_t\left(\frac{M_{t,t+1}}{\mathbb{E}_t[M_{t,t+1}]}\right)$	0.12	0.05	0.30

This second result is due to the combination of two factors. First, pricing schedules in models of sovereign debt are highly nonlinear, and small variations in $[y_t, \chi_t]$ can have sizable effects on interest rate spreads when default risk is non negligible. Second, the lack of discipline on p_t implies that the model can use this shock to fit variation in interest rate spreads that is not accounted by the fundamental shocks. Hence, small degrees of uncertainty over $[y_t, \chi_t]$, generated in our experiment by measurement errors, translates into sizable uncertainty over $\Pr_t\{\mathbf{S}_{t+1} \in \mathcal{S}^{\text{default}}\}$, and ultimately on the probability of a rollover crisis.

The introduction of the weighted-average life of outstanding debt in the set of observables helps resolving this identification problem. This can be seen by looking at the standard errors for the components of the spreads in our benchmark exercise, which are substantially smaller than the specification without duration data. To understand why this is the case, we report in Table 6 the cross-sectional correlation between the filtered probability of a rollover crisis and the implied maturity choices that the government makes at these points in the state space. The Table shows that these two variables are negative correlated: realizations of the state vector in which rollover risk is high are associated to higher debt maturities (lower λ' chosen by the government). Hence, our benchmark estimation assigns low likelihood to these realizations because the implied debt maturity choices of the government are counterfactual.

Table 6: **Correlation between $\Pr_{i,t}\{S_{t+1} \in \mathcal{S}^{\text{crisis}}\}$ and $\lambda_{i,t+1}$**

2011:Q2	2011:Q3	2011:Q4	2012:Q1	2012:Q2
-0.459	-0.453	-0.516	-0.523	-0.434

6.3 The role of risk averse lenders

To be completed.

7 Discussion of Assumptions

In this section we discuss the robustness of our results to some of the assumptions and modeling choices we made. In particular, we discuss the exogeneity of risk premia and output to default risk, and the selection rule considered.

Endogeneity of SDF and output. For tractability, we have assumed that lenders' stochastic discount factor is independent on the probability of a government default. While this assumption may be uncontroversial if one considers a small open economy, this might be problematic for a country like Italy. In fact, it is natural to think that a default of a large economy would have adverse consequences on bondholders, and that the prospect of this event may alter their attitude toward risk.¹⁹ Because of that, one may think that our procedure underestimates the importance of rollover risk: by making a sovereign default more likely, an increase in the probability of a rollover crisis could lead to an increase in the risk aversion of lenders, and impact interest rate spreads through risk premia. Our approach could erroneously misinterpret this indirect effect of rollover risk as a shock to the lenders' stochastic discount factor. We next show that this is not likely to be the case.

To understand why, it is important to stress that the quantitative importance of rollover risk is identified in the model from the joint behavior of debt duration and interest rate spreads. Consider a version of the model in which future default probabilities affect the lenders' stochastic discount factor. In particular, assume that the prospect of a government default makes the stochastic discount factor more volatile,

$$M(s, s') = \begin{cases} \frac{\Pr(s'|s)}{1+r^*} \frac{1}{\mathbb{E}[(1+m)(1-\delta(s'))]} & \text{if } \delta(s') = 1 \\ \frac{\Pr(s'|s)}{1+r^*} \frac{(1+m)}{\mathbb{E}[(1+m)(1-\delta(s'))]} & \text{if } \delta(s') = 0 \end{cases}, \quad m > 0. \quad (18)$$

¹⁹For example, this prediction would arise in a set up where lenders are exposed to government debt and they face occasionally binding constraints on their funding ability, see [Bocola \(2014\)](#) and [Lizarazo \(2013\)](#).

From equation (18) we have that the risk free rate is constant and equal to $1 + r^*$ and the price of risk is increasing in the probability of default. If $\mathbb{E}_t[1 - \delta]$ increases then $M_{t,t+1}$ increases in a second order stochastic dominance sense. When facing the pricing kernel in equation 18 the government has an *extra motive to lengthen* debt maturity because, by doing so, it reduces not only rollover risk, but also its price. This discussion indicates that our calibration would assign a more limited role to rollover risk if we were to incorporate this feedback in the model, because the model would imply an even more counterfactual behavior for debt maturity over the sample.

The same exact argument can be made for output. Previous literature has suggested that the prospects of a future sovereign default are recessionary, see [Bocola \(2014\)](#). One may then argue that expectations of future rollover crises can reduce output, and we could misinterpret this indirect effect as an endowment shock. However, this is ruled out by our identification strategy. If output depends negatively on the probability of future default, then a government facing a higher prospects of rollover risk would have an extra incentive to lengthen the duration of its debt, because that would reduce rollover risk and mitigate the fall in output.

In sum, by looking at the behavior of debt duration, our identification strategy is not likely to underestimate rollover risk because of concerns on the endogeneity of output and of the lenders' stochastic discount factor.

Selection rule. We now briefly discuss the implications of assuming the selection rule in (14) for our results. The particular functional form is not crucial because we identify rollover risk through the comovement of spreads and maturity composition of debt. For example, we could allow $\{p_t\}$ to depend on the fundamental shocks, as it would arise if one applied global game techniques to select an equilibrium. The logic by which the government has incentives to lengthen debt maturity when $E_t[p_{t+1}]$ increases is independent on the specific process for $\{p_t\}$. This is the key economic force that allows us to pin down $\{p_t\}$. Hence, our finding that rollover risk accounts for small/negligible part of spreads in the recent crisis for Italy is determined by the negative commovement between spreads and duration of the stock of Italian debt. [To be completed]

8 Evaluating OMT Announcements

As a response to soaring interest rate spreads in the euro-area periphery, the Governing Council of the European Central Bank (ECB) announced during the summer of 2012 that it would consider outright transactions in secondary, sovereign bond markets. The technical

framework of these operations was formulated on September 6 of the same year. The Outright Monetary Transaction (OMT) program replaced the Security Market Program as a mean through which the ECB could intervene in sovereign bond markets.

OMTs consist in direct purchases of sovereign bonds of members of the euro-area in secondary markets.²⁰ These operations are considered by the ECB once a member state asks for financial assistance, and upon the fulfillment of a set of conditions.²¹ There are two important characteristics of these purchases. First, *no ex ante quantitative limits* are set on their size. Second, the ECB accepts the same (*pari passu*) treatment as private or other creditors with respect to bonds issued by euro area countries and purchased through OMTs.

Even though the ECB never purchased sovereign bonds within the OMT framework, the mere announcement of the program had significant effects on interest rate spreads of peripheral countries. [Altavilla, Giannone and Lenza \(2014\)](#) estimate that OMT announcements decreased the Italian and Spanish 2 years government bonds by 200 basis points. This decline in interest rate spreads was widely interpreted by economists and policy makers as a reflection of the success of this program in reducing non-fundamental inefficient fluctuations in sovereign bond markets of euro-area peripheral countries. Accordingly, OMT has been regarded thus far as a very successful program. In this Section we use our calibrated model to evaluate this interpretation.

We introduce OMTs in our model as a price floor schedule implemented by a Central Bank. Section 8.1 shows that an appropriate design of this schedule i) can eliminate the bad equilibria in our model, and ii) it does not require the Central Bank to ever intervene in bond markets. Therefore, along the equilibrium path the Central Bank can achieve a Pareto improvement without taking risk for its balance sheet. However, we also show that alternative formulations of the price floor may induce the sovereign to ask for assistance in the face of bad fundamental shocks. *Ex-ante*, this option leads the sovereign to overborrow. Under both of these scenarios, interest rate spreads decline once the Central Bank announces the price floor schedule: in the first scenario, the reduction in interest rate spreads is due the elimination of rollover risk. In the second scenario, this reduction reflects the option for bondholders to resell the security to the Central Bank whenever the sovereign is approaching a solvency crises. Section 8.2 proposes a simple

²⁰Transactions are focused on the shorter part of the yield curve, and in particular on sovereign bonds with a maturity of between one and three years. The liquidity created through OMTs is fully sterilized.

²¹A necessary condition for OMTs is a *conditionality* attached to a European Financial Stability Facility/European Stability Mechanism (EFSF/ESM) macroeconomic adjustment or precautionary programs. For a country to be eligible for OMTs, these programs should include the possibility of EFSF/ESM primary market purchases.

procedure to test which of these two hypothesis better characterizes the observed behavior of Italian spreads after the announcements of the OMT program.

8.1 Modeling OMT

We model OMT as follows. At the beginning of each period, after all uncertainty is realized, the government can ask for assistance. In such case the Central Bank (CB) commits to buy government bonds in secondary markets at a price $q_{n,CB}(\mathbf{S}, B', \lambda')$ that may depend on the state of the economy, $\mathbf{S}$, on the quantity of debt issued, B' , and on the maturity of the portfolio, λ' . We assume that assistance is conditional on the fact that total debt issued is below a cap $\bar{B}_{n,CB}(\mathbf{S}, \lambda') < \infty$ also set by the CB. The limit can depend on the state of the economy and on the duration of the stock of the debt portfolio. This limit captures the conditionality of the assistance in the secondary markets. Moreover, it rules out Ponzi-scheme on the central bank. Hence OMT is fully characterized by a policy rule $(q_{n,CB}(\mathbf{S}, B', \lambda'), \bar{B}_{n,CB}(\mathbf{S}, \lambda'))$. We assume that the CB finances such transactions with a lump sum tax levied on the lenders. We further assume that such transfers are small enough that they do not affect the stochastic discount factor $M_{t,t+1}$.

The problem for the government described in (23) changes as follows. We let $a \in \{0, 1\}$ be the decision to request CB assistance, with $a = 1$ for the case in which assistance is requested. Then we have:

$$V(\mathbf{S}) = \max_{\delta, B', \lambda', G, a} \delta \{U(G) + \beta \mathbb{E}[V(\mathbf{S}') | \mathbf{S}]\} + (1 - \delta) \underline{V}(s_1) \quad (19)$$

subject to

$$\begin{aligned} G + B &\leq Y(s_1) + \Delta(\mathbf{S}, a, B', \lambda'), \\ \Delta(\mathbf{S}, a, B', \lambda') &= \sum_{n=1}^{\infty} q_n(s, a, B', \lambda') [(1 - \lambda')^{n-1} B' - (1 - \lambda)^n B] \\ B'_n &\leq \bar{B}_{n,CB}(\mathbf{S}, \lambda') \quad \text{if } a = 1. \end{aligned}$$

The lenders have the option to resell government bonds to the CB at the price $q_{n,CB}$ in case the government asks for assistance. The no-arbitrage conditions for the lenders (6) is

modified as follows: The lender's no-arbitrage condition requires that

$$\begin{aligned} q_1(\mathbf{S}, a, B', \lambda' | \lambda) &= \max\{a q_{1,CB}(\mathbf{S}, B', \lambda' | \lambda); \delta(\mathbf{S}) \mathbb{E}\{M(s_1, s'_1)\} \delta(\mathbf{S}') | \mathbf{S}\} \\ q_n(\mathbf{S}, a, B', \lambda' | \lambda) &= \max\{a q_{n,CB}(\mathbf{S}, B', \lambda' | \lambda); \delta(\mathbf{S}) \mathbb{E}\{M(s_1, s'_1) \delta(\mathbf{S}') q'_{n-1} | \mathbf{S}\} \text{ for } n \geq 2 \end{aligned} \quad (20)$$

where $B'' = B'(s', B', \lambda')$, $\lambda'' = \lambda'(s', B', \lambda')$, $a' = a(s', B', \lambda')$, and $q'_{n-1} = q_{n-1}(s', B'', \lambda'')$. It is important to notice that the bonds prices now depend also on the current and future decision of the government to activate assistance.

Given a policy rule $(q_{CB}, \bar{B}_{CB})$, a *recursive competitive equilibrium with OMT* is value function for the borrower V , associated decision rules δ, B', λ', G and a pricing function q such that V, δ, B', G are a solution of the government problem (19) and the pricing functions satisfy the no-arbitrage condition (20). For exposition, it is convenient to define also the *fundamental equilibrium outcome* $\mathbf{y}^* = \{\delta_t^*, B_{t+1}^*, \lambda_{t+1}^*, G_t^*, q_{n,t}^*\}$ as the equilibrium outcome that maximizes the utility for the government given an initial portfolio of debt. We denote the objects of a recursive competitive equilibrium associated with the fundamental outcome with a superscript “*”.

We now turn to show that an appropriately designed policy rule can uniquely implement the fundamental equilibrium outcome, our normative benchmark.²²

Proposition 1. *The OMT rule can be chosen such that the fundamental equilibrium outcome is uniquely implemented and assistance is never activated along the path. In such case, OMT is a weak Pareto improvement relative to the equilibrium without OMT (strict if the equilibrium outcome without OMT does not coincide with the fundamental equilibrium).*

Proof. An obvious way to uniquely implement the fundamental equilibrium outcome is to set $q_{n,CB}(\mathbf{S}, B', \lambda') = q_n^*(s, B', \lambda')$ and $\bar{B}_{n,CB}(\mathbf{S}, \lambda') \leq (1 - \lambda)^{n-1} B^{*'}(\mathbf{S})$ if $\lambda = \lambda^{*'}(\mathbf{S})$ and zero otherwise. Such construction is not necessary. A less extreme alternative is to design policies such that for all $\mathbf{S}$ for which there is no default in the fundamental equilibrium, $\delta^*(\mathbf{S}) = 1$, there exists at least one (B', λ') with $(1 - \lambda')^{n-1} B' \leq \bar{B}_{n,CB}(\mathbf{S}, \lambda')$ such that

$$U(Y - B + \Delta(\mathbf{S}, 1, B', \lambda')) + \beta \mathbb{E} V^*(B', \lambda', s') \geq \underline{V}(s_1), \quad (21)$$

²²Clearly, the model has incomplete markets and all sorts of inefficiencies (especially when considering an environment with long-term debt). We are going to abstract from policy interventions that aims to ameliorate such inefficiencies. OMT is only targeted at eliminating “bad” equilibria. Such features will also survive in models with complete markets or in environment where some notion of constrained efficiency can be achieved as in [Dovis \(2014\)](#).

and for all (B', λ') such that $(1 - \lambda')^{n-1} B' \leq \bar{B}_{n,CB}(\mathbf{S}, \lambda')$ the fundamental equilibrium is always preferable, in that

$$U(Y - B + \Delta(\mathbf{S}, 1, B', \lambda')) + \beta \mathbb{E} V^*(B', \lambda', s') \leq V^*(\mathbf{S}). \quad (22)$$

Under (21) and (22), it is clear that no self-fulfilling run is possible and there is no over-borrowing. Hence (21) and (22) are sufficient conditions to eliminate runs and to uniquely implement the fundamental equilibrium outcome. $\square$

Note that quantity limits (conditionality) are necessary to uniquely implement the fundamental equilibrium. In absence of $\bar{B}_{CB}$, the government would choose a B' that is larger than the one in the fundamental equilibrium because he acts as a price taker under OMT. So, a limit to B' is necessary to prevent overborrowing.

Proposition 1 gives us the most benevolent interpretation of the drop in Italian spreads after OMT was announced. If OMT follows the rule described in the proof of Proposition 1, it uniquely implements the fundamental equilibrium outcome. In this case the observed drop in spreads is due to the fact that lenders anticipate that no run can happen along the equilibrium path resulting in lower default probability and hence lower spreads.

However, the central bank does not want to support bond prices if they are low because of fundamental reasons. This entails a subsidy from the lenders to the borrower, reducing welfare for the lenders relative to the equilibrium without OMT (assuming lenders are the ones that have to pay for the losses of the central bank). Even in this scenario, bond prices may decline. To see this, suppose that in a given state the fundamental price for the portfolio of debt is q^{*} . Suppose now that the ECB sets an assistance price $q'_{CB} > q^{*}$. It is clear from (20) that the price today increases (the spread drops) relative to a counterfactual world without OMT.

Thus, a decline in the spreads is not informative on whether ECB is following the benchmark rule, or whether it is providing some subsidy to peripheral countries. We now use the calibrated model to test between these two alternatives.

8.2 A Simple Test

We now test for the hypothesis that the ECB did follow the policy described in Proposition 1. To explain our approach, suppose that the Central Bank credibly commits to our normative benchmark. The announcement of this intervention would eliminate all the rollover risk, and the spreads today would jump to their fundamental value, i.e. the value that would arise if rollover crisis were not conceivable from that point onward. This

fundamental level of the interest rate spread represents a lower bound on the post-OMT spread under the null hypothesis that the program was directed exclusively to prevent runs on Italian debt. Our test consists in comparing the spread observed after the OMT announcements to their fundamental value: if the latter is higher than the observed one, it would be evidence against the null hypothesis that the ECB followed the policy described in Proposition 1.

We perform this test using our calibrated model. Our procedure consists in three steps:

1. Obtain decision rules from the fundamental equilibrium.
2. Feed these decision rules with our series for the fundamental shocks $\{\chi_t, y_t\}$. Obtain counterfactual post-OMT fundamental spreads.²³
3. Compare post-OMT spreads with the counterfactual ones.

Table 7 reports the results. In the first column we have the Italian spreads observed after the OMT announcements, while the second column presents the counterfactual spreads constructed with the help of our model. We can verify that the observed spreads lie below the one justified by economic fundamentals under the most optimistic interpretation of OMT. In 2012:Q4, the observed spread on our spread series was 70 basis points, while our model suggests that the spread should have been XXX basis points if the program was exclusively eliminating rollover risk. Therefore, our model suggests that the decline in the spreads observed after the OMT announcements partly reflects the anticipation of a future intervention of the ECB in secondary sovereign debt markets. This is not surprising given our result in Section 6: since rollover risk was almost negligible in 2012:Q2, the observed drastic reduction in the spreads should partly reflect the value of an implicit put option for holders of Italian debt guaranteed by the ECB.

Table 7: **Actual and fundamental sovereign interest rate spreads in Italy**

	Actual spreads	Spreads justified by fundamentals
2012:Q3	142.51	XXX
2012:Q4	70.31	XXX

Clearly, it would be interesting to use our model to dig deeper into the implications of the OMT program. For example, we could try to measure the put option implicit in this

²³ The estimates of the state vector ends in 2012:Q2. For the 2012:Q3-2012:Q4 period, we set y_t equal to linearly detrended Italian output and we filter out $\{\chi_t, v_t\}$ using our pricing model along with the data on the German yield curve and the euro-area price-consumption ratio.

intervention, to calculate the amount of resources that the ECB is implicitly committing under this policy or we could assess the moral hazard implications associated to this policy. This would not be an uncontroversial task, as it would require us to i) specify the policy rule followed by the ECB and to ii) specify how the selection mechanism responds to the policy intervention. The test we have described in this section is robust to these caveats, and we regard it as a first step for the evaluation of this type of interventions in sovereign debt models with multiple equilibria.

9 Conclusion

In this paper, we studied the importance of rollover risk during the euro-area sovereign debt crisis. We argued that observed maturity choices are informative about the prospect of future rollover crises. Our preliminary results indicate that rollover risk accounted for a modest fraction of the increase in interest rate spreads.

Our analysis is limited to belief driven fluctuations that arises from rollover risk as introduced in [Cole and Kehoe \(2000\)](#). We did not consider the type of multiplicity emphasized in [Calvo \(1988\)](#) and recently revived by [Lorenzoni and Werning \(2013\)](#) and [Navarro et al. \(2015\)](#), or the mechanism in [Broner, Erce, Martin and Ventura \(2014\)](#). Future research should investigate which feature of the data can be used to discipline empirically these and other sources of multiplicity.

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APPENDIX

A Numerical Solution

Let $\mathbf{S} = [B, \lambda, y, \chi, p]$ be the vector collecting the model's state variables. Before explaining the numerical solution, it is convenient to rewrite the decision problem for the government as follows

$$V(\mathbf{S}) = \max_{\delta \in \{0,1\}, B', \lambda', G} \delta \{U(G) + \beta \mathbb{E}[V(\mathbf{S}') | \mathbf{S}]\} + (1 - \delta)V(s_1) \quad (23)$$

subject to

$$\begin{aligned} G + B &\leq Y(s_1) + \Delta(\mathbf{S}, B', \lambda'), \\ \Delta(\mathbf{S}, B', \lambda') &= q(s, B', \lambda' | \lambda') B' - q(s, B', \lambda' | \lambda) (1 - \lambda) B, \end{aligned}$$

where $q(s, B', \lambda' | \lambda)$ is the per unit value of a portfolio of ZCBs with decay parameter λ given the realization s for the exogenous state, and given the government's choices for the new portfolio is (B', λ') . The price of this portfolio of ZCBs can be written as

$$q(s, B', \lambda' | \lambda) = \delta(\mathbf{S}) \mathbb{E} \{ M(s_1, s'_1) \delta(\mathbf{S}') [1 + (1 - \lambda) q(s, B'', \lambda'' | \lambda)] | \mathbf{S} \}, \quad (24)$$

where $B'' = B'(s', B', \lambda')$ and $\lambda'' = \lambda'(s', B', \lambda')$.

We define the value of repaying the debt conditional on lenders rolling over the debt, $V_{\text{roll}}^R(\mathbf{S})$, as follows

$$V_{\text{roll}}^R(\mathbf{S}) = \max_{B', \lambda'} \{ U(Y - B + \Delta(\mathbf{S}, B', \lambda')) + \beta \mathbb{E}[V(B', \lambda', s') | \mathbf{S}] \}. \quad (25)$$

The value of repaying conditional on lenders not rolling over the debt, $V_{\text{no roll}}^R(\mathbf{S})$, is

$$V_{\text{no roll}}^R(\mathbf{S}) = \{ U(Y - B) + \beta \mathbb{E}[V(B(1 - \lambda), \lambda, s') | \mathbf{S}] \}, \quad (26)$$

while the value of defaulting, $V^D(y, \chi)$, is

$$V^D(y, \chi) = \left\{ U(Y[1 - \tau(Y)]) + \beta \{ \psi \mathbb{E}[V(0, \bar{\lambda}, y', \chi') | \mathbf{S}] + (1 - \psi) \mathbb{E}[V^D(y', \chi') | \mathbf{S}] \} \right\}. \quad (27)$$

The value function for the government decision problem can then be written as

$$V(\mathbf{S}) = \begin{cases} V_{\text{roll}}^R(\mathbf{S}) & \text{if } V_{\text{no roll}}^R(\mathbf{S}) \geq V^D(y, \chi) \\ V_{\text{roll}}^R(\mathbf{S}) & \text{if } V_{\text{no roll}}^R(\mathbf{S}) < V^D(y, \chi) \text{ and } \xi < p \\ V^D(y, \chi) & \text{otherwise} \end{cases}$$

When coupled with the pricing function q , the knowledge of $\{V_{\text{roll}}^R(\mathbf{S}), V_{\text{no roll}}^R(\mathbf{S}), V^D(y, \chi)\}$ is sufficient to solve for the policy functions of the model. The numerical solution consists in approximating these three value functions and the pricing schedule q .

The inverse duration for the debt portfolio, λ , is assumed to be a discrete variable from the set $\Lambda = \{\lambda_1, \lambda_2, \dots, \lambda_N\}$. The value functions are approximated using piece-wise smooth functions. Specifically, $V_{\text{roll}}^R(\cdot)$, is approximated as follows,

$$V_{\text{roll}}^R(\lambda_j, \tilde{\mathbf{S}}) = \gamma_{\text{roll}, \lambda_j}^R \mathbf{T}(\tilde{\mathbf{S}}),$$

where $\tilde{\mathbf{S}} = [B, y, \chi, p]$ is a realization of state variables that excludes λ , $\gamma_{\text{roll}, \lambda_j}^R$ is a vector of coefficients and $\mathbf{T}(\cdot)$ is a vector collecting Chebyshev's polynomials. The value of repaying conditional on the lenders not rolling over the debt, and the value of defaulting are defined in a similar fashion, and we denote by $\gamma_{\text{no roll}, \lambda_j}^R$ and ψ^D the coefficients parametrizing those values. The pricing schedule q is approximated over a grid of possible debt choices, $\mathcal{B} = [b_1, \dots, b_K]$. Letting $s = [y, \chi, p]$ be a realization of the exogenous states, we have that the price of a λ portfolio given the government's choices (B', λ') is $q(s, B', \lambda' | \lambda)$, as defined in equation (24).²⁴

Letting $\Gamma = [\{\gamma_{\text{roll}, \lambda_j}^R, \gamma_{\text{no roll}, \lambda_j}^R\}_j, \gamma^D]$ collect the coefficients that parametrize the value functions, we can index the model's numerical solution by (Γ, q) . Our procedure consists in using the government's decision problem and the lenders' no arbitrage condition to iterate over (Γ, q) until a convergence criterion is achieved. Specifically, the algorithm for the numerical solution of the model is as follows:

- **Step 0: Defining the grid and the polynomials.** Specify the set of values in Λ . Set upper and lower bounds for the state variables $\tilde{\mathbf{S}} = [B, y, \chi, p]$. Given these bounds, construct a μ -level Smolyak grid and the associated Chebyshev's polynomials $\mathbf{T}(\cdot)$ following Judd, Maliar, Maliar and Valero (2014). Let $\tilde{\mathcal{S}}$ denote the set of points for

²⁴A complication of our approach to maturity choices relative to the set up in Arellano and Ramanarayanan (2012) is that we need to price an arbitrary λ portfolio, given government choices (B', λ') , in order to know the market value of the portfolio repurchased by the government. See Sanchez et al. (2015) for a discussion.

the state variables $\tilde{\mathbf{S}}$.

- **Step 1: Update value functions.** Start with a guess for the value and pricing functions, (Γ^c, q^c) . For each $\mathbf{S}^i \in \Lambda \times \tilde{\mathcal{S}}$, update the value functions using the definitions in equation (25)-(27). Denote by Γ^u the updated guess, and by $[r_{roll}^R, r_{noroll}^R, r^D]$ the distance between the initial guess and its update using the sup-norm.
- **Step 2: Update pricing function.** For each exogenous state s^i in the relevant subset of $\tilde{\mathcal{S}}$ and for each $(B^{i'}, \lambda^{i'}) \in \mathcal{B} \times \Lambda \times \Lambda$, evaluate the right hand side of equation (24) using (Γ^u, q^c) . Denote by $\hat{q}^u(s^i, B^{i'}, \lambda^{i'})$ this value, and by r^Q the distance between q^c and $\hat{q}^u$ under the sup norm. Update the pricing schedule as

$$q^u(.) = \theta \hat{q}^u(.) + (1 - \theta) q^c(.) \quad \theta \in (0, 1).$$

- **Step 3: Iteration.** If $\max\{r_{roll}^R, r_{noroll}^R, r^D\} \leq 10^{-6}$ and $r^Q \leq 10^{-3}$, stop the algorithm. If not, set (Γ^u, q^u) as the new guess, and repeat Step 1-2. $\square$

Regarding the specifics of the algorithm, we generate $\tilde{\mathcal{S}}$ using an anisotropic Smolyak grid of $\mu = 6$ in the B dimension and $\mu = 3$ on the other dimensions. The upper and lower bound for B are $[0, 2\mu_y]$, while the upper and lower bounds for $s = (y, \chi, p)$ are equal to ± 3 times the standard deviation of these stochastic processes. The grid for λ contains 5 values: ± 2 years round an average duration of seven years (the Italian pre-crisis level). The grid for debt choices over which the pricing function is defined, $\mathcal{B}$, consists of 100 equally spaced values between $[0, 2\mu_y]$. Expectations over future outcomes are computed using Gauss-Hermite quadrature, with $n = 15$ sample points on each random variable. The smoothing parameter for the updating of the pricing schedule is set at $\theta = 0.05$.

In the numerical solution, we introduce a small cost for adjusting debt maturity,

$$\alpha \left(\frac{4}{\lambda'} - \bar{d} \right)^2.$$

We set $d = 7$, and $\alpha = 0.001$. We introduce this adjustment cost for two purposes. First, it ameliorates the convergence properties of the algorithm as it breaks down indifference in region of the state space where default risk and risk premia on long term bonds are small.²⁵ Second, we make sure that in this region of the state space the maturity choice is consistent with the pre-crisis level of the weighted average life of Italian outstanding debt.

²⁵Maturity choices in the model are not determined absent default risk and with risk neutral lenders.

B The Lenders' Stochastic Discount Factor

We now derive some results concerning the lenders' stochastic discount factor, and describe in more details our calibration. The stochastic discount factor $M_{t,t+1} = \exp\{m_{t,t+1}\}$ follows the exponentially Gaussian process

$$\begin{aligned} m_{t,t+1} &= -(\delta_0 + \delta_1 \chi_t) - \frac{1}{2} \lambda_t^2 \sigma_\chi^2 - \lambda_t \varepsilon_{\chi,t}, \\ \chi_{t+1} &= \mu_\chi (1 - \rho_\chi) + \rho_\chi \chi_t + \varepsilon_{\chi,t} \quad \varepsilon_{\chi,t} \sim \mathcal{N}(0, \sigma_\chi^2), \\ \lambda_t &= \lambda_0 + \lambda_1 \chi_t. \end{aligned} \tag{28}$$

Let $q_t^{*,n}$ be the log price of a non-defaultable ZCB maturing in n periods. These bond prices satisfy the recursion

$$\exp\{q_t^{*,n}\} = \mathbb{E}_t[M_{t,t+1} \exp\{q_{t+1}^{*,n-1}\}],$$

with initial condition $q_t^0 = 0$. [Ang and Piazzesi \(2003\)](#) show that $\{q_t^{*,n}\}$ are linear functions of the state variable χ_t ,

$$q_t^{*,n} = A_n + B_n \chi_t,$$

where A_n and B_n satisfy the recursion

$$\begin{aligned} B_{n+1} &= -\delta_1 + B_n \phi^*, \\ A_{n+1} &= -\delta_0 + A_n + B_n \mu^* + \frac{1}{2} B_n^2 \sigma_\chi^2, \end{aligned} \tag{29}$$

with $A_0 = B_0 = 0$, $\phi^* = [\phi - \sigma_\chi^2 \lambda_1]$ and $\mu^* = [\mu(1 - \phi) - \sigma_\chi^2 \lambda_0]$. It is important to highlight that A_n and B_n are implicitly functions of $\theta_1 = [\delta_0, \delta_1, \lambda_0, \lambda_1, \mu_\chi, \rho_\chi, \sigma_\chi]$.

We now discuss in details the restrictions described in [Section 5.3.1](#), and the calibration of θ_1 .

B.1 Log forward rates and χ_t

By definition, the log forward rate at time t for loans between $t + n - 1$ and $t + n$ equals

$$f_t^n = q_t^{*,n-1} - q_t^{*,n} = \underbrace{(A_{n-1} - A_n)}_{\tilde{A}_n} + \underbrace{(B_{n-1} - B_n)}_{\tilde{B}_n} \chi_t. \tag{30}$$

Given equation (30), we can express $\hat{\gamma}_0 + \hat{\gamma}'\mathbf{f}_t^{\text{model}}$ as

$$\hat{\gamma}_0 + \hat{\gamma}'\mathbf{f}_t^{\text{model}} = \hat{\gamma}_0 + \sum_{j=1}^6 \hat{\gamma}_j \tilde{A}_{4(j-1)} + \left(\sum_{j=1}^6 \hat{\gamma}_j [\tilde{B}_{4(j-1)}] \right) \chi_t.$$

Therefore, one has that $\chi_t = \hat{\gamma}_0 + \hat{\gamma}'\mathbf{f}_t^{\text{model}}$ if the following conditions hold

$$\begin{aligned} \hat{\gamma}_0 + \sum_{j=1}^6 \hat{\gamma}_j \tilde{A}_{4(j-1)} &= 0, \\ \left(\sum_{j=1}^6 \hat{\gamma}_j [\tilde{B}_{4(j-1)}] \right) &= 1. \end{aligned} \tag{31}$$

B.2 Cochrane and Piazzesi (2005) regressions

By definition, holding period excess log returns on a ZCB maturing in $n = 20$ quarters equal $rx_{t+1}^{20} = q_{t+1}^{*,19} - q_t^{*,20} + q_t^{*,1}$. Substituting the expression for log prices, we can rewrite it as

$$rx_{t+1}^{20} = \underbrace{[A_{19} + B_{19}\mu(1 - \phi) - A_{20} + A_1]}_{a_{20}^{\text{model}}} + \underbrace{[B_{19}\phi - B_{20} + B_1]}_{b_{20}^{\text{model}}} \chi_t + B_{19}\varepsilon_{\chi,t+1}. \tag{32}$$

If the restrictions described in the previous subsection hold, Equation (32) has the same form of the C-P predictive regressions we have estimated in Section 5.3.1, and it will exactly reproduce the results reported in Table 3 if the following conditions hold

$$\begin{aligned} a_{20}^{\text{model}} &= \hat{a}_{20}, \\ b_{20}^{\text{model}} &= \hat{b}_{20}, \\ B_{19}\sigma_{\chi}^2 &= \hat{\sigma}_{\eta^{20}}^2. \end{aligned} \tag{33}$$

B.3 The risk free rate

By definition, log-yields on a bond maturing next quarter equal $y_t^1 = -q_t^{*,1}$. We can express it as

$$y_t^1 = \delta_0 + \delta_1 \chi_t. \tag{34}$$

The mean and variance of y_t^1 can then be easily derived as a function of deep model parameters

$$\mathbb{E}[y_t^1] = \delta_0 + \delta_1 \mu \quad \text{var}[y_t^1] = \delta_1^2 \frac{\sigma_\chi^2}{(1 - \phi^2)}. \quad (35)$$

B.4 Calibration of θ_1

The parameters in θ_1 are chosen so that i) the conditions in (31) are satisfied, ii) the model reproduces the predictive regressions in Table 3 (the equations in (33) hold), and iii) the mean and volatility of the risk free rate in the model, defined in equation (35), equal the associated sample moments reported in Table 2.