Demographic Origins of the Startup Deficit

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PRELIMINARY

Abstract

This paper studies the causes of the declining startup rate over the past three decades. The stability of firms' lifecycle dynamics throughout this period along with the widespread nature of the declining startup rate place strong restrictions on potential explanations. We show that declines in the growth rate of the labor force, which peaks in the late 1970s, explain an important share of the startup rate decline while leaving incumbent dynamics unaffected. Using cross-sectional demographic variation we estimate a quantitatively and statistically significant labor supply growth elasticity of the startup rate, which is robust to alternative explanations. Finally we show that the equilibrium response to a permanent demographic shift in a standard Hopenhayn (1992) setting matches the steady decline in the U.S. startup rate over the last 30 years without additional frictions. This success also has implications for a debate over scale effects in endogenous growth models, suggesting the fixed costs must scale with aggregate productivity. Overall, our findings suggest that the decline in the growth rate of the working age population, reinforced by steadying female labor force participation through their general equilibrium effects on firm dynamics are an important driver of the decline in firm entry.

*We thank Rob Dent, Sam Kapon and Harry Wheeler for their superb research assistance. We are also indebted to Felipe Schwartzman for his insightful comments on a previous paper and suggestion to consider the effect of labor supply shocks on firm entry. Any opinions and conclusions expressed herein are those of the author(s) and do not necessarily represent the views of the U.S. Census Bureau, Federal Reserve Bank of New York or the Federal Reserve System. All results have been reviewed to ensure that no confidential information is disclosed.

1 Introduction

Recent empirical work has identified an unmistakable shift in U.S. firm dynamics since the late 1970s. Principally, the startup rate, measured as the share of new employer firms out of all employer firms, declined by about 30 percent from 1979 to 2007, before further declines in the Great Recession. Remarkably, this steady decline occurred relatively uniformly within geographic areas and narrow industry groups and without disturbing the lifecycle dynamics of incumbent firms.

Startups are both a vital source of job creation and productivity gains, and an important question is what explains this apparent decline in the rate of business formation. Bartelsman and Doms (2000), for example, find about 25 percent of within-industry gains in TFP occurs from new entrants in the manufacturing sector.[1] Understanding the source (or sources) of the declining startup rate is crucial to understanding whether the decline is an efficient response to technological shifts or escalating misallocation that could be eased through policy reforms. For example, fixed costs of starting or running a business may have risen from increased regulations. One difficulty with many potential explanations based on costs or technology is that ceteris paribus they impact the value of an incumbent firm and thus its lifecycle dynamics, in contrast with experience in the U.S. over this period. The widespread nature of the startup rate declines poses a similar challenge.

In this paper we propose and evaluate an alternative hypothesis for the declines in the entry rate. Demographic shifts in the growth rate of the labor supply feedback into the pace of business formation. Figure 1 shows that trend labor force growth peaks in the mid 1970s. Its subsequent decline coincides with a gradual reduction in the startup rate (smoothed here with an HP filter), which is plotted against the right axis. We show that the 30-year decline in the startup rate evident in the aggregate time series is primarily the slow convergence to a new balanced growth path for an economy with permanently lower expected labor supply growth. We then test this hypothesis using cross sectional variation in the demographic component of labor supply growth, and we find that this channel explains a large share of the declines in the startup rate.

Our paper is closely related to the emerging literature on the declining dynamism in the U.S. economy. Reedy and Strom (2012) first called attention to a decline in the aggregate entry rate of new employers. Using more disaggregated data, recent papers by Pugsley and Şahin (2014), Decker, Haltiwanger, Jarmin, and Miranda (2014), Hathaway and Litan (2014a), Gourio, Messer, and Siemer (2014) and Davis and Haltiwanger (2014) all document that declines in the entry rate are pervasive within geographic areas and relatively narrow industry aggregations. In addition to our paper, Hathaway and Litan (2014b) and Liang, Wang, and Lazear (2014) note a correlation between declining startup rates and population growth as well as business consolidation. All of these papers have also drawn attention to the relevance of the declining startup rate for the ongoing health in labor market.

The declining entry rate is a significant macroeconomic development. Although entrants are a small fraction of aggregate employment Haltiwanger, Jarmin, and Miranda (2013) have shown

[1] In a structural estimation of an endogenous growth model, Lentz and Mortensen (2008) find a similar share of aggregate TFP growth due to the entry margin.

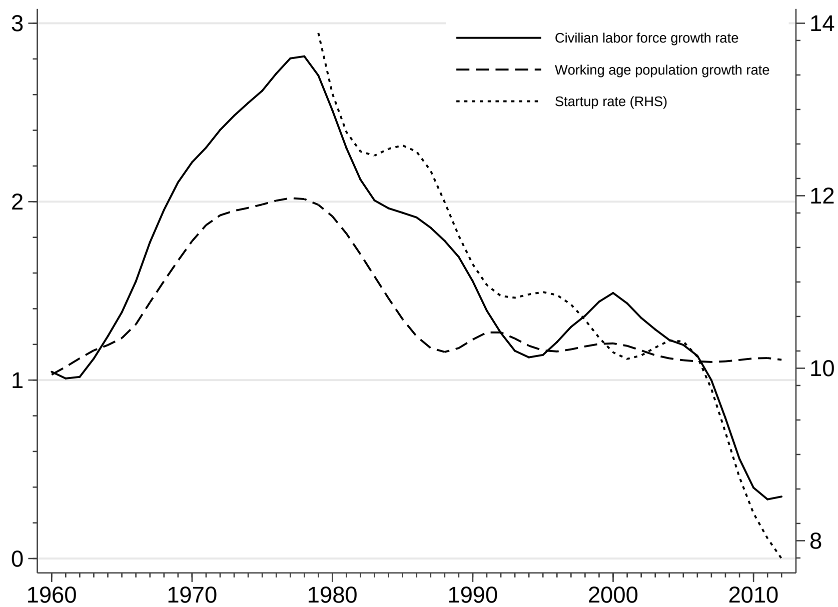


Figure 1: Trend declines of startup rate and labor supply growth coincide

Note: BLS, Annual Census Bureau population estimates, and Business Dynamics Statistics. Annual data, HP filtered with smoothing parameter 6.25. See appendix Figure A1 for unfiltered rates. Working age population is defined here as population ages 20 to 85. Civilian Labor Force is measured by the BLS for the adult (16+) civilian non institutional population. Startup rate is number of age 0 employers as share of the total number of employers within a year.

that new and young businesses are a key input into net employment creation. Taking the trend decline or "startup deficit" as given, Pugsley and Şahin (2014) show that both its direct effect on startup job creation and its indirect cumulative effect through shifts in the employer age distribution partly explain the emergence of slower employment recoveries with each business cycle. Startups are also a source of aggregate productivity gains from the advances in technology embodied in the new firms. A slowdown in business entry may portend an overall slowing of employment and labor productivity growth. Changes in laws and regulations, market concentration, education and licensing requirements, and shifts in economies of scale might discourage firm entry by creating higher barriers to enter and/or a higher fixed cost of operating. Noting this possibility, Davis and Haltiwanger (2014) among others highlight the introduction of several forms of labor market and occupation regulations in the U.S. that are a potential source of these slowdowns in the entry margin.

Without offsetting effects, these individual changes to the economy are difficult to reconcile with the empirical evidence. We show generically that changes to costs and technology will change the value of an operating firm and by extension its lifecycle dynamics. However, Pugsley and Şahin (2014) show that both in the time series and the cross section that conditional on age, expected firm dynamics have been unaffected by the net forces causing the decline in the startup rate. This incumbent stationarity of expected survival and growth along with the widespread nature of the entry declines lead us to consider changes to the supply of labor input as a potential general equilibrium source of the decline.[2] Specifically we study the role of slow moving demographic shifts in labor supply in driving the decline in startups.

There are various reasons why demographic shifts would affect business formation. Most directly, an older population might be associated with a lower rate of business formation in the economy if younger workers are more likely to engage in entrepreneurial activity. Ouimet and Zarutskie (2014) show that new and young firms hire a disproportionate share of young workers and show the share of young workers in a state is predictive of the startup rate, especially in high-tech sectors. Liang, Wang, and Lazear (2014) show that countries with older workforces have lower rates of entrepreneurship. In the U.S., the age group with the highest propensity to form businesses, ages 35-54, grows over this period so this is unlikely to fully explain the decline in startups. We are interested in a different channel.

Changes in the growth rate of the working age population and also diminishing increases in female labor force participation affect the expected expansion in labor supply and could have important effects on business formation through a general equilibrium channel. Positive shocks to the labor supply put downward pressure on wages and create incentives for incumbent firms to expand, but they also create opportunities for potential entrants. Any effect on the entrant share will depend on how the shocks to the labor supply are accommodated by expanding incumbents and new firms. A lesson from models of firm dynamics, starting with Hopenhayn (1992), is that in the long run, the free entry of new firms make labor demand infinitely elastic. Ultimately, shifts in labor supply are absorbed not at the intensive margin by incumbent firms, but at the extensive margin by adjusting the quantity of entrants. Crucially, labor supply changes have no direct effect on the value of an incumbent firm, except indirectly through transient effects on the real wage.

We consider the balanced growth path of a standard Hopenhayn and Rogerson (1993) economy where population and thus labor supply grows at some rate η. We show that as population growth slows, the entry rate along the balanced growth path must fall. Even without additional frictions, the long-run properties of this balanced growth path, as well as the convergence to a new balanced growth path given a permanent demographic shift in population growth characterize well the firm dynamics in the U.S. since the 1980s.

The success of the model also has implications for endogenous growth. With growth driven by the introduction of new varieties, if fixed costs are constant, larger economies will grow faster. This "scale effect" vanishes when costs are in units of required labor, since advances in productivity also increase real wages. The units of these costs is an ongoing debate. The balanced growth path of our model with entry and exit requires that costs scale proportionally with changes in productivity. One mechanism to deliver this automatically is if fixed costs are specified in terms of required labor. To the extent the model is successful in describing the patterns in the U.S. it lends further strength to specifying costs in labor. This pattern reinforces the findings of Bollard, Li, and Klenow (2016) for the manufacturing sector.

In the U.S. the declining startup rate coincides with an abrupt decline in the growth rate of

[2] We thank Felipe Schwartzman for his insightful suggestion to consider this margin

the labor force in the late 1970s. Driven in part by a decline in fertility many years prior and compounded by diminishing increases in female participation, this decline put upward pressure on wages. In response, incumbents may hire fewer workers and the declines in profitability of potential firms reduces the gains from entry. As the flow of entrants eventually adjusts to the reduction in labor market growth, any pressure on wages is relieved and incumbent dynamics such as age-specific survival rates, average firm sizes, and employment growth rates are completely unaffected. Figure 1 shows the change in the growth rate of the labor force and the slow transition to a startup share consistent with the long run reduction in labor market growth. While the strong correlation between labor force growth and the startup rate in the aggregate time series makes the demographic channel we describe a compelling explanation, we turn to an alternative source of variation to validate this hypothesis.

We use cross sectional demographic variation in labor supply growth to measure its effects on firm dynamics. In particular using cross state and industry data pooled over time, we estimate a linear model of the startup rate and several incumbent margins on two measures of labor force growth, controlling for state, time, and industry effects. To generate variation in labor force growth from fully anticipated demographic shifts we instrument with 20-year lags of each state's fertility rate. The goal is to see whether changes in the growth rate of the working age population and lower frequency component of the civilian labor force growth, our two measures, predicted by purely demographic forces have an effect on firm dynamics.

This strategy relies on two identifying assumptions. The first and most important is the exclusion restriction that conditional on state and time effects, whatever determines the cross sectional variation in the fertility has no other long lasting effects that would still affect business dynamics 20 years later. We argue this is likely to be the case. Second, and less important, is that the mobility costs are large enough to prevent geographic mobility from completely equating the real wage across these segmented markets. To the extent this is not true, our estimates will understate the effect of demographics on startups and incumbent dynamics. One side-effect of the fertility instrument is that it also changes the age composition of the labor force. We use an alternative instrument for labor force growth that relies on historical migration patterns (as in Altonji and Card, 1991 and Card, 2001) that does not systematically change the age composition and find nearly identical effects. As an overidentifying restriction, this instrument also validates our fertility instrument. The effects we estimate follow from changes in the growth rate of the labor supply and not its composition.

We find that the demographic changes have a large effect on the startup rate. We estimate a startup rate semi-elasticity of labor supply growth of roughly 1 to 1.5. Given these estimates, the declines in the working age population growth and civilian labor force growth over this period in the aggregate time series and plotted in Figure 1, the demographic shifts explain an important share of the decline in the startup rate. Further, we find that the demographic shifts have little effect on incumbent dynamics. A demographically induced increase in the growth rate of labor supply causes a short run increase in incumbent firm size, which diminishes over longer horizons. This is consistent with the transient adjustment of incumbent firms as the entry margin fully adjusts to its long run levels. Although these patterns are consistent with a broader class of models, we show in a calibrated version of the Hopenhayn (1992) model set in general equilibrium with growth, the same gradual adjustment to a fully anticipated shift in the growth rate of the labor supply.

Since the elasticities are identified only from the cross section, we use the cross sectional regressions as an auxiliary model to estimate our structural model. In the model a demographic shock is a perfect foresight revelation of a path of future population growth. Demographics are slow moving, and a decline in the growth rate of the working age population in the 1980s was forecastable as early as the end of the baby boom. Because entry is costly and the shifts to labor supply are anticipated, changes in entry are smoothed over time, and the estimated model delivers a "macro elasticity" of entry to a demographic shock that is larger than the one estimated in the cross section. For a range of estimation strategies the structural model explains anywhere from 50 to 100 percent of the decline in entry since 1980s.

Overall, our results show that a large fraction of the declining startup rate is consistent with the demographic shifts over this same period. At first, this may be a surprising result since changes in the entry costs or other technological explanations would have a more direct effect on entry. However, the stability of the incumbent survival and growth margins combined with the widespread nature of the declines in entry point to an alternative channel. Generically, shifts in costs and technology affect incumbent dynamics through their effects on the value of a firm, whereas in standard formulations labor supply changes do not.[3] In the long run with free entry the labor supply shifts are completely absorbed by the entry margin. In the U.S. the aggregate time series and cross state evidence strongly supports this explanation of the declining startup rate.

The paper is organized as follows. Section 2 presents a simple framework to evaluate potential explanations for the decline in business formation in the spirit of Hopenhayn (1992) and Hopenhayn and Rogerson (1993) and shows that aggregate time series evidence is consistent with our hypothesis that a negative labor supply shock is the main driver of the decline in firm entry. Section 3 explains our data and uses cross-state variation in labor supply growth to evaluate the labor supply channel and finds strong evidence in favor of this explanation. Section 4 presents various robustness experiments. Section 5 concludes.

2 Time series evidence and evaluating potential explanations

In this section we document the widespread decline in business formation and present a simple firm dynamics framework. We then use this framework to evaluate alternative explanations for the decline in firm entry using aggregate time series data on firm dynamics. First we describe the data we use to characterize the decline in the startup rate and test our demographic-based hypothesis.

[3] With a frictional labor market, decreasing returns in the aggregate matching function would be one example where changes in the labor supply would effect the value of the firm through their effects on the match surplus.

2.1 Data description

To analyze the determinants of the declining startup rate we combine historical data on both firms and demographics in the U.S.

Firm level data For firms, we use firm-level data from the U.S. Census Longitudinal Business Database (LBD) and its public use data product the Business Dynamics Statistics (BDS). The LBD is a longitudinally-linked database covering nearly all private-sector establishments in the U.S with paid employees. Using additional information on the organizational structure, the Census is able to identify firms, which may span multiple establishments. This is an important detail since we are interested in true firm startups rather than new establishments of an existing firm. The data contain at the establishment and firm level, annual measures of employment (for the week containing March 12) from 1976 to 2012.[4]

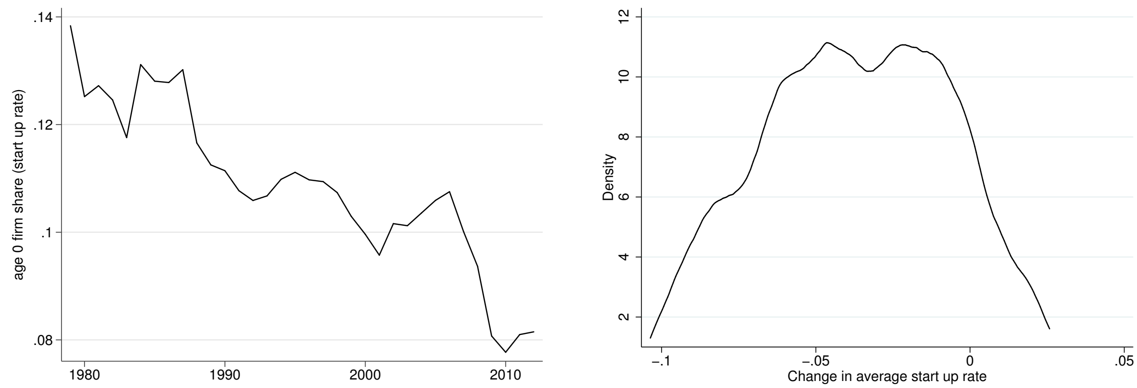
We use the data to classify firms by location, industry, size, and age. Firm age is assigned to new firms in the database as the age of a firm's oldest establishment. Establishments are assigned age 0 in the year they first report positive employment. Both firms and establishments age naturally after the initial assignment. Done in this way, we ensure that age 0 firms, which we refer to as "startups", are truly new entrants. Other characteristics are relatively straightforward to measure and described in the data appendix.

For the years 1979 to 2012 we measure several margins of firm dynamics by firm age group, size group and also by industry and state.[5] First is the startup rate, which we measure as the number of age 0 firms as a fraction of the total number of firms. Figure 1 plots this measure for the U.S. overall. We measure average employment for both incumbent age groups $N_t^{1+}$and startups N_t^0 for each state and industry. We measure firm survival x_t for young and mature incumbents as one minus the fraction of firms in that cohort the previous year that exit, and the conditional growth rate for the cohort as the increase in average employment $1 + n_t = N_t^a / N_{t-1}^{a-1}$. Defined this way, overall growth rate of employment within the cohort, or unconditional growth rate, is the product of the survival rate and the conditonal growth rate $x_t\,(1 + n_t)$.

Demographic data Our demographic measures include national- and state-level measures of the working age population and the civilian labor force. We use population data from the Census Bureau's decennial census and annual American Community Survey. We use the Current Population Survey (CPS) to measure the size of the civilian labor force. We define the working age population as the non-institutional population between the ages of 20 and 65 and the civilian labor force as the non-institutional population 16 years or older that is either employed or looking for work. To instrument for cross state variation in the growth rate of the working age population and labor force, we use lagged measures of fertility within the state. We construct state-level fertility rates

[4]For a detailed description of the LBD, see Jarmin and Miranda (2002).

[5]Firm age is left censored in 1977, because this is the first year we can measure age 0 firms. We do not use years 1977 and 1978 because of difficulties in measuring entry in the early years of the database. We thank Rob Shimer for this suggestion.



(a) Startup rates

(b) Change in within-group startup rates

Figure 2: Declines of startup rate in the aggregate and by state-industry pairs

Note: Business Dynamics Statistics and Longitudinal Business Database. Startup rate is number of age 0 employers as share of the total number of employers within a year within BDS. The left panel displays the aggregate startup rate and the right panel shows the density plot of the change in this measure across groups defined by state and 4-digit industry. The change is computed by taking the difference between the average startup rate in the period 1980-1984 and 2003-2007.

from historical county level data on the number of live births from the Department of Health and Human Services.

2.2 The decline in entry

Figure 2a shows the startup rate, which is the number of newborn firms as a fraction of the overall stock of firms, for the period 1979 to 2012. The startup rate has declined steadily from an average of roughly 13 percent in the early 1980s to around 10 percent before the Great Recession and eventually to 8 percent by 2012. Moreover, this decline is not concentrated in a particular set of states or industries, but is a widespread phenomenon. To exhibit this across the board decline, we compute the change in startup rates for each state 4-digit industry cell during the sample period, by taking the difference between the average startup rate in the period 1980-1984 and 2003-2007. Figure 2b displays the density of these differences. Startup rates fell in about 85 percent of state-industry pairs. If the differences were computed over a period that includes the Great Recession this share increases to about 95 percent. This across the board decline suggests that it should be a common factor causing a lower pace of business formation in all sectors.

2.3 Understanding the decline in entry

There are several potential explanations that might have caused the decline in firm entry. One hypothesis is that the U.S. economy faced serious impediments over this episode, causing a decline in labor market fluidity. Changes in regulations, market concentration, education and licensing requirements, and shifts in economies of scale might discourage firm entry by creating higher barriers to enter and/or a higher fixed cost of operating. A recent literature including Davis and Haltiwanger (2014) emphasize several forms of labor market and occupation regulations as potentially important sources of the slowdowns in the entry margin. Yet another possibility is increased uncertainty about demand or policy that has discouraged entrepreneurial activity (Davis and Haltiwanger (2014)).

A common implication of such factors is a change in the lifecycle dynamics of incumbent firms, which we show to be in contrast with the data. This feature is simply illustrated in the canonical Hopenhayn (1992) equilibrium model of firm entry and exit, but it applies generally to wider variety of environments. Changes in the entry or operating costs affect the value of the firm, which in turn alters equilibrium prices, and ultimately the exit and growth behavior of incumbent firms. To see this, consider the simple environment in Hopenhayn (1992) in which firms have to pay a fixed cost pc_e, where p is the price of output (relative to wages), to enter the economy. Upon entry, they draw a random initial productivity, s, from an exogenous distribution G. Staying in business requires the payment of a fixed cost of c_f, so that firms exit when their expected value falls below c_f. Let $V(s,\psi)$ denote the value of a firm with productivity s, where ψ is a vector containing parameters entering the firm's production and exit decisions. The parameter c_f is one example.[6] The free entry condition in this model is given by:

$$pc_e = \int V(s,\psi)\, dG(s)\,. \tag{1}$$

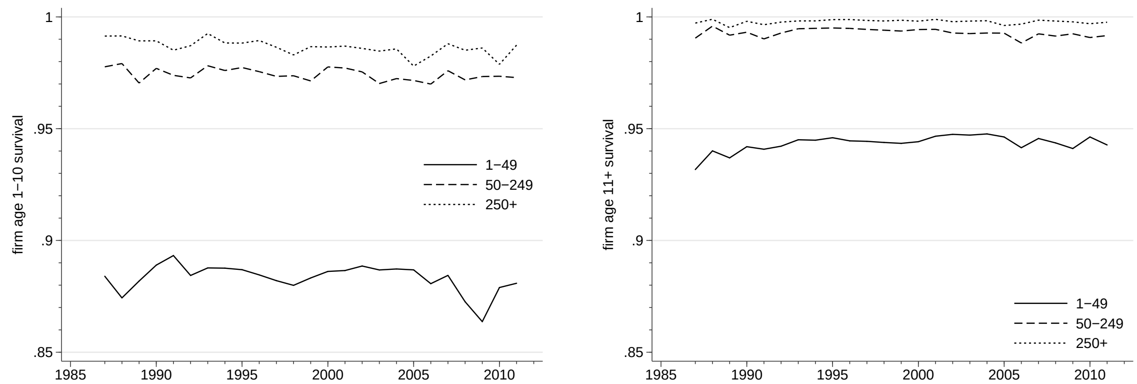
This equation equates the cost of entry to the expected value of a new firm. In this framework, one can think of changes in regulations, market concentration, or financial frictions as innovations to entry costs c_e or parameters ψ. Equation (1) implies that such factors affect equilibrium prices, thereby altering the value of a firm at any level of productivity. This change in firm value affects the exit behavior as incumbent firms remain in business only if the value of doing so exceeds fixed operating costs c_f. Consequently, the quality of incumbent firms is different in equilibrium, implying a different growth rate for incumbents. In short, by changing the value of remaining in business, these factors affect the behavior of incumbents.[7]

To study the plausibility of the explanations mentioned above, we look at the survival rates and employment growth rates of incumbent firms for several firm size categories. Figures 3 and 4 plot the survival and growth rates of incumbents by three size categories, respectively: 1–49, 50–249 and 250+. We present each rate separately for young firms (left panel) ages 1 to 10 and mature firms (right panel) ages 11+ since the declining startup rate has changed the age composition of each size group.[8] While there are notable business cycle fluctuations in these rates, overall they

[6] The contents of ψ depends on the details of the model and may include firm size, the standard deviation of idiosyncratic shocks to firm-level productivity, and so on.

[7] This logic carries forward to more factors than fixed costs and would hold for any factor affecting firm value such as uncertainty shocks or financial frictions. In fact, Ates and Saffie (2014) argue that financial frictions hurt the entry margin but improve the quality of incumbents, making them *fewer but better*.

[8] Haltiwanger, Jarmin, and Miranda (2013) show that even conditional on size, firm age has a significant effect on firm dynamics.



(a) Young Firms

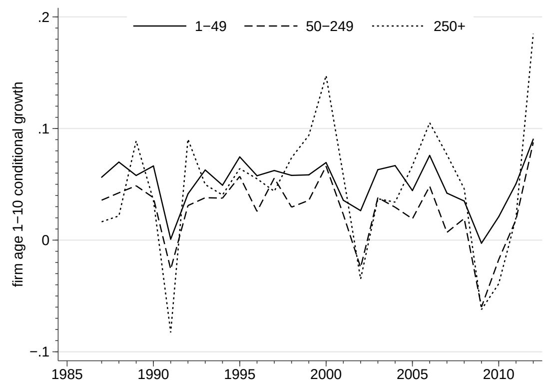
(b) Mature Firms

Figure 3: Incumbent survival and conditional growth rates by firm size from 1980 to 2012

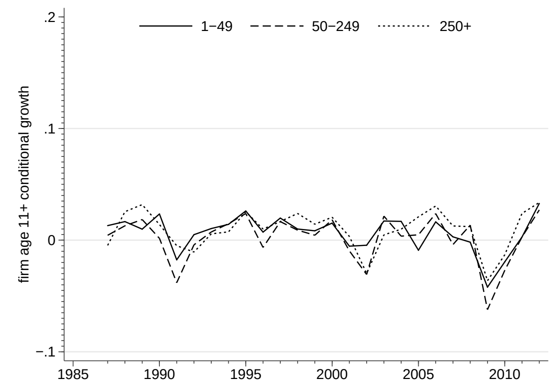
Note: Business Dynamics Statistics. Survival rate is the number of incumbent firms within each size group as a fraction of the number of firms in the size group cohort in the previous year. Year 2012 is omitted because of difficulties measuring firm exits in the final year.

have remained remarkably stable amid a period of substantial declines in the entry rate. Pugsley and Şahin (2014) show that conditional on firm age, there are no trends in survival or growth of incumbent firms using both aggregate time-series and cross-sectional evidence. Thus, the evidence on incumbent behavior is inconsistent with the explanations that imply changes in the value of incumbent firms, such as increase fixed costs. We should note that it is not possible to rule out these explanations altogether just by the time series evidence, because a combination of various factors may still be consistent with a lower startup rate and a stable incumbent behavior.

This paper emphasizes a different factor—decline in the growth rate of labor supply over this episode—and argues that this factor is quantitatively very important. When adjusted to include population growth, a stationary equilibrium in per-capita terms depends on the population growth rate. Economies with a quickly growing population, but no change in firm lifecycle dynamics, require a larger share of entrants to keep pace with the population growth. Free entry implies that shifts in labor supply are absorbed in the long run by the entry margin and thus have no lasting effects on equilibrium prices or incumbent behavior. This result follows from (1) and holds precisely because labor supply has no direct effect on firm value. Thus, labor supply shifts as a driver of declining entry are also consistent with the time series evidence on stable incumbent behavior. Figure 1 shows that the long-run decline in startups has coincided with a shift in two measures of labor supply. The growth rate of the labor force has declined from around two percent to about 0.8 percent. Similarly, the growth rate of the working age population fell from over two percent to a low of just over one percent. These changes in growth rates if permanent have substantial long run effects on the equilibrium share of entrants or startup rate.



(a) Young Firms



(b) Mature Firms

Figure 4: Incumbent conditional growth rates by firm size from 1987 to 2012

Note: Business Dynamics Statistics. Conditional growth rate is the growth rate of average employment size within each size group (excludes all startups) relative to the average employment of the size group cohort in the previous year.

3 Model

The model closely resembles the firm dynamics model in Hopenhayn (1992). The economy is populated by a representative household of size H_t that grows at a constant rate *eta* and a continuum of potential firms that produce a homogeneous good. Time is discrete. The household has preferences over per-capita leisure l_t and consumption c_t. The labor and output markets are competitive with prices w_t and p_t. We choose labor as the numeraire so that $w_t = 1$. We first study the balanced growth path of this economy with a constant population growth rate *eta* and later consider the transitional dynamics associated with unanticipated changes in *eta*.

3.1 Firms

All firms have production technology

$$f\left(s_t, n_t\right) = s_t n_t^{\theta} - c_f \, . \tag{2}$$

They observe s_t, choose n_t and produce, and then decide whether or not to continue operating given their expectations of s_{t+1}. Idiosyncratic productivity evolves as an AR(1) in logs

$$\log s_{t+1} = a + \rho \log s_t + \sigma \varepsilon_{t+1} \, . \tag{3}$$

Given output price p_t in terms of wages (so that $w_t = 1$), the value of an operating firm solves the following Bellman equation

$$W_t\left(s_t\right) = \max_{n_t \geq 0} \left\{ p_t s_t n_t^{\theta} - p_t c_f - n_t + \frac{1}{1 + r_t} \max_{X_t \in \{0,1\}} \left\{ E_t\left[W_{t+1}\left(s_{t+1}\right)\right], 0 \right\} \right\} .$$

Let $n_t(s)$ be the optimal policy for n. In this case with no adjustment costs n solves a static profit maximization problem

$$n_t(s) = \underset{n}{\operatorname{argmax}} \left\{ p_t s n^\theta - p_t c_f - n \right\} ,$$

with solution

$$n = (\theta p_t s)^{\frac{1}{1-\theta}} .$$

Gross of fixed costs, profit is

$$\begin{aligned} \pi_t(s) &= p_t s (\theta p_t s)^{\frac{\theta}{1-\theta}} - (\theta p_t s)^{\frac{1}{1-\theta}} \\ &= (\theta p_t s)^{\frac{1}{1-\theta}} \left(\frac{1-\theta}{\theta} \right) \\ &= (\chi p_t s)^{\frac{1}{1-\theta}} \end{aligned}$$

where $\chi \equiv \theta^\theta (1-\theta)^{1-\theta}$. Given the optimal choice of n the value of a firm simplifies to

$$W_t(s_t) = (\chi p_t s_t)^{\frac{1}{1-\theta}} - p_t c_f + \frac{1}{1+r_t} \max_{X_t \in \{0,1\}} \{E_t W_{t+1}(s_{t+1}), 0\} . \tag{4}$$

Let $X_t(s)$ be the optimal exit decision, which is 0 if the firm decides to exit and 1 otherwise. Entering firms draw initial productivity from a distribution ν so that the expected value of an entering firm gross of entry costs is

$$W_t^e = \int W_t(s)\, \nu(ds) .$$

The cost of entry is c_e denoted in units of output or $p_t c_e$ in terms of the numeraire.

Firm Distribution Let $\mu_t(S)$ be the measure of firms active in period t (after entry, but before exit) over Borel sets of productivity levels S.[9] Let $F(s'|s)$ be the conditional distribution of s' given s implied by the law of motion for $\log s$. Let M_t be the measure of new entrants in period t that draw from ν. The measure over all firms follows a law of motion

$$\mu_{t+1}(S') = \int (1 - X_t(s))\, \mu_t(s)\, F(S'|ds) + M_{t+1} \nu(S') . \tag{5}$$

The RHS defines a linear operator that is homogeneous in μ and M.

[9] This is a slight deviation from HR, where it appears μ_t is the measure of only incumbents in period t. Letting μ_t be the measure over all firms is consistent with Hopenhayn (1992), and will make the notation in this note easier to follow. So μ_t is the measure of all firms (both entrants and incumbents) at the beginning of the period, before firms make their end of period exit decisions.

3.2 Households

A representative household of size H_t and growing at rate η_t has preferences over per-capita consumption c_t and leisure l_t ordered by

$$U = \sum_{t=0}^{\infty} H_t \beta^t \left(\log C_t + A \frac{l_t^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} \right) . \tag{6}$$

Parameter ψ indexes the Frisch elasticity of household labor supply. When $\psi \to \infty$ preferences match Hopenhayn and Rogerson, and when $\psi \to 1$ preferences are Cobb-Douglas. The household chooses c_t, $n_t = 1 - l_t$, and b_t to maximize (6) subject to its budget constraint

$$p_t c_t + \frac{b_t}{1+r_t} = n_t + \frac{\Pi_t}{H_t} + \frac{b_{t-1}}{1+\eta_t}$$

where Π_t / H_t is the per-capita dividend issued by firms.

With multiplier λ_t, optimal choices of c_t, n_t, and $b_t$require

$$\begin{aligned} \frac{1}{c_t} &= \lambda_t p_t \\ A \left(1-n_t\right)^{\frac{-1}{\psi}} &= \lambda_t \\ \frac{\lambda_t}{1+r_t} &= \beta \lambda_{t+1} \end{aligned}$$

These require that consumption and labor satisfy

$$c_t A \left(1-n_t\right)^{\frac{-1}{\psi}} = \frac{1}{p_t} \tag{7}$$

$$\frac{1}{c_t} = \beta \left(1+r_t\right) \frac{p_t}{p_{t+1}} \frac{1}{c_{t+1}} \tag{8}$$

The real wage is $1/p_t$. The Frisch elasticity, which holds marginal utility of income constant is

$$\begin{aligned} \log c_t + \log A - \frac{1}{\psi} \log \left(1-n_t\right) &= \log \frac{1}{p_t} \\ \frac{1}{\psi} \frac{1}{1-n_t} dn_t &= d \log \frac{1}{p_t} \\ \frac{d \log n_t}{d \log \left(1/p_t\right)} &= \psi \frac{1-n_t}{n_t} . \end{aligned}$$

With $n_t \approx 1/3$ then the Frisch elasticity is approximately 2ψ.

The MRS between consumption and labor along with the budget constraint require

$$\frac{1}{A} \left(1-n_t\right)^{\frac{1}{\psi}} + \frac{b_t}{1+r_t} = n_t + \frac{\Pi_t}{H_t} + \frac{b_{t-1}}{1+\eta_t} , \tag{9}$$

which even when $b_t = 0$ does not in general have a closed form solution for n_t for $\psi > 0$. If n_t^* solves equation (9), we can define

$$N_t^s \equiv \max\left\{\min\left\{1, n_t^*\right\}, 0\right\} \,.$$

There are at least two special cases where n_t^* can be computed in closed form.

Quasi-linear preferences When $\psi \to \infty$ preferences are linear in labor and equivalent to the preferences in Hopenhayn and Rogerson. In this case, and using the equilibrium condition that bonds are in zero net supply then

$$c_t = \frac{1}{p_t A}, \quad n_t = \frac{1}{A} - \frac{\Pi_t}{H_t}, \quad \frac{1}{1+r_t} = \beta \,.$$

Quasi-linearity and the choice of bonds denoted in units of labor ensures that the real interest rate is constant.

Cobb-Douglas preferences When $\psi \to 1$ preferences are Cobb-Douglas. In this case and again using the equilibrium condition that $b_t = 0$, consumption and leisure are constant expenditure shares of total income $1 + \frac{\Pi_t}{H_t}$

$$c_t = \frac{1}{p_t}\frac{1}{1+A}\left(1 + \frac{\Pi_t}{H_t}\right), \quad n_t = 1 - \frac{A}{1+A}\left(1 + \frac{\Pi_t}{H_t}\right), \quad \frac{1}{1+r_t} = \beta\frac{1+\frac{\Pi_t}{H_t}}{1+\frac{\Pi_{t+1}}{H_{t+1}}} \,.$$

The real interest rate fluctuates with changes in the growth rate of the aggregate dividend.

3.3 Aggregation

Total labor demand is

$$N_t^d \equiv \int \left(\theta p_t s\right)^{\frac{1}{1-\theta}} \mu_t\left(ds\right)$$

Total output is

$$Y_t \equiv \int \left(s\left(\theta p_t s\right)^{\frac{\theta}{1-\theta}} - c_f\right) \mu_t\left(ds\right) \,.$$

Total profits are

$$\begin{aligned}\Pi_t &\equiv \int \left(\left(\chi p_t s_t\right)^{\frac{1}{1-\theta}} - p_t c_f\right) \mu_t\left(ds\right) - M_t p_t c_e \\ &= p_t Y_t - N_t^d - M_t p_t c_e\end{aligned} \tag{10}$$

To simplify the algebra, note that the aggregates N and Y define linear operators on μ

$$N_t^d = \mathbf{N}_t \mu_t \quad Y_t = \mathbf{Y}_t \mu_t$$

and aggregate profits defines a linear operator homogenous in μ and M

$$\Pi_t = (p\mathbf{Y}_t - \mathbf{N}_t)\,\mu_t - p_t c_e M_t$$

The resource constraint is

$$H_t c_t + M_t c_e = Y_t\,,$$

which would also follow from aggregating over the representative household's budget constraint after substituting in for profits and labor market clearing

$$p_t C_t = N_t^s + \Pi_t\,.$$

3.4 Recursive Stationary Equilibrium with No Population Growth

Look for an equilibrium with constant population $H_t = 1$, constant prices $p_t = p$ and $r_t = r$, and with entry/exit, i.e., $M > 0$ where the measure of firms μ is stationary.

Step 1 The value of an incumbent firm is

$$W(s) = (\chi p s)^{\frac{1}{1-\theta}} - p c_f + \frac{1}{1+r} \max\left\{\int W(s')\,F(ds'|s)\,,0\right\}$$

The solution to the dynamic program generates stationary policy function $X(s) \in \{0,1\}$. With $M > 0$ the expected value of an entrant is equal to the entry costs

$$p c_e = W^e\,. \tag{11}$$

We can locate p by finding a fixed point of $W^e(p)/c_e$. In practice we can fix $p = 1$ and solve for the entry costs consistent with this price.

Step 2 Given p and the optimal policy rules, we locate a stationary measure μ that satisfies the law of motion in equation (5)

$$\mu(S') = \int (1 - X(s))\,F(S'|ds)\,\mu(s) + M\nu(S')\,.$$

The RHS defines a linear operator homogenous in μ and M, so that the solution for μ is indexed by M. Define $\hat{\mu}$ as the solution when $M = 1$, then the homogeneity implies that $\mu = \hat{\mu}M$.[10]

[10] With a discrete number of states: (i) let column vector X be the optimal exit policy over each state; (2) let P be a transition matrix P , and (iii) let column vector ν be the p.m.f. of the initial distribution of entrants, then

$$\hat{\mu} = P' \operatorname{diag}(1 - X)\,\hat{\mu} + \nu$$

Step 3 Finally, given p and X, then satisfying labor market clearing (or equivalently the resource constraint) determines M. When $\psi \to \infty$ then

$$\begin{aligned}\frac{1}{pA} + Mc_e &= Y \\ \frac{1}{pA} + Mc_e &= \mathbf{Y}\hat{\mu}M \\ M &= \left[pA\left(\mathbf{Y}\hat{\mu} - c_e\right)\right]^{-1}\end{aligned}$$

The second equality follows from $\mu = \hat{\mu}M$.

When $\psi \to 1$ then using the resource constraint

$$\begin{aligned}\frac{1}{1+A}\frac{1}{p}\left(1+\Pi\right) + Mc_e &= Y \\ \frac{1}{1+A}\frac{1}{p}\left(1 + \left(p\mathbf{Y} - \mathbf{N}\right)\hat{\mu}M - pc_eM\right) + Mc_e &= \mathbf{Y}\hat{\mu}M \\ 1 + \left(p\mathbf{Y} - \mathbf{N}\right)\hat{\mu}M + Apc_eM &= \left(1+A\right)p\mathbf{Y}\hat{\mu}M \\ M &= \left[\left(Ap\mathbf{Y} + \mathbf{N}\right)\hat{\mu} - Apc_e\right]^{-1}.\end{aligned}$$

For arbitrary $\psi > 0$ then the budget constraint requires

$$pc + \left(pcA\right)^{\psi} = 1 + \Pi$$

and along with the requirement from the resource constraint that $c = Y - Mc_e$ then M must solve

$$\begin{aligned}p\left(\mathbf{Y}\hat{\mu} - c_e\right)M + \left(pA\left(\mathbf{Y}\hat{\mu} - Mc_e\right)\right)^{\psi} &= 1 + \left(p\mathbf{Y} - \mathbf{N}\right)\hat{\mu}M - pc_eM \\ \mathbf{N}\hat{\mu}M + \left(pA\left(\mathbf{Y}\hat{\mu} - c_e\right)M\right)^{\psi} &= 1\end{aligned}$$

3.5 Balanced Growth Path Restriction

If H_t grows at rate η_t the problem is non stationary. We want to find for a constant population growth rate η a balanced growth path where per-capita consumption and labor and prices p and r are constant. The incumbent firm's problem is unchanged with population growth. If prices are constant, then the exit policy X will also be constant. To have a balanced growth path with constant prices p, per-capita consumption c and labor supply n, we need Π_t to grow at rate η.

Profits from (10) are

$$\Pi_t \equiv \int \left(\left(\chi p s_t\right)^{\frac{1}{1-\theta}} - pc_f\right)\mu_t\left(ds\right) - M_t pc_e\,,$$

so

$$\begin{aligned}\hat{\mu}\left(I - P'\,\mathrm{diag}\left(1-X\right)\right) &= \nu \\ \hat{\mu} &= \left(I - P'\,\mathrm{diag}\left(1-X\right)\right)^{-1}\nu\,.\end{aligned}$$

and

$$\Pi_{t+1} = \int \left((\chi p s_{t+1})^{\frac{1}{1-\theta}} - p c_f \right) \mu_{t+1}(ds) - M_{t+1} p c_e \, .$$

If $M_{t+1} = M_t (1+\eta)$ then $\Pi_{t+1} = \Pi_t (1+\eta)$ if and only if

$$\int \left((\chi p s_{t+1})^{\frac{1}{1-\theta}} - p c_f \right) \mu_{t+1}(ds) = (1+\eta) \int \left((\chi p s_t)^{\frac{1}{1-\theta}} - p c_f \right) \mu_t(ds) \, . \tag{12}$$

Note that $\mu_{t+1} = (1+\eta)\mu_t$ violates the law of motion for μ equation (5), but the profit restriction still holds if the shares are constant and the total measure of firms grows at rate η

$$\int \left((\chi p s_{t+1})^{\frac{1}{1-\theta}} - p c_f \right) \frac{\mu_{t+1}(ds)}{\int d\mu_{t+1}} = (1+\eta) \int \left((\chi p s_t)^{\frac{1}{1-\theta}} - p c_f \right) \frac{\mu_t(ds)}{\int d\mu_t} \frac{\int d\mu_t}{\int d\mu_{t+1}} \, ,$$

i.e., if

$$\frac{\mu_{t+1}}{\int d\mu_{t+1}} = \frac{\mu_t}{\int d\mu_t} \quad \text{and} \quad \frac{\int d\mu_t}{\int d\mu_{t+1}} = \frac{1}{1+\eta} \, .$$

A weaker condition to satisfy balanced growth restriction (12) is that a normalized measure of firms is constant, where the normalizing constant grows at rate η. The above example is normalizing by the total measure of firms. Another example is normalizing by the measure of the population.

3.5.1 Stationary per capita measure

Define measure of firms normalized by total population

$$\bar{\mu}_t \equiv \frac{\mu_t}{H_t}$$

and the entrants per capita

$$\bar{M}_t = \frac{M_t}{H_t} \, .$$

Reformulate the law of motion (5) in terms $\bar{\mu}$ and $\bar{M}$

$$\begin{aligned} \frac{\mu_{t+1}}{H_{t+1}} (S') \frac{H_{t+1}}{H_t} &= \int (1 - X_t(s)) F(S'|s) \frac{\mu_t(ds)}{H_t} + \frac{M_{t+1}}{H_{t+1}} \frac{H_{t+1}}{H_t} \nu(S') \\ \bar{\mu}_{t+1}(S') &= \int (1 - X_t(s)) F(S'|s) \frac{\bar{\mu}_t(ds)}{1+\eta} + \bar{M}_{t+1} \nu(S') \, . \end{aligned}$$

Look for a solution where

$$\bar{\mu} = \int (1 - X(s)) F \frac{\bar{\mu}}{1+\eta} + \bar{M}\nu \, .$$

As in the case with no population growth the RHS defines a linear operator $\mathbf{T}(\bar{\mu}, \bar{M}; \eta)$ homogeneous in $\bar{\mu}$ and $\bar{M}$, so the law of motion can be written as

$$\bar{\mu} = \mathbf{T}(\bar{\mu}, \bar{M}; \eta) = \mathbf{T}\left(\frac{\bar{\mu}}{\bar{M}}, 1 \right) \bar{M} \, .$$

Letting $\hat{\bar{\mu}} = \bar{\mu}/\bar{M}$ then

$$\hat{\bar{\mu}} = \mathbf{T}\left(\hat{\bar{\mu}}, 1; \eta\right)$$

has a unique fixed point.

Closing the model We close the model with the resource constraint (or clearing the labor market) determining $\bar{M}$. The solution is exactly the same as the case when $H = 1$. The only difference is the new stationary measure $\bar{\mu}$

For example when $\psi \to 1$ using the resource constraint again in terms of shares

$$\begin{aligned}
\frac{1}{p_t}\frac{1}{1+A}\left(H_t + \Pi_t\right) + M_t c_e &= Y_t \\
\frac{1}{1+A}\left(H_t + p_t \mathbf{Y}_t \mu_t - \mathbf{N}_t \mu_t - p_t c_e M\right) + M_t c_e &= \mathbf{Y}_t \mu_t \\
1 + \left(p\mathbf{Y} - \mathbf{N}\right)\bar{\mu} + A p c_e \bar{M} &= \left(1+A\right) p \mathbf{Y} \bar{\mu} \\
\bar{M} &= \left(Ap\left(\mathbf{Y}\hat{\bar{\mu}} - c_e\right) + \mathbf{N}\hat{\bar{\mu}}\right)^{-1} ,
\end{aligned}$$

where the third line uses the fact that once normalized by H_t the variables are constant. Now given $\bar{M}$,

$$\bar{\mu} = \hat{\bar{\mu}}\bar{M} ,$$

closing the model.[11]

If $\psi \to \infty$, then using the resource constraint

$$\begin{aligned}
H_t c + M_t c_e &= Y_t \\
\frac{H_t}{Ap} + M_t c_e &= \mathbf{Y}_t \mu_t \\
\frac{1}{Ap} + \tilde{M} c_e &= \mathbf{Y}\bar{\mu} .
\end{aligned}$$

and

$$\bar{M} = \frac{1}{Ap}\left(\mathbf{Y}\hat{\bar{\mu}} - c_e\right)^{-1} .$$

Here it's clear the role the Frisch elasticity plays in determining the long run measure of entrants

[11] As above, with a discrete number of states: (i) let column vector X be the optimal exit policy over each state; (2) let P be a transition matrix P , and (iii) let column vector ν be the p.m.f. of the initial distribution of entrants, then

$$\bar{\mu} = P' \operatorname{diag}\left(1 - X\right)' \frac{\bar{\mu}}{1+\eta} + \bar{M}\nu$$

so

$$\bar{\mu} = \hat{\bar{\mu}}\bar{M}$$

where $\hat{\bar{\mu}}$ is a vector defined as

$$\hat{\bar{\mu}} \equiv \left(I - \frac{P'}{1+\eta} \operatorname{diag}\left(1 - X\right)'\right)^{-1} v .$$

per capita $\bar{M}$. The startup rate, which is

$$SR_\infty = \frac{\bar{M}}{\int d\bar{\mu}} = \frac{1}{\int d\hat{\bar{\mu}}}$$

is unaffected by ψ.

3.6 Recursive Stationary Equilibrium with Population Growth

Since the firm's decisions are invariant to the constant population growth rate η, the solution is the same as in section (3.4). The only adjustment is to steps 2 and 3. The normalized entrant share $\bar{M}$ and stationary measure $\bar{\mu}$ are solved as above. [WORK IN PROGRESS]

3.7 Empirical Strategy

As we discussed above, shifts in labor supply may have general equilibrium effects on firm dynamics. Our strategy for measuring these effects exploits cross-sectional demographic variation in labor supply growth. In particular, using cross-state data pooled over time, we estimate a linear model of the startup rate and several incumbent adjustment margins in state s and year t on labor supply growth, g_{st}, controlling for state and time fixed effects. We estimate the following baseline specification for various outcomes related to firm dynamics:

$$y_{st} = \mu_s + \lambda_t + \beta g_{st} + \epsilon_{st}. \tag{13}$$

In this specification, the outcome variable, y, will be the startup rate, average startup size and several incumbent margins described above in section 2.1, such as average incumbent size $N_t^{1+}$and survival- and conditional growth rates x_t and n_t. The terms. μ_s and λ_t capture state s and year t fixed effects, respectively, and ϵ_{st} captures other sources of variation in measure y_{st}. In estimating (13), we use two measures of labor supply growth g_{st}: working age population growth and labor force growth.

The key identification problem is that there are possibly unobservable factors that determine firm dynamics and also affect labor supply. Permanent characteristics that affect firm entry and labor supply would not create an identification issue as they would be absorbed by state fixed effects. However, workers might relocate to states that temporarily look better for firms and thus have more firm entry, causing a spurious correlation due to an omitted variable problem. A successful empirical strategy must deal with such endogeneity issues.

To generate variation in labor force growth that is exogenous to other factors affecting firm dynamics at the state level, we instrument labor supply shifts with 20 year lags of each state's fertility rate. Lagged birth rates are positively correlated with current labor supply measures at the state level, because of low mobility rates across states. Together with the fixed effects, the first-stage explains 70 and 40 percents of total variation in the growth rates of the working age population and labor force.[12]

This strategy relies on two identifying assumptions. The first and most important is the exclusion restriction that conditional on state and time effects, fertility rates are not correlated with business dynamics 20 years later, except indirectly through labor supply. This exclusion restriction requires fertility decisions not to be driven by long-term expectations. For example, the assumption would be violated if people in a given state had a higher fertility rate 20 years ago in anticipation of, for example, a persistently stronger labor market relative to other states. The restriction also requires that higher fertility or its determinants have no other long lasting (but not permanent) effects that would still affect business dynamics 20 years later. Second, and less importantly, to justify using states as the unit analysis, our empirical strategy requires that the mobility costs should be large enough to prevent geographic mobility from completely equating differences across these segmented markets. To the extent this is not true, our estimates will likely understate the effect of demographics on startups and incumbent dynamics. Drawing on the literature on mobility costs, we argue that this is likely to be the case.[13]

To better understand the cross sectional variation behind our estimation, Figure 5 plots the reduced form for residual startup rates on residual fertility, where both the startup rate and the fertility rate have been purged of state and time fixed effects. Each data point, labeled by its corresponding state is a state year for years 1980 through 2007. The x-axis plots the state's lagged fertility relative to its state average and the cross state average that year. Similarly the y-axis plots the state's startup rate relative to both its state average and the cross state average for that year. Although there is no test for our exclusion restriction we can see from the reduced form that the results are not driven by states that we know a-priori to have had unusual shocks to both fertility and the business environment.

4 Results

Tables 1 through 5 report the estimated effects of labor supply shifts on various dimensions of firm dynamics. All the estimates use the growth rate of working age population instrumented with the lagged fertility rate as the proxy for labor supply shifts. Table 1 shows the effect of the growth rate of working age population on the contemporaneous startup rate as well as the startup rates one andtwo year ahead. According to this estimate, a 1 percentage point slow down in the working age population growth rate leads to an almost a full percentage point decline in firm entry. The effect gets stronger when we consider firm entry in subsequent years. Since the working age population growth rate slowed down roughly by a full percentage point in our sample period, the results suggest that this labor supply shift can explain about 1 percentage point decline in startup rates. This effect corresponds to more than 25 percent of the overall decline and is economically significant.

[12] The results of the first stage are reported in the appendix.

[13] In an influential paper, Kennan and Walker (2011) find an average moving cost of $312,000 (in 2010 dollars). This cost encompasses psychic as well as monetary costs. This estimate suggests that labor market differences across states are unlikely to be entirely offset by geographical mobility.

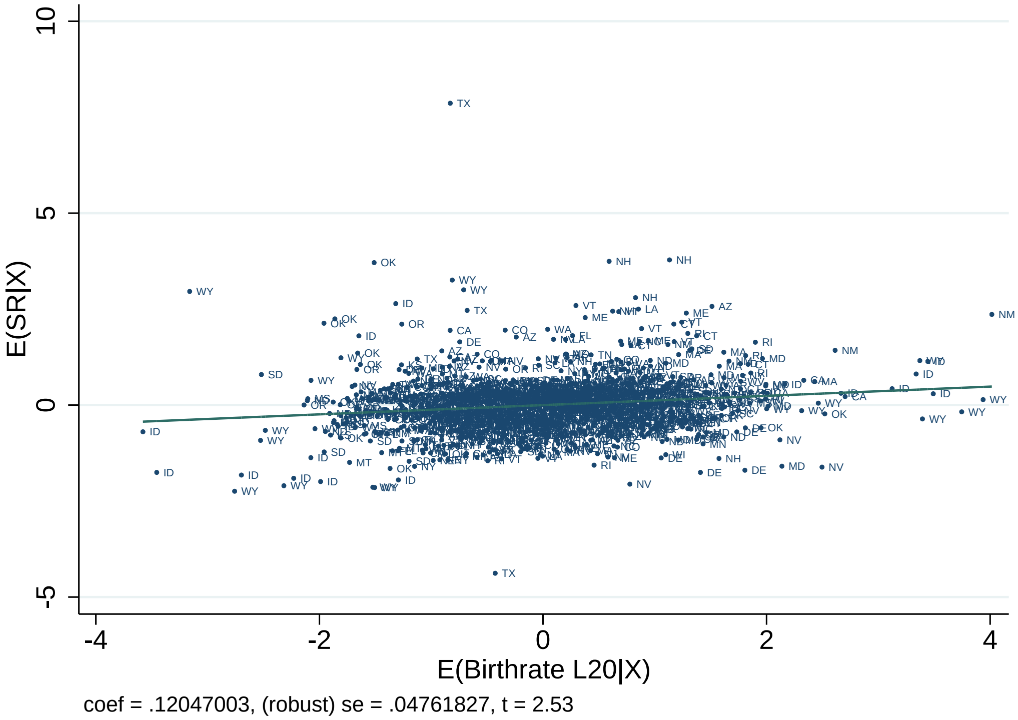


Figure 5: Reduced form partial regression

One immediate question is whether the decline in firm entry in response to labor supply shifts is associated with a change in the quality or behavior of entrants. Documenting how other margins are affected by demographic shifts is crucial for understanding the economic forces through which startups are affected. For example, if such shifts affect the relative price of labor, they may also affect the selection margin of entrants and the survival and growth rates of incumbents. Instead, if the only long run impact is on the entry margin, as predicted by workhorse models of firm dynamics with free entry of firms, one would expect all other margins to be unaffected in the long run.

Motivated by these considerations, we turn to the response of other entrant and incumbent margins. Table 2 reports the effect of the growth rate of working age population on the size of startups. The estimated coefficients are negative but economically and statistically insignificant. For example, with a one percentage point decline in the growth rate of working age population over our sample period, these estimates imply a rise in startup size of about 0.3 workers. Given an average startup size around 6, this effect corresponds to an elasticity of -0.08.

Turning to the effects of labor supply shifts on incumbents, tables 3 and 4 show the response of young incumbents' survival and conditonal employment growth rates. Between 1980 and 2007, the estimated coefficients imply an increase in the survival rates of young incumbents by less than 0.1 percentage point. With a baseline survival rate of young incumbents about 89 percent, this effect implies a negligible elasticity (-0.001). Similarly, the effects on young incumbent conditional growth rates are economically small, with an elasticity of roughly 0.2. Finally, table 5 shows the effect

Table 1: Working age population and startup rates

VARIABLES	(1) SR_t	(2) SR_{t+1}	(3) SR_{t+2}
WAP GR (20+, %)	0.925***	1.042***	1.152***
	(0.292)	(0.262)	(0.263)
Constant	10.76***	10.36***	10.05***
	(0.550)	(0.546)	(0.577)
Observations	1,316	1,316	1,316
R-squared	0.887	0.876	0.862
Instrumented	IV	IV	IV
State FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Cluster	State	State	State
State Trend	-	-	-
Years	1980-2007	1980-2007	1980-2007
F-stat	23.53	27.31	27.09

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Note: Business Dynamics Statistics and Current Population Survey. Stand errors clustered by state in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Dep var: startup rate (%). Excludes AK, HI, UT and DC.

on incumbent firms' average employment. The effect is positive on impact and in the two years following the shift. However, the estimates are not statistically or economically significant and, interestingly, they get smaller over time.[14] This declining pattern along with the increasing effect for startup rates in table 1 suggests that some of the increase in labor supply might be absorbed by existing firms at least within the first few years. Overall, our results suggest that shocks to labor supply affect firm dynamics mainly along the entry margin changing the startup rate but leaving other margins unaltered.

Changes in the working age population might fail to fully capture labor supply shifts since labor force participation rate also responds to changes in the economy. In fact, our sample period has witnessed important changes in the participation behavior of young and old workers.[15] To tackle this issue, Tables 6 through 10 repeat the analysis using the labor force growth rate. All the estimates use the growth rate of labor force instrumented with the lagged fertility rate as the proxy for labor supply shifts. Our findings are qualitatively similar: a one percentage point decline in labor force growth rate is associated with roughly one percentage point decline in the startup rate and quantitatively insignificant changes in young firms' post entry performances.

[14] The elasticity associated with the effect on impact is around 0.07.
[15] See, for example, Aaronson and Davis (2012).

Table 2: Working age population and startup size

VARIABLES	(1) N^0_t	(2) N^0_{t+1}	(3) N^0_{t+2}
WAP GR (20+, %)	-0.358	-0.200	-0.164
	(0.238)	(0.195)	(0.217)
Constant	6.342***	6.084***	6.022***
	(0.487)	(0.426)	(0.481)
Observations	1,316	1,316	1,316
R-squared	0.455	0.496	0.503
Instrumented	IV	IV	IV
State FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Cluster	State	State	State
State Trend	-	-	-
Years	1980-2007	1980-2007	1980-2007
F-stat	23.53	27.31	27.09

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Note: Business Dynamics Statistics and Current Population Survey. Standard errors clustered by state in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Dep var: startup rate (%). Excludes AK, HI, UT and DC.

Table 3: Working age population and young incumbent survival rates

VARIABLES	(1) X^{1-10}_t	(2) X^{1-10}_{t+1}	(3) X^{1-10}_{t+2}
WAP GR (20+, %)	-0.0709	-0.0682	-0.0396
	(0.288)	(0.222)	(0.194)
Constant	88.40***	88.40***	88.37***
	(0.280)	(0.232)	(0.200)
Observations	987	987	987
R-squared	0.681	0.690	0.695
Instrumented	IV	IV	IV
State FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Cluster	State	State	State
State Trend	-	-	-
Years	1987-2007	1987-2007	1987-2007
F-stat	28.92	40.45	53.66

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Note: Business Dynamics Statistics and Current Population Survey. Standard errors clustered by state in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Dep var: startup rate (%). Excludes AK, HI, UT and DC.

Table 4: Working age population and young incumbent conditional growth rates

VARIABLES	(1) G_t^{1-10}	(2) G_{t+1}^{1-10}	(3) G_{t+2}^{1-10}
WAP GR (20+, %)	-1.386	-0.845	-0.647
	(0.852)	(0.670)	(0.543)
Constant	9.895***	9.428***	9.209***
	(0.798)	(0.627)	(0.502)
Observations	987	987	987
R-squared	0.318	0.395	0.416
Instrumented	IV	IV	IV
State FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Cluster	State	State	State
State Trend	-	-	-
Years	1987-2007	1987-2007	1987-2007
F-stat	28.92	40.45	53.66

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Note: Business Dynamics Statistics and Current Population Survey. Standard errors clustered by state in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Dep var: startup rate (%). Excludes AK, HI, UT and DC.

Table 5: Working age population and incumbent size

VARIABLES	(1) N_t^{1+}	(2) N_{t+1}^{1+}	(3) N_{t+2}^{1+}
WAP GR (20+, %)	0.955	0.869	0.864
	(0.608)	(0.554)	(0.559)
Constant	20.89***	20.90***	20.85***
	(1.090)	(1.084)	(1.142)
Observations	1,316	1,316	1,316
R-squared	0.945	0.944	0.941
Instrumented	IV	IV	IV
State FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Cluster	State	State	State
State Trend	-	-	-
Years	1980-2007	1980-2007	1980-2007
F-stat	23.53	27.31	27.09

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Note: Business Dynamics Statistics and Current Population Survey. Standard errors clustered by state in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Dep var: startup rate (%). Excludes AK, HI, UT and DC.

Table 6: Labor force growth rate and startup rates

VARIABLES	(1) SR_t	(2) SR_{t+1}	(3) SR_{t+2}
CLF GR (20+, %)	0.775***	0.961***	1.082***
	(0.257)	(0.285)	(0.284)
Constant	11.12***	10.05***	9.191***
	(0.444)	(0.732)	(0.891)
Observations	1,316	1,316	1,316
R-squared	0.773	0.691	0.601
Instrumented	IV	IV	IV
State FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Cluster	State	State	State
State Trend	-	-	-
Years	1980-2007	1980-2007	1980-2007
F-stat	20.05	17.04	18.12

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Note: Business Dynamics Statistics and Current Population Survey. Standard errors clustered by state in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Dep var: startup rate (%). Excludes AK, HI, UT and DC.

Table 7: Labor force growth rate and startup size

VARIABLES	(1) N^0_t	(2) N^0_{t+1}	(3) N^0_{t+2}
CLF GR (20+, %)	-0.300	-0.184	-0.154
	(0.193)	(0.176)	(0.201)
Constant	6.203***	6.142***	6.143***
	(0.384)	(0.475)	(0.637)
Observations	1,316	1,316	1,316
R-squared	0.365	0.461	0.473
Instrumented	IV	IV	IV
State FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Cluster	State	State	State
State Trend	-	-	-
Years	1980-2007	1980-2007	1980-2007
F-stat	20.05	17.04	18.12

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Note: Business Dynamics Statistics and Current Population Survey. Standard errors clustered by state in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Dep var: startup rate (%). Excludes AK, HI, UT and DC.

Table 8: Labor force growth rate and incumbent survival rates

VARIABLES	(1) X_t^{1-10}	(2) X_{t+1}^{1-10}	(3) X_{t+2}^{1-10}
CLF GR (20+, %)	-0.0520	-0.0591	-0.0340
	(0.213)	(0.194)	(0.168)
Constant	88.35***	88.39***	88.36***
	(0.136)	(0.209)	(0.174)
Observations	987	987	987
R-squared	0.680	0.685	0.695
Instrumented	IV	IV	IV
State FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Cluster	State	State	State
State Trend	-	-	-
Years	1987-2007	1987-2007	1987-2007
F-stat	17.06	21.20	30.85

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Note: Business Dynamics Statistics and Current Population Survey. Standard errors clustered by state in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Dep var: startup rate (%). Excludes AK, HI, UT and DC.

Table 9: Labor force growth rate and incumbent conditional growth rates

VARIABLES	(1) G_t^{1-10}	(2) G_{t+1}^{1-10}	(3) G_{t+2}^{1-10}
CLF GR (20+, %)	-1.015	-0.732	-0.555
	(0.635)	(0.594)	(0.470)
Constant	9.090***	9.328***	9.105***
	(0.450)	(0.590)	(0.451)
Observations	987	987	987
R-squared	0.254	0.355	0.401
Instrumented	IV	IV	IV
State FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Cluster	State	State	State
State Trend	-	-	-
Years	1987-2007	1987-2007	1987-2007
F-stat	17.06	21.20	30.85

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Note: Business Dynamics Statistics and Current Population Survey. Standard errors clustered by state in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Dep var: startup rate (%). Excludes AK, HI, UT and DC.

Table 10: Labor force growth rate and young incumbent average firm size

VARIABLES	(1) N_t^{1+}	(2) N_{t+1}^{1+}	(3) N_{t+2}^{1+}
CLF GR (20+, %)	0.800	0.801	0.812
	(0.521)	(0.524)	(0.529)
Constant	21.26***	20.65***	20.20***
	(0.892)	(1.276)	(1.545)
Observations	1,316	1,316	1,316
R-squared	0.910	0.906	0.900
Instrumented	IV	IV	IV
State FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Cluster	State	State	State
State Trend	-	-	-
Years	1980-2007	1980-2007	1980-2007
F-stat	20.05	17.04	18.12

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Note: Business Dynamics Statistics and Current Population Survey. Standard errors clustered by state in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Dep var: startup rate (%). Excludes AK, HI, UT and DC.

5 Robustness of demographic channel

5.1 Labor Force Composition

[VERY PRELIMINARY]

5.1.1 Age Composition

One side effect of the fertility rate instrument is that by manipulating the future inflows of individuals into the working age population holding outflows constant, it also affects the age composition of the labor supply. In a recent paper, Ouimet and Zarutskie (2014) show that young firms disproportionately hire young workers. If young workers are a necessary input for new and young firms then changes in the age composition of the labor supply could also affect the startup rate.

We consider two approaches to differentiate the growth rate of the labor supply from this separate composition channel. First we re-estimating equation (13) further controlling for the share of young workers. The idea is if there is a separate composition channel states with a relatively low share of young workers may be more affected by the same lagged fertility shock, which would have a proportionally larger compositional effect. The results in Table (11) show that the effects of the demographic changes to the growth rate of the labor supply are robust to controlling for the state's young worker share. In fact, the estimated semi-elasticities are slightly larger.

A second approach is to use an alternative demographic shocks that affect outflows from the working age population, such as the lagged share of the population that would exit the working age population. [IN PROGRESS]

5.1.2 Immigration Shocks

[IN PROGRESS]

5.2 State Level Trends

A frequent concern with state level fixed effects empirical strategy is the state level fixed effects may not adequately control for differences across states in their trend changes in startup rate or fertility relative to the national averages. We re-estimate equation ((13)) further including linear state trends. Table 12 shows that the results are robust to further controlling for state specific trends.

5.3 Industry Controls

One possibility is that the demographic shocks may be correlated with the industry composition within a state. Using the tabulations from the LBD that allow us to use cross industry and cross state variation we show that the labor supply growth elasticities are robust to including industry fixed effects at both the 2-digit and 4-digit level.

[TABLE TO BE DISCLOSED]

Table 11: Working age population growth rate and startup rate controlling for age composition

VARIABLES	(1) SR_t	(2) SR_{t+1}	(3) SR_{t+2}
WAP GR (20+, %)	1.389***	1.189***	1.163***
	(0.297)	(0.243)	(0.250)
Share of Population 25-34 (%)	0.265***	0.148***	0.0275
	(0.0605)	(0.0567)	(0.0654)
Constant	5.348***	7.523***	9.550***
	(1.318)	(1.113)	(1.095)
Observations	1,316	1,316	1,316
R-squared	0.852	0.865	0.861
Instrumented	IV	IV	IV
State FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Cluster	State	State	State
State Trend	-	-	-
Years	1980-2007	1980-2007	1980-2007
F-stat	19.86	27.49	28.61

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Note: Business Dynamics Statistics and Current Population Survey. Standard errors clustered by state in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Dep var: startup rate (%). Excludes AK, HI, UT and DC.

Table 12: Working age population growth rate and startup rate controlling for state trends

VARIABLES	(1) SR_t	(2) SR_{t+1}	(3) SR_{t+2}
WAP GR (20+, %)	1.434***	1.382***	1.295***
	(0.302)	(0.232)	(0.208)
Constant	10.07***	9.918***	9.902***
	(0.584)	(0.516)	(0.499)
Observations	1,316	1,316	1,316
R-squared	0.867	0.861	0.864
Instrumented	IV	IV	IV
State FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Cluster	State	State	State
State Trend	Yes	Yes	Yes
Years	1980-2007	1980-2007	1980-2007
F-stat	25.30	21.59	16.10

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Note: Business Dynamics Statistics and Current Population Survey. Stand errors clustered by state in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Dep var: startup rate (%). Excludes AK, HI, UT and DC.

5.4 Differentiating Supply and Demand Shocks

We argue that the effects on the startup rate work primarily through a labor supply channel. To the extent the fertility shock may also affect labor demand, we want to distinguish these effects. To do this we compare the labor supply growth rate elasticity of startup for tradable and nontradable industries. The motivation for this exercises is that both sectors will be exposed to the same labor supply growth shock, but demand for tradables extends beyond the local labor market.

[IN PROGRESS: TRADABLE VERSUS NON TRADABLE]

[IN PROGRESS: CONTROLLING FOR CROSS SECTIONAL VARIATION IN CAPITAL INTENSITY AND MINIMUM EFFICIENT SCALE]

5.5 Anticipation Effects and Partial Adjustment

The demographic shocks occur well before any effect on the labor supply. Changes in fertility will take 20 plus years to effect the working age population and so the eventual demographic shifts are fully anticipated. We show in more detail below that an intertemporal substitution motive smooths the reduction in the startup rate well in advance of any direct effects on the working age population.

6 Interpreting Cross-sectional Regressions through the Lens of a Model of Firm Dynamics

A decline in the fertility rate today causes lower labor supply growth in 20 years. Thus, fertility shocks work as news shocks to labor supply, meaning that the variation in labor supply that we exploit is potentially fully anticipated as early as 20 years ago. If firms act upon such advanced information, a sizable part of the decline in the entry rate may realize before any change occurs to labor supply. An immediate corollary for us is that the cross-sectional semi-elasticities reported in table 1 are biased downward, as they use the contemporaneous relationship between labor supply and the startup rate. In this section, we quantify this bias through the lens of a simple model of firm dynamics with entry and exit.

The model we consider is a version of Hopenhayn (1992), modified to accommodate population growth and admits a balanced growth path. We calibrate this model to mimic several features of U.S. firm dynamics in the early 1980s. The details of the model as well as its calibration are explained in detail in appendix B.

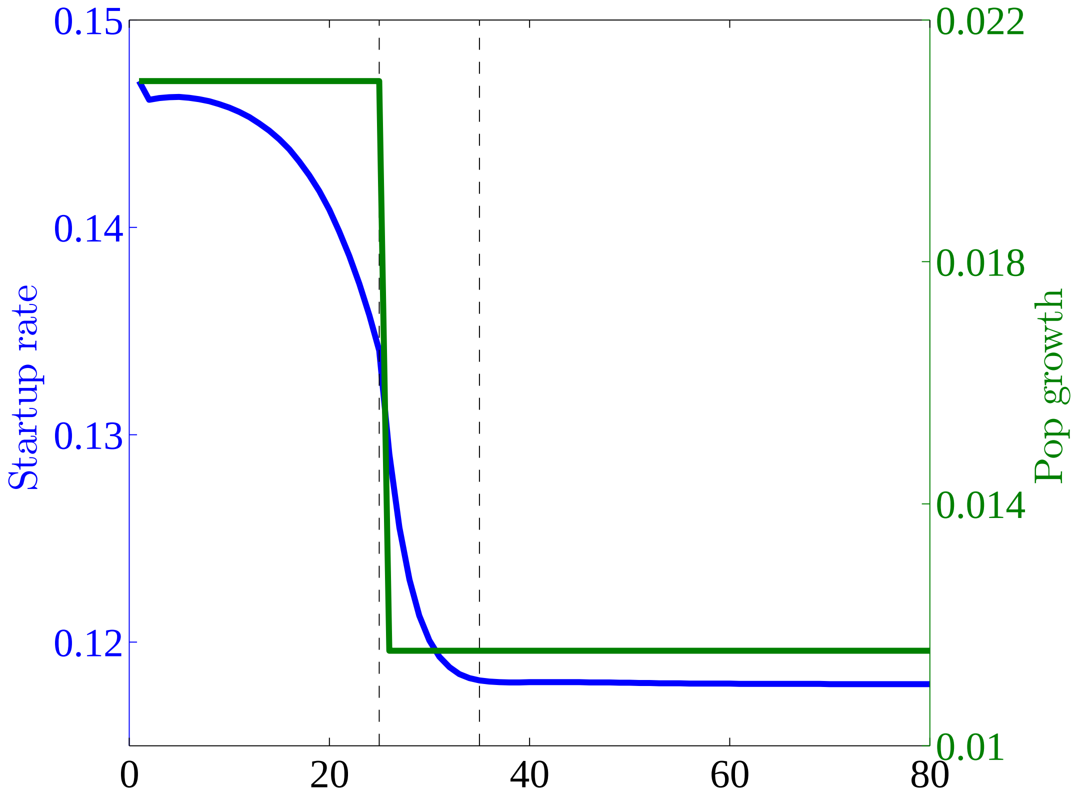
[IN PROGRESS: SHORT MODEL DESCRIPTION]

We start by studying the effects of a permanent reduction in the population growth rate on the equilibrium startup rate. Figure (6) shows the impulse response of the startup rate to an unexpected fertility shock that will permanently reduce the growth rate of the working age population from 2.1 to 1.25 percent in 25 years.

The effects are gradual. It's apparent that the startup rate adjusts well before the fertility shock affects the labor supply and even after this point, the startup rate continues to decline for an additional 10 years for a total decline of more than 3 percentage points. The gradual adjustment is due to two forces, the first is an intertemporal motive for the households to smooth the changes in the startup rate. In the model this is primarily because changes in the startup rate have significant effects on aggregate profits and thus aggregate consumption, which the household prefers to smooth over time. Second, even absent any smoothing motive, the distribution of firms takes many years to stabilize around the lower startup rate because of the large mass of incumbent firms.

6.1 Quantifying the Anticipation Bias

Labor supply moves abruptly, whereas the adjustment to the startup rate is gradual and preceeds the labor supply shift. These observations imply that the contemporaneous co-movement that we measure through cross-sectional regressions likely capture a fraction of the true effect. To quantify how large this bias could be, we simulate a panel of states from the model by varying the magnitude of the labor supply shifts. We then estimate a panel model, as in equation 13, on this artificial panel. Using the estimated regression coefficient, we estimate the effect on the startup rate of a decline in population growth from 2.1 to 1.25 percent and compare this to the effect in figure (6). The difference between this figure and the one implied by the cross-sectional estimates quantifies the anticipation bias.



$\left|\frac{x(T)-x(t)}{max(x)-min(x)}\right| < 0.01$ at t=35

Figure 6: Impulse response of startup rate to an anticipated permanent reduction in labor force growth

6.2 Correcting the Empirical Estimates of Labor Supply on the Startup Rate

7 Conclusion

[TO BE WRITTEN]

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A Additional Tables and Figures

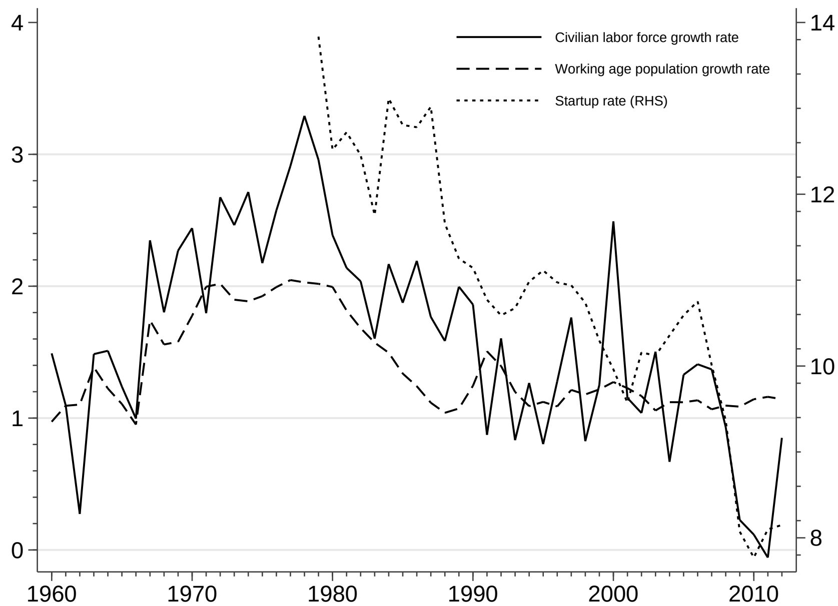

Figure A1: Unfiltered data: declines of startup rate and labor supply growth coincide

Note: BLS, Annual Census Bureau population estimates, and Business Dynamics Statistics. Annual data. Working age population is defined here as population ages 20 to 85. Civilian Labor Force is measured by the BLS for the adult (16+) civilian non institutional population. Startup rate is number of age 0 employers as share of the total number of employers within a year.

B A general equilibrium framework

The cross-state evidence points to a tight link between changes in the growth rate of the working age population and the entry behavior of firms. The link also appears strongest through a labor supply channel rather than a shift in the quantity of potential entrepreneurs or new business. Although the cross-state evidence is weaker, we also see some link between proxies for firm-level costs and declines in the entry rate. To assess these relative contributions quantitatively and in general equilibrium, we turn to an equilibrium model of firm dynamics.

Our starting point is the workhorse Hopenhayn (1992) model set in general equilibrium as in Hopenhayn and Rogerson (1993). In addition to the stationary equilibrium being well-understood, the model's implied behavior of incumbent firms is consistent with the results in Pugsley and Şahin (2014) and further developed in Section 2. Specifically, incumbent firm's conditional life-cycle dynamics appear remarkably stable during over the thirty year period we study.

The key divergence between the predictions of the Hopenhayn (1992)model and its variants is in the behavior along the entry margin. To address this margin, to the standard model we introduce non-stationarity through both population and productivity growth. With an appropriate normalization, we can characterize a balanced growth path in a stationary equilibrium, but the equilibrium allocation will not be invariant to demographic shifts. The model allows the entry rate and other key indicators of firm dynamism to shift with demographics and various costs to business operation. These predictions are consistent with the reduced-form evidence, and we use the model to identify and quantify the sources of the secular shifts in these measures of firm dynamism.

B.1 Entrant and incumbent dynamics

Entrants draw an initial productivity from distribution G and are able to operate immediately.[16] The expected value of an entering firm gross of entry costs $p_t c_{et}$ is

$$W_t = \int V_t(s)\, G(ds) \; .$$

We let M_t be the measure of entrants in period t. We only consider distributions G compatible with entry and exit in equilibrium so that $M_t > 0$ and

$$p_t c_{et} = W_t \, . \tag{14}$$

As in Hopenhayn (1992), let $\mu_t(S)$ for Borel subsets S be the measure of firms producing in period t with idiosyncratic productivity $s \in S$. This measure of firms includes the new entrants M_t and incumbent firms, which are firms that chose not to exit at the end of the previous year. This implies that

$$\mu_t(S') = \underbrace{\int (1 - X_{t-1}(s))\, F(S'|s)\, \mu_{t-1}(ds)}_{\text{incumbents}} + \underbrace{M_t G(S')}_{\text{new entrants}} \quad , \tag{15}$$

where F is the conditional distribution for productivity implied by the stochastic process for productivity s_t in equation (**??**). We define the startup rate as the share of new entrants out of all firms

$$SR_t \equiv \frac{M_t}{\int d\mu_t} \, .$$

B.2 An equilibrium with demographic and productivity shifts

We first define a potentially non-stationary competitive equilibrium that admits shifts in population and productivity growth as well as fixed and entry costs.

Definition 1. Given an initial population of measure H_{-1}, technology A_{-1}, a measure of firms μ_{-1}, sequences of population growth, productivity growth, fixed costs and entry costs $\{\eta_t, g_t, c_{ft}, c_{et}\}$ for $t \geq 0$ an equilibrium is a constant real interest rate $r = \beta^{-1} - 1$, a sequence of prices $\{p_t\}$, sequences of per-capita consumption and leisure $\{c_t, l_t\}$, sequences of individual labor demand and exit rules $\{n_t, X_t\}$ for operating firms, and a sequence of measures $\{\mu_t\}$ such that given prices (i) $\{c_t, l_t\}$ are

[16] Given the fixed operating costs, this timing admits the possibility for initial s sufficiently low that entering firms produce and immediately exit at the end of the period with a loss.

optimal, (ii) $\{n_t, X_t\}$ are optimal, (iii) $\{\mu_t\}$ satisfies the the law of motion given by equation (15), and (iv) markets clear for all $t \geq 0$.

This definition of an equilibrium allows the number of firms to grow as needed along with the population, as in the data. To compute the comparative statics of a stationary version of this equilibrium we make the necessary assumptions to define a balanced growth path.

B.2.1 Balanced growth path

Assumption 1. *Fixed costs and entry costs are proportional to productivity so that* $c_{ft} = c_f A_t$ *and* $c_{et} = c_e A_t$.

If fixed costs and entry costs grow at the same rate as productivity, a balanced growth path just requires reformulating the law of motion in equation (15) in terms of a per-capita measure of firms $\bar{\mu}_t = \frac{\mu_t}{H_t}$, then

$$\bar{\mu}_t\left(S'\right) = \int \mathbf{1}\left\{X_{t-1}\left(s\right) = 0\right\} F\left(S'|s\right) \frac{\bar{\mu}_{t-1}\left(ds\right)}{1+\eta_t} + \bar{M}_t G\left(S'\right) , \tag{16}$$

where $\bar{M}_t$ is defined the measure of entrants per capita. From this law of motion it is clear with constant η and X that the long run startup rate will depend positively on the population growth rate η. Since

Definition. Given a constant population growth rate η and aggregate productivity growth g a stationary recursive equilibrium with balanced growth is a set of prices r and $p\left(A\right)$ a value function $W\left(s\right)$ and individual labor demand $n\left(s\right)$, allocations $c\left(A\right)$ and l for each household, a measure $\bar{\mu}$, and a startup rate $\overline{M}$ such that (i) given relative price $\tilde{p}\left(A\right)$, $n\left(s\right)$ solves $W\left(s\right)$, (ii) $c\left(A\right)$ and l are optimal, (iii) $\bar{\mu}$ satisfies equation (16) and (iii) markets clear.

The solution strategy is the same as in Hopenhayn (1992) and Hopenhayn and Rogerson (1993). First note that the solution to an incumbent firm's problem is invariant to shifts in the population growth rate. An incumbent firm's behavior and value are also invariant to changes in aggregate productivity when $p\left(A\right) = \frac{\bar{p}}{A}$, which will turn out to be the equilibrium relative price. Since entry costs are proportional to productivity in units of output, with entry and exit this price satisfies the free entry condition in equation (14) for any value of A. Similarly, household consumption must be $c\left(A\right) = \bar{c}A$ where $\bar{c} = \frac{1}{\alpha\bar{p}}$. With constant $\bar{\mu}$ and $\overline{SR}$ the total measure of firms and of entrants grow at rate η insuring that profits per household Π/H are constant.

It remains only to determine the equilibrium startup rate $\overline{SR}$ and with it the distribution of firms $\bar{\mu}$. As in Hopenhayn (1992) the right hand side of equation (16) defines a linear operator homogeneous in $\bar{\mu}_{t-1}$ and $\overline{SR}_t$. With both constant then the constant measure $\bar{\mu}$ that satisfies (16) can be as

$$\bar{\mu} = f\overline{SR},$$

where f satisfies

$$f\left(S'\right)=\int\mathbf{1}\left\{X\left(s\right)=0\right\}F\left(S'|s\right)\frac{f\left(ds\right)}{1+\eta}+G\left(S'\right)$$

Clearing the labor market in equation (**??**)

$$\overline{SR}=\left(\int\left(s\left(\theta ps\right)^{\frac{\theta}{1-\theta}}-c_f\right)f\left(ds\right)-c_e\right)^{-1}\frac{1}{\alpha\bar{p}}.$$

determines $\overline{SR}$ and with it $\bar{\mu}=f\overline{SR}$.

C Transition Path for Changes in Population Growth

C.1 Non Stationary Equilibrium Path

Given an initial normalized measure of firms $\bar{\mu}_0$, and a (possibly stochastic) sequence of population growth rates $\{\eta_t\}_{t=1}^{\infty}$ that converges to η_T for some $T<\infty$, a non-stationary equilibrium with entry and exit is a sequence of prices $\{p_t,r_t\}_{t=1}^{T}$, a sequence of per-capita allocations $\{c_t,n_t\}_{t=1}^{T}$, a sequence of entrants $\left\{\bar{M}_t\right\}_{t=1}^{T}$, and a sequence of re-scaled measures $\{\bar{\mu}_t\}$ such that (i) $p_t c_e=W_t^e$ (ii) $\bar{\mu}_t$ satisfies its law of motion (iii) the allocations satisfy the household's optimality conditions (7) and (8), and (iii) and markets clear for all $t\geq 1$.

C.2 Transition Solution

In general the equilibrium must be computed numerically. We sketch a solution here

Step 1 Fix T as some large number and solve for balanced growth path stationary equilibrium with $\eta=\eta_T$. This yields prices p_T and r_T, firm value W_T, and household allocations c_T and n_T.

Step 2 Rather than iterate on a sequence of prices $\{p_t,r_t\}_{t=1}^{T}$, it is more stable to iterate on a sequence of consumption allocations $\{c_t\}_{t=1}^{T}$. This allows us to reduce the dimensionality by computing the sequence of free-entry consistent prices when solving backwards and is more flexible for values of ψ that do not have a closed form solution for consumption/hours.

Step 2 For any given sequence $\{c_t\}_{t=1}^{T}$, compute the required sequence of prices and real interest rates $\{p_t,r_t\}_{t=1}^{T}$ period by period. The values p_T and r_T are known from the new balanced growth path. For period $T-1$, given W_T and $\frac{c_T}{c_{T-1}}$, solve for the values of p_{T-1} and r_{T-1} that satisfy both the Euler equation and the free entry condition

$$\int_s\frac{\left(\chi p_{T-1}s_t\right)^{\frac{1}{1-\theta}}}{p_{T-1}}-c_f+\beta\frac{1}{p_T}\frac{c_{T-1}}{c_T}\max_{X_{T-1}\in\{0,1\}}\left\{E_{T-1}W_T\left(s_T\right),0\right\}\nu\left(ds\right)-c_e=0\,.$$

and given p_{T-1}

$$\frac{1}{1+r_{T-1}} = \beta \frac{p_{T-1}}{p_T} \frac{c_{T-1}}{c_T} \, .$$

Formulating the free entry condition in terms of output instead of the numeraire makes it easier to solve numerically. It has the added benefit that we can compute an analytical gradient since p_{T-1} does not enter the normalized continuation value. The gradient is

$$\int_s (\chi s)^{\frac{1}{1-\theta}} \frac{\theta}{1-\theta} (p_{T-1})^{\frac{\theta}{1-\theta}-1} \nu(ds)$$

simplified further to

$$\int_s (\theta s)^{\frac{1}{1-\theta}} p_{T-1}^{\frac{\theta}{1-\theta}-1} \nu(ds) \, .$$

With r_{T-1} and p_{T-1} then we solve for W_{T-1} and X_{T-1}. using the firm's Bellman equation (4) given W_T. This yields a sequence of firm values and exit rules $\{W_t, X_t\}_{t=1}^T$

Step 3 It will be useful to represent the law of motion as a linear operator $\mathbf{T}_t$

$$\mathbf{T}_t \left(\bar{\mu}, \bar{M}\right) \left(S'\right) \equiv \left(\int_s (1 - X_{t-1}(s)) \frac{F(S'|s)}{1+\eta_t} \bar{\mu}(ds) + \bar{M} \nu \left(S'\right) \right)$$

that may be further decomposed into

$$\mathbf{T}_t^I \bar{\mu} \left(S'\right) = \int_s (1 - X_{t-1}(s)) \frac{F(S'|s)}{1+\eta_t} \bar{\mu}(ds)$$

and the measure of entrants ν so that

$$\mathbf{T}_t \left(\bar{\mu}, \bar{M}\right) \left(S'\right) = \mathbf{T}_t^I \bar{\mu} \left(S'\right) + \nu \left(S'\right) \bar{M} \, .$$

Per-captia output is homogenous in μ_0 and $\bar{M}_1$ and may be written as

$$\begin{aligned} \frac{Y_1}{H_1} &= \mathbf{Y}_1 \bar{\mu}_1 \\ &= \mathbf{Y}_1 \mathbf{T}_1^I \bar{\mu}_0 + \nu \bar{M}_1 \, . \end{aligned}$$

Then with p_1 and c_1 the resource constraint (per-capita) determines $\bar{M}_1$

$$\begin{aligned} c_1 + \bar{M}_1 c_e &= \frac{Y_1}{H_1} \\ c_1 + \bar{M}_1 c_e &= \mathbf{Y}_1 \mathbf{T}_1^I \bar{\mu}_0 + \mathbf{Y}_1 \nu \bar{M}_1 \\ \bar{M}_1 &= \frac{\mathbf{Y}_1 \mathbf{T}_1^I \bar{\mu}_0 - c_1}{(c_e - \mathbf{Y}_1 \nu)} \end{aligned}$$

Given $\bar{\mu}_0$, p_1 and now $\bar{M}_1$ determine $\bar{\mu}_1$

$$\begin{aligned}\bar{\mu}_1\left(S'\right) &= \int_s \left(1-X_1\left(s\right)\right)\frac{F\left(S'|s\right)}{1+\eta_1}\bar{\mu}_0\left(ds\right)+\nu\left(S'\right)\bar{M}_1 \\ &= \mathbf{T}_t^I\bar{\mu}_0\left(S'\right)+\nu\left(S'\right)\bar{M}_1\end{aligned}$$

and repeat to determine $\left\{\bar{M}_t,\bar{\mu}_t\right\}_{t=1}^T$.

Step 4 With $\{\bar{\mu}_t\}_{t=1}^T$ compute residuals from the implied consumption given aggregate profits

$$c_t A\left(1-n_t\right)^{\frac{-1}{\psi}}=\frac{1}{p_t}$$

$$\begin{aligned}p_t c_t^{'} &= 1-\left(p_t c_t \kappa\right)^{\psi}+\frac{\Pi_t}{H_t} \\ p_t c_t^{'}+\left(p_t c_t^{'}\kappa\right)^{\psi} &= 1+\left(p_t\mathbf{Y}_t-\mathbf{N}_t\right)\bar{\mu}_t-\bar{M}_t c_e\nu\,.\end{aligned}$$

This may need to be solve numerically. Then

$$R_t \equiv c_t-c_t^{'}$$

Step 5 Use sequence of residuals $\{R_t\}_{t=1}^T$ to choose new sequence of prices.

D Calibrating the early 1980s economy

We calibrate our model at the annual frequency to match various statistics in early 1980s. We calibrate the time discount rate $\beta^{-1}-1$ and the parameter θ directly to match the real interest rate and labor's share of total revenue. Population growth rate η is set to 0.023, its level in 1980. Innovations to productivity are assumed to follow a log-normal disribution. The initial productivity distribution G is assumed to be Pareto. The fixed cost c_f, entry cost c_e, average idiosyncratic productivity $a/\left(1-\rho\right)$, and the disutility of labor α, along with the productivity parameters (persistence ρ and volatility σ of the productivity shock as well as the Pareto parameter λ) are jointly estimated using the simulated method of moments to match the employment-to-population ratio (0.6), the 2-year exit rate of startups (0.38), the startup rate (0.13), average size (21), average startup size (6), average incumbent size (23) and the 20th percentile of incumbent size ($\sim$10). These parameters are summarized in tables A1 and A1.

Table A1: Fixed Parameters

Parameter	Value
Time discount rate, r	0.064
Elasticity of output relative to labor, θ	0.64
Population growth rate, η	0.021

Note: Table A1 reports the values for parameters fixed outside the model.

Table A1: Parameters Calibrated with MSM

Parameter	Value
Fixed cost, c_f	2.505
Entry cost, c_e	31.93
Average log productivity, a	0.111
Disutility of labor, α	0.468
Persistence of productivity shock, ρ	0.930
Standard deviation of productivity process errors, σ_e	0.251
Incumbent survival, ξ	0.921
Mean of log-normal entry productivity distribution	-1.473
Standard deviation of log-normal entry producivity distribution	1.323

Note: Table A1 reports the values of parameters calibrated through Simulated Method of Moments (SMM).

Table A1 shows the outcome of the model for the distribution of firms and distribution of employment across different firm sizes and compares them with the average in the data for the 1977-1980 period. The model captures the distributions of firms and employment across firm sizes well.

Table A1: Firm and employment distributions across firms sizes in the data and the model

	Distribution of Employment		Distribution of Firms	
	Data	*Model*	*Data*	*Model*
1-49	0.32	0.16	0.96	0.96
50-249	0.16	0.15	0.03	0.03
250+	0.52	0.69	0.01	0.01

E Data Appendix

E.1 Longitudinal Business Database Firm Characteristics

We classify firms by age, size, location, and industry. Firm age is assigned to new firm records as the age of each new firms firm's oldest establishment. Establishment age is measured from the year the establishment first reports employment. Because the LBD is populated from an IRS database of businesses with employer identification numbers (EIN) whether or not they have employees, an establishment may exist in the database for many years without any employees. Assigned in this way, age 0 firms are truly new entrants. We further aggregate age into new (age 0), young (ages 1 to 10), and mature (ages 11+) categories. Firm size is measured by summing employment across all establishments within a firm. Firm location is assigned by the physical location of the firm's establishments. In the case of firms with multiple locations, firm location (state or CBSA) is assigned as the location with the modal employment. Industry is assigned at the NAICS 6-digit level. We only use the 4-digit and 2-digit level of aggregation. Prior to 2002 when NAICS was introduced we use Theresa Fort's database of retroactive NAICS classifications at the plant level. These assignments are also compatible with the SIC to NAICS bridge at the appropriate level of aggregations.

With those classifications, we aggregate firm employment and growth by state, industry (2-digit and 4-digit), location and age group, computing the employment weighted averages. We also compute a survival-weight, as one minus the firm weighted average of an indicator for a firm that had employment the previous year, but has exited the database.

[TO BE COMPLETED]

E.2 Business Dynamics Statistics Measures

[TO BE COMPLETED]